Computational Electromagnetics
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2D Finite Element Method
Lecture - 11.05
Scattered field formulation

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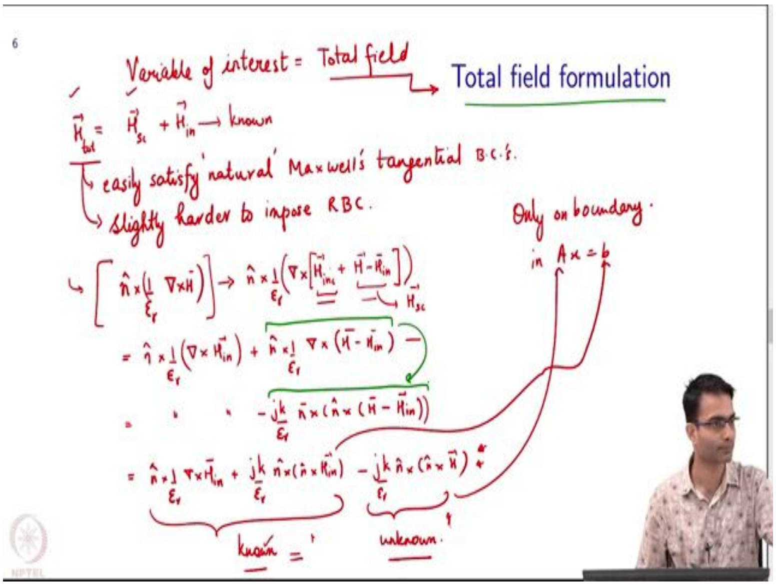

So, we continue our discussion of edge based or vector 2D Finite Element Method and what we have seen so far is the most natural way of formulating the problem, unknown in terms of the total field right. To summarize the key thing to keep in mind here is that, in order to apply the boundary condition I had to add and subtract the incident field and apply the boundary condition only to the scattered field term. So, of the two terms that come on the right hand side one remains, which has a known information and the rest goes back right.

So, the other way of formulating this problem is what is called the scattered field formulation ok.

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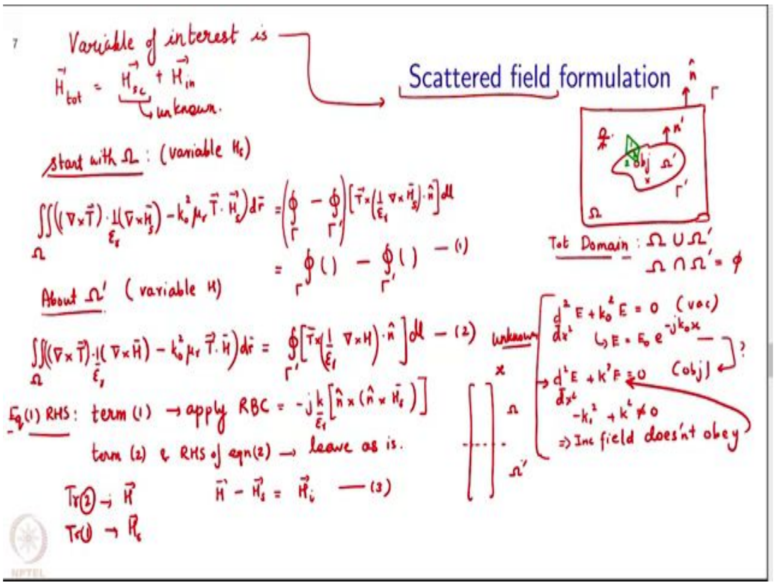

So, that is what we look at next. So, here the variable of interest, is the scattered field ok. So, we have already seen that, $\vec{H}_{tot} = \vec{H}_{sc} + \vec{H}_{in}$ ok, what kind of a computational domain are we talking about. So, now, we need to get a little bit particular.

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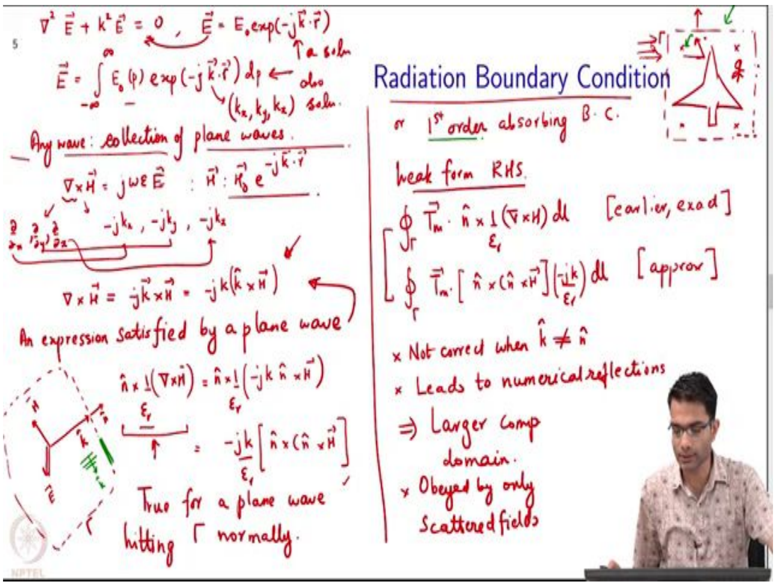

So, far we just said, when we looked at for example, the previous formulations over here let us see, the right hand side term right, this is the right hand side term that I had.

So, it is the boundary enclosing the entire domain, we did not have to ask about where is the object where is the I mean just outside-most boundary mattered. In the scattered field formulation, the problem changes a little bit. So, let us see what happens here. So, let us take our domain like this and this is some object ok, we have said that the notation we have is Γ is the outer most boundary and Γ' is the object ok. So, we can call this as Ω and call this as Ω' ok. So, domain is the union of both total domain.

Student: Sir Ω is the domain then what is Γ.

Γ is ok; Γ is the outer most boundary.

Student: (Refer Time: 02:56) Γ.

Γ' is the boundary of the object.

Student: Ω is the.

And Ω so, Ω is the part of the domain that does not include the object ok. So, for example, this object could be a aircraft and it is surrounded by air. So, air is Ω, Ω' is the object. So, I mean if you want be very particular if I do the intersection of this, this is the null set ok. So, that makes it clear there is no intersection, I have broken up the domain into 2 disjoint boundaries, a disjoint regions and they share a common boundary Γ' ok. So, there should be no confusion about this ok.

So, now that I have broken up this domain into 2 parts ok, let us start with omega. So, when I start with Ω, I can go back to my Maxwell's equations and repeat the same steps that I did for the weak form right, what would change. The right hand side where I had 1 line integral or contour integral, I would have 2 ok.

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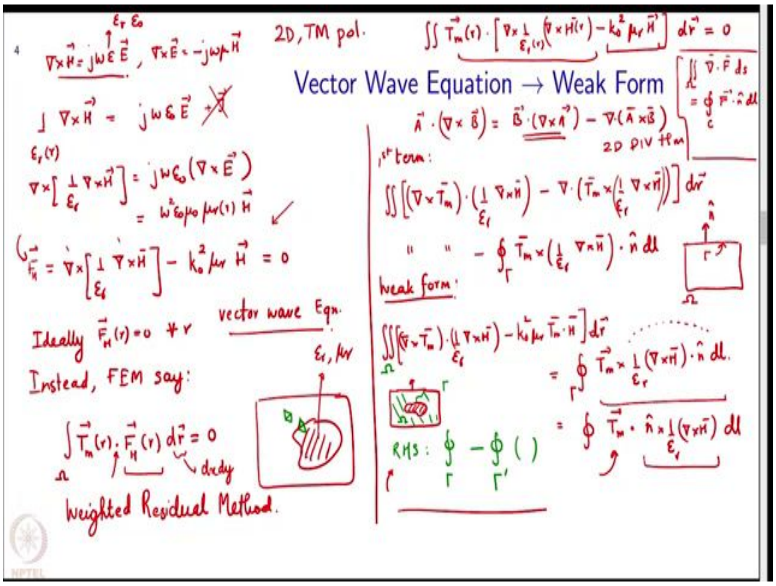

Let us again go back and refresh our memory here, over here when I wrote down this weak form, I got one term on the right hand side but, if I had this green case over here then, there were 2 integrals right. So, that is why I had mentioned this and this is just a simple consequence of your divergence theorem whatever, at the boundaries and closing the region those many line integrals are there because, that is the outward flux that is a principle of divergence theorem.

So, those many line integrals or contour integrals are going to be there ok. So, what we will do, what is different now in the case of the scattered field formulation is we will write two different weak forms, one for each region, that is how we will break it up and by the way this total field and scattered field formulation, this is even common to other methods in the electromagnetics.

So, finite difference time domain again, we will visit this over here right. So, it is a actually a general principle not specific to finite element finite element. So, if I start with omega:

$$\int_{\Omega}\int[(\nabla\times\vec{T}).(\frac{1}{\varepsilon_r}\nabla\times\vec{H}_s)-k_0^2\mu_r\vec{T}.\vec{H}_s]d\vec{r}=(\oint_{\Gamma}-\oint_{\Gamma'})[\vec{T}\times(\frac{1}{\varepsilon_r}\nabla\times\vec{H}_s).\hat{n}]dl$$

So, this is two different integrals, is the notation clear what I mean over here, the integrand is the same for both the integral. So, I am just writing it once and there are two different contour integrals all right. Now, what is the I have chosen the variable of interest to be the scattered field right. So, I should make one clarification here wherever, I write H I actually mean $\vec{H}_s$, the scattered field right. So, just to be even more explicit, this is what I made.

Student: Sir in the testing function is same in both the cases.

Testing function is the which two cases. So, far there is only one case.

Student: (Refer Time: 07:39).

So question is what should be the form of the testing function, the testing function is going to be the same because, even though my variable is now the scattered field, I am going to still expand the unknown in terms of the same basis function, the basis function is going to be the same. So, testing function whatever I did previously is what continues to be the case. So, so far you know, there is no problem over here, where might where are the issues where are the places where we would have to take care on these boundary terms, we have to see which ones of which one of these is to be handled carefully ok.

But, that will become clear once I start with, once I go to the next revision right. So, about now next is about Ω' ok. So, I am writing two different weak forms, what happens to the left hand side? Before we go into this, physically what does the scattered field mean; it is a field that is produced.

Student: (Refer Time: 08:42).

Due to the object, if there were no object there would be no scattered field correct. So, where does it make sense, it make sense for the observer standing out here, who can see an incident field and can compare with and without object right, that is the physical meaning of scattered field, if the observer.

Now, the object is there and the observer is inside the object, is there any is there any sense of mentioning the scattered field, what is the physical meaning of scattered field now, mathematically I can define a scattered field, as total minus incident and I know incident in analytical form let us say then, I can mathematically define a scattered field but, inside the object there is no meaning really, there is no physical meaning of a scattered field because, what am I comparing with.

If there is no object right, I mean that case does not arise, I am I am inside the object the object is there is only 1 case, when I am outside the object there are 2 cases object no object. So, I can compare total I mean incident and total. Once I am inside the object, there is no real physical meaning to make this distinction, what is scattered field, what is incident field. So, inside the inside the object, we go back to the total field as the variable of interest ok.

So, another way of putting it is the incident field, does it obey Maxwell's equations in free space, yes. Does total field obey Maxwell's equations in free space yes therefore, scattered field also will obey superposition principle, that is how I was able to write down the equation 1 because, equation 1 has finally, come from Maxwell's equations. The scattered field obeyed the Maxwell's equations because, total field obeyed incident field obeyed.

However, in the case of when I am inside the object, can I say that scattered field obeys Maxwell's equations; I cannot because, even the incident field does not obey Maxwell's equations inside the object, if I have a plane wave simple case, I have a plane wave travelling through free space ok. So, the form is $e^{j(\omega t - k_0 x)}$, vacuum wave number.

Now, I put a object inside it, will the form of the plane wave be the same inside, we know for example, k itself will change, the wave number inside the medium will change, the wavelength will become I mean it will alter, frequency remains the same, speed will change wavelength change we know all of that. So, we know that the incident field is not a solution to Maxwell's equations inside the medium it will be.

Student: Then you please changing the equation the.

Ok, so let us let us have a look at that. So, 1D case ok. So, what is Maxwell's equations outside the medium, $\frac{d^2}{dx^2}E + {k_0}^2 E = 0$ right, this is vacuum, this gave me the solution of $E = E_0 e^{-jk_0 x}$ ok, consider inside the medium, is this guy a solution to this, what happens to the left hand side.

Student: (Refer Time: 12:01).

k is different from k_0. So, this will become $-k^2 + {k_0}^2 \neq 0$, when is it equal only when $k = k_0$ means, no object. So, implies that incident field, does not obey, incident field does not obey but, what obeys Maxwell's equations in the medium.

Student: (Refer Time: 12:29).

The total field right, that is why it makes no sense to break it up into incident and scattered inside the object, inside the object there is 1 field which is the solution to this equation ok. Simple 1D example gives us this proof right. So, that is why inside. So, here the variable is H_s, here I am going to keep the variable as H because, always remember this equation 1 was derived starting from Maxwell's equation whatever, variable I choose has to satisfy Maxwell's equation, if H_s does not satisfy Maxwell's equation I cannot arrive at this right.

$$\int_{\Omega}\int [(\nabla \times \vec{T}).(\tfrac{1}{\varepsilon_r}\nabla \times \vec{H}) - {k_0}^2\mu_r\vec{T}.\vec{H}]d\vec{r} = \oint_{\Gamma'} [\vec{T} \times (\tfrac{1}{\varepsilon_r}\nabla \times \vec{H}).\hat{n}]dl$$

So, what happens now, left hand side will remain exactly the same except H_s will become H ok, this $d\vec{r}$ vector just remember that this is a two dimensional can be dxdy ok, that is why I put a vector on top of r, what happens to the right hand side? What happens to the right hand side, how many terms are going to be there.

Student: (Refer Time: 13:59).

Only one term over.

Student: Γ'.

Right. So now, here is it seems like I have two different sets of equations. So, the main sort of concept now is how do I tie these two things together. So, how do you think we should do it, where will the incident. So, far I do not see any sign of incident field, where does the incident field coming, at the boundary well yeah, which boundary there are 2 boundaries.

Student: (Refer Time: 14:42).

Γ' but, I do not see it in the equation so far. So, let us deal with the equations 1 by 1. In equation 1, I have 2 terms on the right hand side right. So, what approximations or what boundary conditions need to be applied; so, equation 1 right hand side.

Student: Yeah we just mean that e cross (Refer Time: 15:09).

Yeah. So, we can do some algebraic manipulation and.

Student: A boundary condition.

Yeah, so, we need to apply a boundary condition on the right hand side, question is there are 2 terms on the right hand side now in equation 1, one from each boundary. So, for the first term, what should I do, what can I do about the first term, what boundary condition, can I apply the radiation boundary condition here? For the first term, is it a scattered field? That I am talking about, first equation is it a scattered field appearing in the variable edge.

Student: Yeah.

Yes, will it obey radiation boundary condition yes, at the remember the boundary gamma is a mathematical imaginary boundary; we do not want any field to come back from it. So, the natural place to apply this is.

Student: Γ.

Γ itself right. So, term 1, apply a radiation boundary condition, our aim is to formulate a system of equations with some of the form $Ax = b$, I want to solve and get the fields everywhere.

Student: (Refer Time: 16:27) this is the boundary condition.

Boundary condition.

Student: We are doing that.

We are doing the same thing, we are trying to somehow get it into a, you know $Ax = b$, where b is known and I can solve for x right. Except now, that x vector, which is the vector of unknowns has now been broken up into edges in Γ and edges sorry edges in Ω and edges in

Ω' and there is something common between the 2 ok. So, a term 1 now, I can apply a RBC, does will the incident field appear inside there, like in the total field, when I applied the radiation boundary condition on this term, I split it into 1 2.

And, incident field appeared on the right hand side that is how this term up came over here, will that happen here? No it wont the incident field will not come here because, it is already the scattered field, there is nothing to subtract from it, what is the need to subtract think physically, the boundary condition applies to the scattered field, my variable is scattered field. So, all I have to do over here for this second term over here, this is what will happen right. So, it will become $-j\frac{k}{\varepsilon_r}[\hat{n}\times(\hat{n}\times\vec{H}_s)]$, just like over here right, same logic we are applying over here, remember what we did, this is the logic that we are applying right.

So, this term is becoming this term right, except now $H_{tot}-\vec{H}_{in}=\vec{H}_{sc}$, which is already there right ok. So, this term has been dealt with, what about the second term, what should I do about the second term, is there any particular boundary condition I can apply there? Actually the answer is no, we will leave it as it is. So, this term 2 and the RHS of equation 2, both deal with the same boundary Γ' correct, we are going to leave these terms as they are ok.

So, and I will where shortly you will see why. So, the main question now, that has that has come about is what happened to the incident field? Where does the incident field enter inside the system of equations?

Student: (Refer Time: 19:01).

Equation 1 can be written in terms of total field that is the total field formulation.

Student: That intended tabulated will (Refer Time: 19:07) becoming to play.

Yeah. So, the yeah that is what we did previously but, in the scattered field formulation, where the variable of interest is the scattered field. Now, where has the incident field vanished or has it vanished, where am I switching between variables, at the boundary. So, if I look at one particular boundary over here right. So, over here what can I write? So, imagine 2 triangles over there right.

So, let us take 1 triangle over here inside like this and 1 triangle over here inside ok. For the triangle outside over here, let us call it triangle 1 and this call it triangle 2. So, in triangle 1, if I look at the edge that is I mean the edge of the triangle on the boundary, the variable is H_s right. So, H_s and inside the triangle the variable is H. So, now, the same edge, has 2 different values. So, what are what and what are they what is the difference between them, incident field right.

So, if I look at it like this, triangle 2, the variable is H triangle 1, the variable is H_s but, we know that at the boundary, the difference between the 2 is simply the incident field right. So, the third equation that I get from here is going to be $\vec{H} - \vec{H}_{sc} = \vec{H}_{in}$, that is the third equation that I get ok. So, how does that translate into a system of equations?

Student: (Refer Time: 21:20).

So, yeah so, if we go about it in a slightly should I say lengthier way but, easier to understand, what I would do is let us say this is my column vector x unknowns, I can break up x as edges from Ω, edges from Ω'. So, I can let us say that this is from Ω and this is from Ω' but, there are some edges that are common correct.

But, what we will do is to keep it very simple, we will do a double counting, we will count these variables 2 time. I mean we will count these edges 2 times and we will say that the variables are different in 1 case it is scattered field, in 1 case it is incident field but, I will have, I can use this equation 3 to get, one more set of equations only for the interface edges and that is where the right hand side term will become nonzero, I will get a incident field from there from this equation.