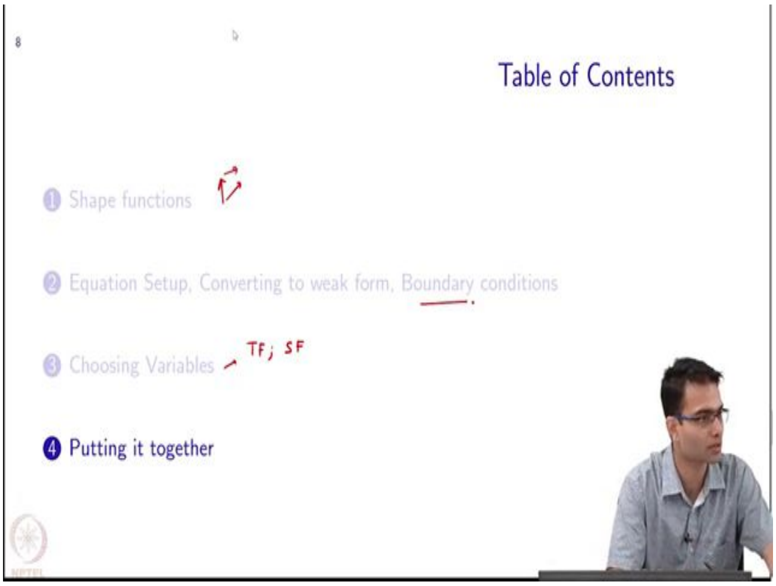
Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering (EE)
Indian Institute of Technology, Madras

2D Finite Element Method
Lecture – 11.07
Matrix assembly – Part 1

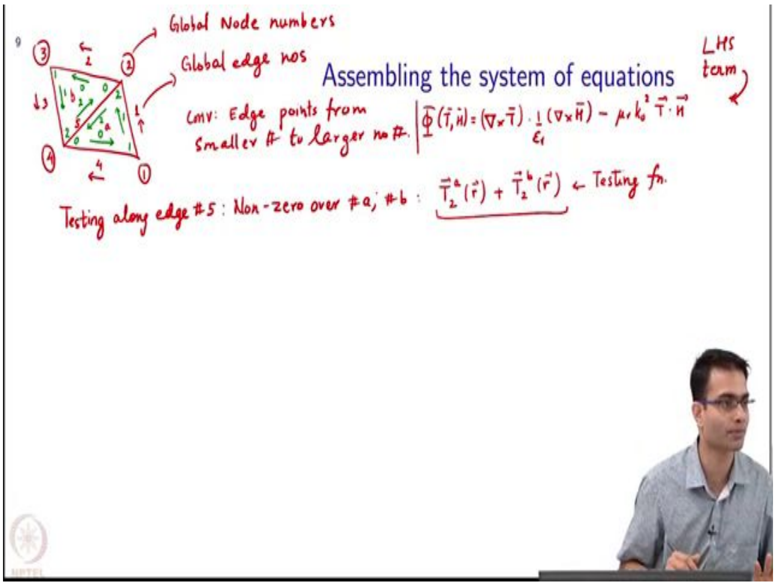
(Refer Slide Time: 00:13)



Right so now we come to the final section on the 2D edge based Finite Element Method. So, so far we have looked at shape functions, weak form, boundary conditions and choice of formulation- total field or scattered field formulation. The final step is to show you how to put it all together into a system of equations, at the end of the day we want to $Ax = b$ linear system of equations which we can solve ok.

So, let us try to put it together. So, what we will do is going to be very similar to what we did in the case of a 1D FEM. What did we do? We took a line and we chopped into elements and formed the system of equations. So, let us get into that ok.

(Refer Slide Time: 00:55)



So, the minimum building unit to talk about this is going to be 2 elements. Because an edge except if it is on the boundary will always be shared by 2 elements ok. So, let us draw that. So, these are 2 adjacent elements ok. Now, one very important thing to keep in mind in FEM in general is the mapping between global nodes to local nodes local nodes to global nodes ok.

Now, I just nodes now we talk talking about edges right this is an edge based FEM. So, we also have to keep a table which allows me to go from a local edge to a global edge numbering. So, each of these we will use a different color for, let us use the green color for local ok. So, these are the 2 elements, elements let us call this element ‘a’ let us call this element ‘b’.

Each of these triangles has three nodes and there will be some local numbers. So, the local numbers of the nodes will be for example, 012 012 ok. So, this node will be 0 1 2 similarly I can say that 0 1 2 these are the local nodes. So, let us be very clear what determines which one is 0, which one is 1 which one is 2. Is there anyone secret about which is 0, 1 or 2? Now these are node numbers we only talking about node numbers there is nothing secret what is so, so how do how do I fix which is 0 1 and 2?

Student: (Refer Time: 02:49).

You can choose any way you want, but in practice you will not get to choose anywhere you want. The software which meshes the domain will give you a list of for each element it will give a list of nodes. So, at least some I mean it will list of three nodes and some order. So, you will choose the first one to be 0, the second one to be 1, the third one to be 2 ok; so, this coming to you from the mesh from the CAD software which we will use to mesh the domain ok.

So, it is sort of given to you, but that is nothing very secret about it right you can remap it if you really wanted. Well the machine software may internally try to follow a convention of clockwise, but clockwise still will not tell you where on the clock you are starting. You are starting at 6 o' clock, 12 o' clock, 9 o' clock ok.

So, and it may violate that conventional also. So, we should not count or dependent on any particular convention, we just want the three nodes right. Now corresponding to these local node numbers, I get to choose how to decide to fix the global node numbers ok. Because once I start reading and I will see if there is any node will be repeated, I will not give a new number to it right.

So, that is a part of code which I have to write. So, let us call this guy global node number 1, global node 2 global node 3 and global node 4.

Student: (Refer Time: 04:10) position being of the.

Yeah, as let us say when I read in the CAD file, it will give me element an element b and the nodes for element 'a' and the nodes for element 'b', but 2 nodes are common. So, I should not double count when I am doing the global numbering that is all ok.

So, if we were doing node based FEM we would stop here, but we are not doing a node based we are doing an edge base FEM. So, the next thing to take into account is the edge the edges right. So, edges let us see how to do. So, there are many conventions, one conventions simply is in the following way from smaller local node number to greater local node number right that is that could be 1 convention.

So, for example, these can be I will call this local edge number 0, local edge number 1, local edge number 2 ok. So, up to you how do you defining it, but you because you are going to code it, you need some logic right. So, the logic can just be this right the next is over here, over here and over here.

Student: Can you just all the continuity equation.

Yeah. So, the natural question will come how do you fix the continuity equation right. So, these are as I told you these are local node numbers, local edge numbers and there is a 1 to 1 mapping to unique edge number. So, it seems that this edge between node 2 and 4 has the vectors in the opposite direction, but that can be fixed by taking care of this local to global we will see how would happen.

So, we will fix the local edges what remains is global edges right. So, the edges need some kind of a unique thing right. So, I can say edge number 1 is this, edge number 2 is this, edge number 3 is here and edge number 4 comes here and finally, I have edge number 5 ok.

These are. So, let us write this over here these are global yeah. So, now, just as I had a convention for local edges, that let us say from smaller node number to larger node number, similarly I can have a convention for the direction in which a global edge should point. So, I can say that global edge number 1 is from smaller global node number to larger global node number. So, this edge for example, can be assigned to be pointing in this way, this can be assigned to be pointing in this way, similarly in this way and in this can be this land up being this way and 5 will be from 2 to 4 ok.

So, the only thing I have to take care of is that on the common edge if the global and the local edges differ by a direction and I have to add a minus sign in the equations ok. Because the convention I have taken is, convention edge points from a smaller number to a larger number. So, you see this edge number 4 is pointing from global node number 1 to global node number ‘4’. The direction is from a smaller number to a larger number.

Student: So, in 2 to 0 there is 1 evaporates right.

2 to 0 there is no 0 number over here in terms of the global nodes.

Student: Ok.

Is this part clear? So, these are the things which you have to be very careful about when you code. There is a set of global convention there are set of local convention then a 1 to 1 mapping over here. So, 5 is pointing from 2 to 4 ok. When I talk about global edges, the convention comes from global node numbers when I talk about local edges the convention comes from local node numbers ok. So, all of this you did not have to really worry about that much when you did integral equation methods here it is important to do this very good.

The next thing is I mean so far when we looked at the boundary conditions and the total field versus scattered field formulation, we were paying a lot of attention to the right hand side, but now it is time to also take a look at the left hand side right. So, left hand side term which appears again and again is of this form right. So, it was there was a $(\nabla \times \vec{T}) \cdot 1/\varepsilon_r(\nabla \times \vec{H}) - \mu_r k_0{}^2(\vec{T} \cdot \vec{H})$. So, this was the left hand side term ok. So, now, this expression will keep appearing again and again when I look at element 'a' element 'b' and so on.

So, I am going to make a write a shortcut for this I am going to call this $\Phi(\vec{T} \cdot \vec{H})$ just a shortcut. So, when you see capital $\Phi(\vec{T} \cdot \vec{H})$ I mean this expression. So, what was what was the FEM procedure to get a system of equations take one one $\vec{T}$. So, supposing this very simplistic example of just 2 elements, supposing my mesh was just made of 2 elements then how many how many edges do I have in total?

Student: 5

5 edges right.

And I could take the testing function to be each one of the edges, the shape function corresponding to each edge and get 5 equations in 5 variables right. So, that is going to be a general procedure.

So, just let us take let us try to derive the system of equations I get if I do testing with using edge number 5. Its now testing with let me say not with, but along and when I say edge 5; obviously, I am referring to the global edge number 5 ok. Now edge number 5 is shared as a reason I have taken Φ is because it is more interesting it is common to both the elements right.

So, again like in 1 D case I had a choice whether the basis function should be belonging only to 1 element or linear combination of something that belongs to both elements right. Here also we will do the same thing, my testing function is going to be the sum of the basis function corresponding to element 'a' and element 'b' right it is like the triangle function which I had in 1 D right. So, this is non-zero over element 'a' and element 'b'.

So, you can think of it as supposing I wanted to use the convention. So, $\vec{T}$ belonging to which element now? Say element a and local node the local edge number what? 2 plus $\vec{T}$ of element b local edge number.

Student: 0.

Local edge number.

Student: 2 2.

Two right. So, this is my.

Student: What about that plus sign that directions is not seeking theme get because seen three we have used to job (Refer Time: 13:16).

It does not. So, the question is that T these 2 functions $\vec{T}_2{}^a$ and $\vec{T}_2{}^b$ they have a different sign on the edge it will not matter. It will I mean I will get a minus sign in the system of equations. So, once again I must mentioned that I could go in a in a slightly slower manner, I could just take $\vec{T}_2{}^a$ ones that 1 equation, take $\vec{T}_2{}^b$ ones get another equation this is like a shortcut that we do we will take at the same time.

Student: Sir now we are working local (Refer Time: 13:55).

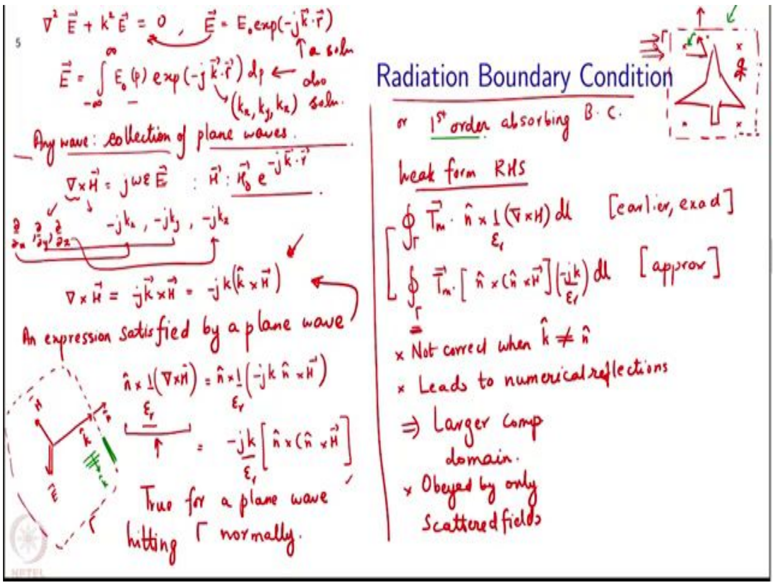
No these are well this is a testing function so.

Student: We are taking T 2 a 2.

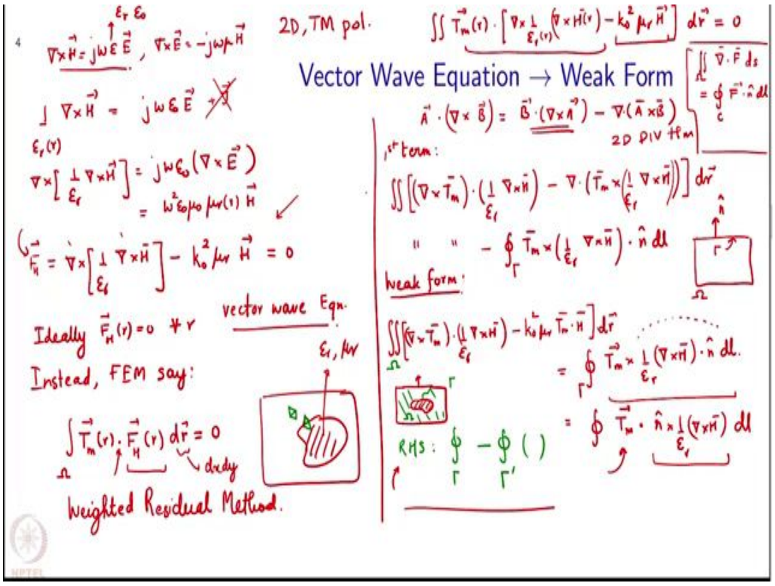
Yeah, $\vec{T}_2^{\,a}\ \vec{T}_2^{\,b}$: yeah because I have to make the testing function as a composite sum of two things. So, there I have to somehow refer to the local things right. The reason I am doing this is because we said there are 5 global edges. So, there will be 5 variables. So, I should get 5 equations.

How many local triangles I mean local triangles there are six are there right. So, I will get one extra equation I will get six equation and five variable. So, avoid that I have combined these two all right. So, now, let us see what happens when I take my weak form. So, everyone is let us go back to our weak form over here right further still yeah here is the weak form right.

(Refer Slide Time: 14:43)



(Refer Slide Time: 14:46)



This is the left hand side and the right hand side is that contour integral ok. So, let us just come back to this $\vec{T}_0^{\,b}$ not $\vec{T}_0^{\,b}$.

Student: T t b.

$\vec{T}_2^{\,b}$ we can take 2 so, in element b edge number 2 corresponds to global edge number 5 right no right. No I am not naming it by the opposite node number there are various conventions you can choose whichever convection you are. Here I have taken that the global edge number sorry the local edge number corresponds to the starting local edge number like and I am I have taken the slightly different convention from earlier, because various reference books I have different convention. So, you should not think that there is some universal agreement or how to name number these things fine.