Computational Electromagnetics
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2D Finite Element Method
Lecture - 11.06
Comparing total and Scattered field formulation

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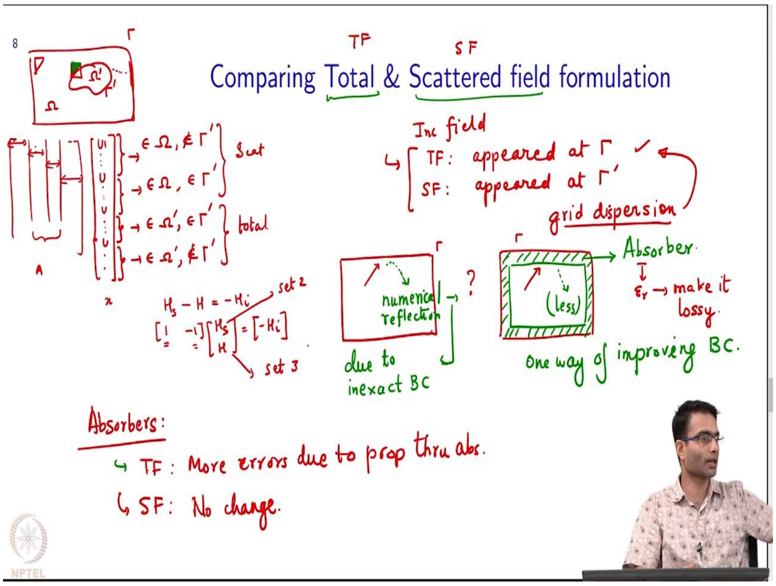

So, this is my ok now x is my set of unknowns which is the tangential component along the triangle we have seen that the U_i's right. So, I can write this as. So, $U_1, \ldots$ right. So, these are all U's ok. So, I have written 4 sets of U's ok. So, these sets of U's are first set of U's I can say interior sorry like this. So, this is Ω this is Ω'. So, the first set of edges are all the edges that belong to Ω, but do not belong to Ω' fine. The next set of edges they belong to Ω and they also belong to the Ω'. So, the first two sets of unknowns have captured everything that belongs to Ω.

Student: They gave Ω.

Yeah to get H total I add incident yeah

Student: But.

Yeah that is why we are doing we are that is yeah so, to clarify that we are doing this breaking up. First two break ups is clear ok. So, all the boundary edges boundary I mean the Γ' edges have come into the second set. Now what about the third set? Third set I am going to say belongs to Ω' and belongs to Γ'. So, which edges will come over here? The same edges as the second set we are talking about these U's are the edges and the final set is belongs to Ω' does not belong to I mean belongs to yeah Ω' does not belong to Γ'.

So, these are the ones that are purely inside the object. Now so, it seems that I have counted the Γ' edges 2 times, but that is all right because in one case the edges are representing the total field in the other case they are presenting the.

Student: Scattered field.

Scattered field, but there is also a relation between the U's of the second set and the U's of the third set they differ by incident field right. So, when I have my matrix my A over here. So, this was my x when I have my A matrix over here right. So, again this I can break up as how many parts? 4 parts right; so, these will multiply the first set, these will multiply the second set, third set and fourth set right. So, when it comes to these two sets of edges 2 and 3 what kind of coefficients will they be? 1, -1 and the right hand side will have incident field because these are scattered these are total because I am trying to represent.

So, what is the equation that links scattered and total it is going to be of this form H s minus H is equal to. So, in terms of if I want to write it like this it becomes $H - H_s$ right so, a very nice way to link up the H_s right. So, here you can see H_s is H scattered that is one of the variables H is H total that is one of the variables. So, this guy is going to come from set 2, this guy is going to come from set 3 and correspondingly the coefficients are simply 1 and -1 and they give me a $-H_i$.

No we have to construct it; we have to construct it. So, when I do the testing what will I do the testing we will go edge by edge right. So, for the when I start taking edges belonging to Γ' that is when these equations will come into being. Remember the kind of equations that I get there will be sparse. So, if I have a edge let us say over here this is not going to involve the edges far away those all going to be 0 we have seen that right. So, the boundary will come into play when I have some elements over here like this.

So, this is that is why I mean in at least in the first course in finite element or FDTD they do not talk about scattered field formulation very much, but you can see it is not that difficult. Only thing book keeping you have to do that at the edge make sure that you are taking care of what is your variable is it scattered field or total field? If they are different, make sure that you subtract them by the correct amount ok. So, your system of equations becomes a little bit larger, but it does not matter much ok.

There are there are ways of being clever about it and combining set 2 and set 3 into one variable itself, but basically you are doing the same thing once you identify this. Well we cannot independently solve it because those edges finally, like the last look at this green triangle over here we. The reason we cannot independently solve it is this green triangle has edges belonging to the set 1 and set 2. So, they are not decoupled.

I cannot solve set 1 completely because set 1 will has edges belonging I mean if I look at this triangle it has edges belonging to both set 1 and set 2. So, they become coupled equations I cannot get a system of equations purely in set 1 is what I am saying set 1 is purely interior I mean edges. So, what is set 1 belongs to.

Student: Ω.

Ω does not belong to Γ'.

Student: That is.

The first equation means which equation?

Student: That mean the.

This is your Ax this is that set there is no other set of equations. No, this is the full set of equations where the number of variables I am doing a double counting on the edge ok. So, there is no way for me to separately find out just you know first set and second set and third.

Student: They all.

Because they are coupled.

Student: Yeah.

Yeah, there will always be one variable which belongs to the other set somewhere. So, when we try to form the system of equations this will become even more clear this coupling that is happening. So, is it clear? So, what has happened is the incident field right. So, now here we can get into the comparison between the total and the scattered field total and scattered field. So, incident field in the total let us call it total F let us call it TF and SF ok. So, this is TF and SF ok so, TF where did the incident field appear? So, incident field appeared at Γ.

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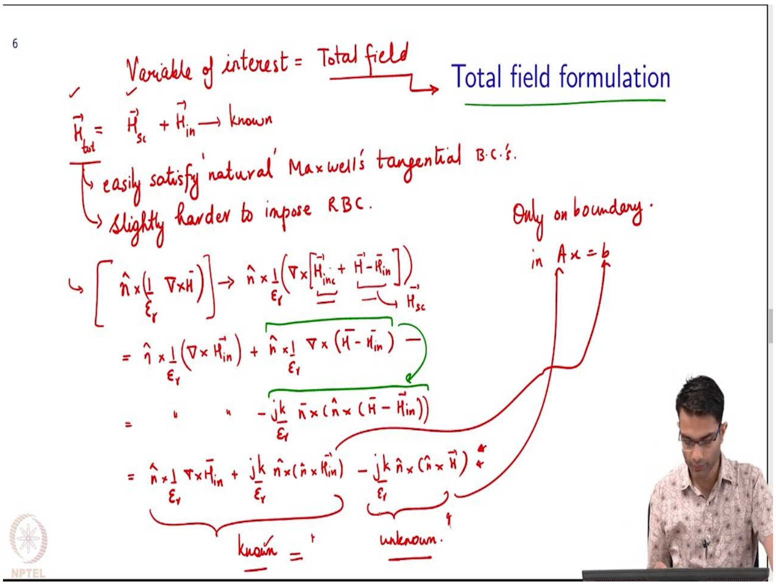

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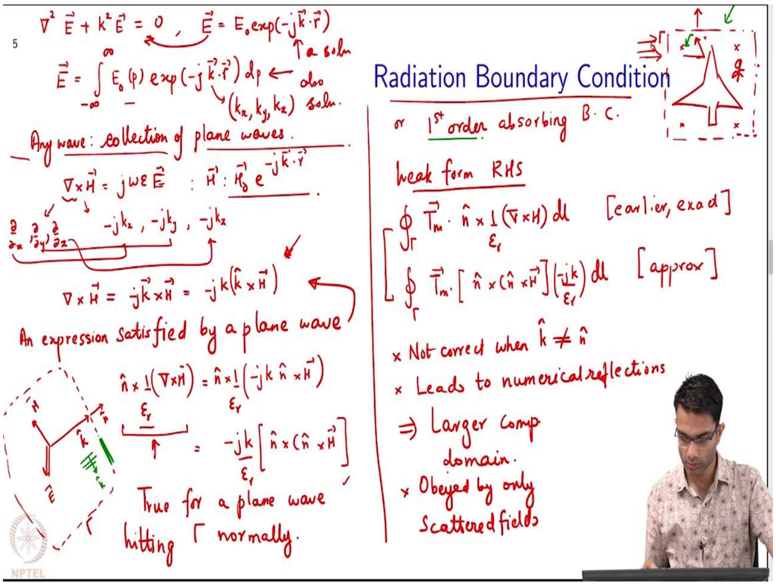

If you look back here this total field formulation all of this is happened this all of this happened on Γ itself outermost boundary right. So, the total field appeared at Γ. In the scattered field where did it appear where is it being introduced into the system of equations? At Γ' not at Γ; at Γ nothing the incident field did not appear because the scattered field was the variable because no chance for the incident field to appear ok. So, do you think that this is the big difference or a significant difference where the incident field is being introduced into the formulation?

At Γ' we will have lower number of edges. So, what? Counting the number of non-zero entries in b does not really tell us anything is there something else that is happening. So, here is an aspect of numerical computations which you should know that supposing I have a boundary and I introduce a no object and I introduce a plane wave at one point a plane wave source at one point. I know analytically the form of what the field should be because there is no object plane wave, but I can choose to solve this on the same finite element mesh I should get the same answer as the analytical solution, but will you get the same answer?

You will get the same answer as you make the discretization finer and finer because the variable is being expressed on the basis function of those triangles right and those triangles you saw they were linear in x and y, but the actual field is what exponential sine and cos. So, I am approximating sine and cos by linear in x and y that approximation gets better and better as the size of the triangles is made smaller sine x becomes x cos x whatever other approximation you want to do.

So, this approximation of the incident field on the mesh is it more or less accurate as I go away from the source location. Its less accurate the farther I go away because the errors will accumulate as I go further away right very close to where I am specifying the boundary condition it will be accurate because the boundary conditions right there fields will try to match the boundary condition as much as possible.

As I go further away the approximations the error of the approximations will begin to grow because I am finally, trying to represent a sin and cos by linear terms right. So, now do you can you answer which is better between TF and SF? SF is much better because?

Student: Because TF is the incident field.

Because in TF the incident field has appeared over here on this boundary then it has to numerically propagate to the object and gets scattered right, but in the scattered field in the system of equations its being introduced on the surface of the object there is no what is called grid dispersion right. So, this is this term is called grid another term that we will encounter in finite difference time domain.

So, the incident field I know it analytically, but once I have put it into the system of equations it is the system of equations they do not; they do not know what the form is anywhere else unless I give it to them. So, the scattered field has this advantage that it appears directly at Γ' what else can you think of? So, this is the main advantage of scattered field.

The other advantage is this let us say that you we have we have spoken about this radiation boundary condition being inexact correct and its inexact when the field that is hitting it is non-normal not coming at 90 degrees then something reflects back correct. Now as an engineer if you wanted to make that situation better ok. So, let us draw it once over here. So, this is so, right now it does not matter total field or scattered field.

Some wave has come over here this is true in total field or scattered field. Even in the scattered field formulation I am imposing the boundary condition the RBC which is correctly true only for normal incidence not for non normal incidence right. So, some amount of this is going to get reflected. Now as an engineer wanting to reduce this what other option might you do, this is reflecting this is the numerical reflection why due to due to inexact boundary condition ok.

I want to reduce this. So, what do I do? Transpose the situation into a real life situation; real life situation you have an antenna here and you do not the its sending out some fields you do not want the fields to come back to you basically that is the reflection I want to cut it out what would you do?

Student: Will increase the boundary.

Supposing I cannot increase the boundary I mean I am in a room I cannot make my room bigger put some absorber. If I put an absorber then nothing will come back to me and we already have seen examples of this is right if you go to acoustic chambers anechoic chambers they are called it is a very funny thing because nothing is reflecting from the walls it feels very I do not know if you have been inside any of these rooms nothing comes back to you right. So, that is physically creating a absorbing situation.

All microwave and radar studies if you go to their labs they have these anechoic chambers where they if you want to characterize a antenna you do not want some spurious reflection from the wall. Supposing I do a measurement in this room I am trying to measure the radiation pattern of an antenna, but without realizing something is hit the roof and come back and my measurement is corrupted by these reflection I want to cut it out. So, how do I do?

I fill the room with the I line the wall with an absorber ok. So, this was an absorber in real life can I make an absorber mathematically?

Student: Sir.

Student: Put some extra padding.

Put some extra padding all right fine and what how do I implement mathematically an absorber?

Student: (Refer Time: 14:56).

How in physically you are saying measure the field, but mathematically what do I do?

Student: Thought like measure as a.

So, now, you have to use the word compute not measure. How do I compute without solving the system of equations first and once I solve it with a inexact boundary I do not know I mean what I have got is everything put together reflections everything put together. So, and let us go take the intuition from the physical problem to the mathematical problem physical problem I put some layer over here which absorbs; absorbs means the first word that should hit your mind from your undergrad EM courses skin depth what is skin depth? If you have some conductor.

Student: Alright.

Wave goes inside gets absorbed and dies after skin depth, but what made that property to have different levels of skin depth.

The loss in the medium right conductivity basically if there was no conductivity in the medium what would happen would there be any skin depth.

What is the skin depth of vacuum? It is a observed question the skin depth is infinity the wave never decays in vacuum right vacuum has no loss it goes on forever. The moment I introduce some loss into the thing there is conductivity the wave begins to decay. So, so that is the hint how do I implement this absorber mathematically? Well I just create in this in this green shaded region I introduce loss in the epsilon r mu r of those triangle. So, over here I can say epsilon r make it lossy exactly epsilon r will become complex.

In free space epsilon r is 1 now what I will do is, I will make it 1 minus j something right. So, this is this is one way of implementing it. So, what will happen now even if the wave comes at a non-normal direction what will happen?

Student: (Refer Time: 16:59).

Some part will still get reflected right if you look at your reflection coefficient Fresnel reflection coefficients there is going to be some as long as there is a difference between n_1

and n_2 between two mediums refractive index some reflection happens, but that reflection I can control by controlling the refractive index of this green region right.

So, that is that is the sort of a intuition over here. So, this is hopefully less. So, this is one way of improving boundary condition and in this course we will look at further on specific examples of these absorbing layers as they are called ok. The most successful and popular wave absorbers are called PML; Perfectly Matched Layers we will talk about that subsequently in the course ok, but is an example of an absorber it absorbs the field and does not reflect back.

All of this having being said; do you see any differences as far as absorbers go in total field and scattered field. So, I have put an absorbing layer because I wanted to improve boundary conditions very good. When I go to implement this using the total field formulation what will happen, the place where I introduced the incident field will not change it has to be on Γ.

Now, this Γ has to travel through a lossy layer. So, I will get further errors in the propagation of this wave it was bad enough to propagate it through vacuum, but now it has to travel through this lossy layer.

Student: Yeah.

The source is outside let us say some radar far away think of it like that. So, I have no control over where the source is, but now if I introduce the if I introduce a lossy layer incident will continues to be at Γ nothing changes right. If I look at my total field formulation over here well let us look at the right hand side over here right.

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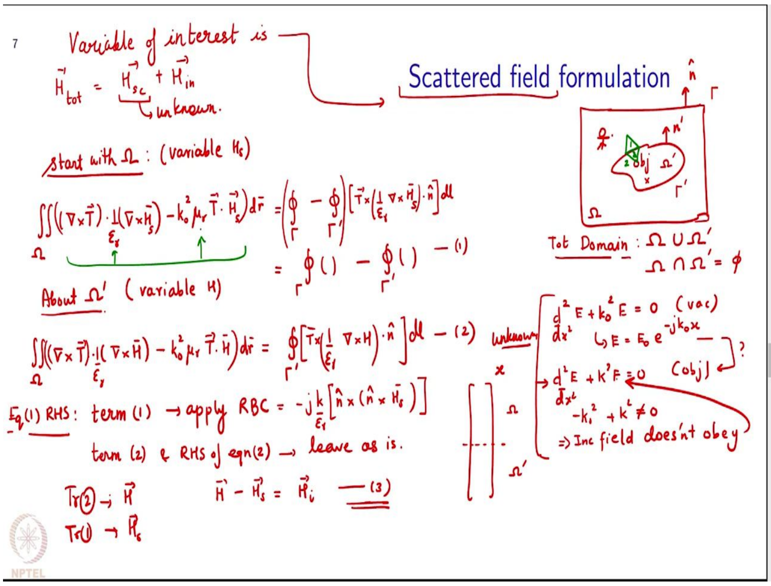

So, whether total field or scattered field the left hand side term is the same right your material properties are entering here and here. So, the fact that I have now introduced some lossy materials here does not alter the left hand side of the equation right. So, that incident field will have to travel through this lossy layer in order to get there right. So, the in terms of absorbers the total field has more errors due to propagation through absorber what happened to scattered field formulation?

Perfect no change at all because incident field is being introduced at Γ' and your absorber is far away. In fact, it made everything improve because the scattered field would be absorbed even better incident field suffers no degradation in propagation across the mesh right. So, that is the win-win situation over here. So, there are two strong advantages for scattered field formulation over here ok, you can improve the boundary condition and no errors in the field propagation.

So, given a choice it is easier to code total field formulation. So, many many times that is the first thing you would code and that is useful like as a reference solution then if you want to actually use your code for some serious work it is better to switch to scattered field formulation because then you can keep improving your boundary condition right. Adding a absorbing layer is trivial what is there in a in code right you have to introduce some loss and this design of this absorbers is a another separate area of research altogether.

You know that if I introduce if you introduce a loss abruptly what will happen you will get a strong reflection. So, there is a lot of art in this you know you make the absorbing the loss component gradually or adiabatically increase from the boundary that makes the reflection to be very very less. So, all of these things are tricks used in physics electromagnetics to make high quality factor resonators and so on, but that is a separate topic.

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So, what we have done is that we have spoken about. So, we started with the actually this is a good thing we started with the kind of shape functions these were are vectors in space then we took our equations converted into weak form. And weak form and we spoke about our boundary condition radiation boundary condition which is the applicable only to scattered field.

And then we chose the two variables either total field or scattered field these are two different ways of solving a problem right. Any questions yeah because the computational domain is ending at Γ that is the place where I do not want any spurious reflections. So, that is the place where I apply the RBC.

Student: Triangles near Γ which are these lossy factor.

Yeah.

Student: (Refer Time: 22:49).

Yeah the triangles near Γ are where I will begin to impose the loss if I were implementing a absorber. Incident field will undergo further distortion

Student: (Refer Time: 23:03) that is why total field is.

Yeah in the total field formulation.

Student: Td.

H incident is defined to be the field in the absence of object.

Student: Yeah absence of everything.

Yeah.

Student: So, yeah so.

It has to travel through the absorber right.

Student: Yeah, but just.

Because in analytical form I am defining it only on the outermost boundary the rest of the propagation which we are not actually doing, but the equations are doing it. The information of the incident field has to reach the object in order to scatter that is happening mathematically through the mesh automatically we are not doing anything to it, but when we impose Maxwell's equation that is what is happening.

But, now those incidence fields have to they are anyway getting distorted by the mesh because of the approximation of basis function they get further messed up because of these absorbing layers what will be the value of H incident inside.

Student: In Ω.

In Ω in some integrand.

Student: Outside Ω.

What will be the well you would have to calculate it. You are telling the in a system of equations you are telling the value of H incident only on Γ you are asking what is H inside at some point what are you asking H or H incident or H scattered?

Student: H incident.

H incident.

Student: I hope it that we know.

That we know analytically.

Student: In the whole domain.

In the whole domain we know, but that does not enter into the system of equations. The system of equations accepts it only at Γ in the total field formulation. How does it enter because its $e^{-j\vec{k}.\vec{r}}$ what value of r are you putting only boundary edges nowhere in between around. The left hand where I have spoken only about the RHS so, far because that is where some amount of subtlety is there next when we talk about putting everything together we will come to the left hand side and put it.

So, that is this last part which is putting it all together we will look at left hand side we will look at right hand side and assemble a system of equations all there yeah all the. So, far we have been concentrating on the boundary because that is the more sort of critical crucial part next we will look at the left hand side.