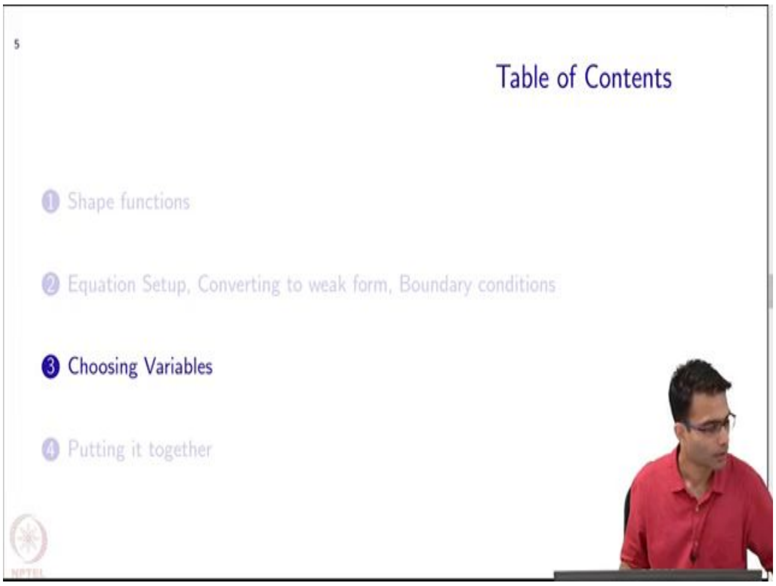
Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

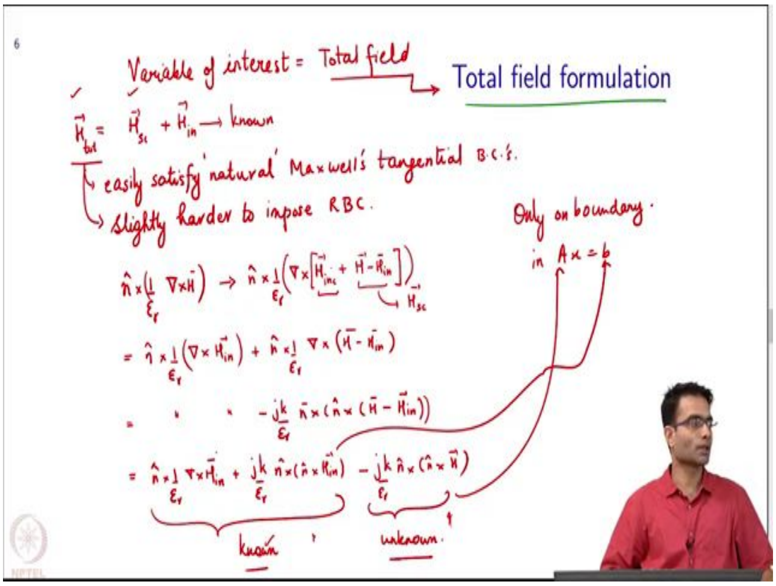
2D Finite Element Method
Lecture - 11.04
Total field formulation

(Refer Slide Time: 00:14)



Right so; now we will come to how do you choose variables ok? We did not have much of a choice in the case of 1D or we did not spend too much time on it but so, we will have a look at it over here.

(Refer Slide Time: 00:25)



So, what is called the Total field formulation? This will look like a very obvious thing because so far we have been only talking about total field formulation. So, what does it mean, it means that the variable of interest equals total field and that is why it is called the total field formulation?

So, the $\vec{H}$ over here can be written as so, $\vec{H} = \vec{H}_{inc} + \vec{H}_{scat}$; in scattering problems $\vec{H}_{inc}$ is known. You could choose to have your variables be either the total filed or the scattered field it is your choice, total your choice each method will have some advantages and some disadvantages. So, the first and the most intuitive choice is to allow the total field to be the variable ok. So, once I can get the total field if I want the scattered field I have to subtract of the incident field and vice versa for the scattered field vice right.

What is the when I choose to total field over here what is the sort of one conceptual simplicity you can see of doing that? boundary conditions are obeyed by whom?

Student: Scattered field.

No the boundary conditions in Maxwell's equations.

Student: Whereas true.

Boundary condition which is tangential E and tangential H are conserve they are obeyed by total field right. So, it becomes easy to impose a natural boundary conditions in the total field formulation right. So, we can say that easily satisfy let us call them natural.

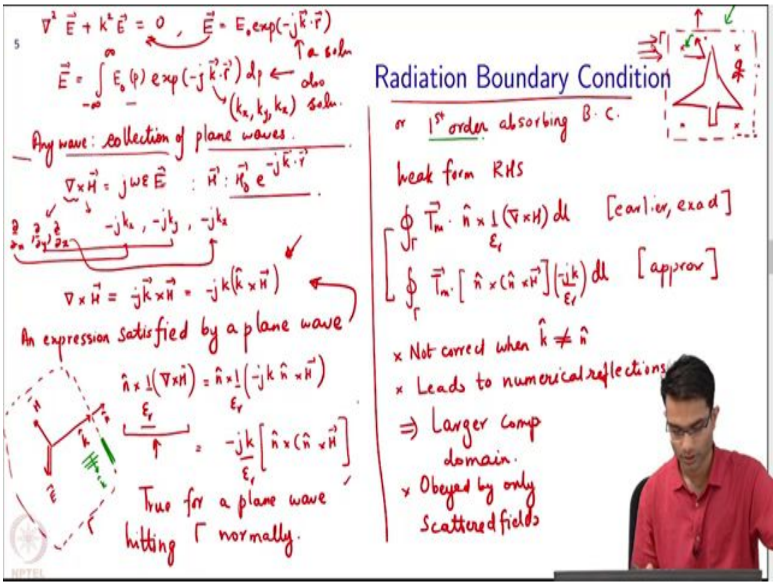
Student: Reporting if these are the obey the object obey.

Yeah. So, we are I am calling them natural Maxwell's tangential boundary condition right/ Basically E tan is conserved, H tan is conserved. But on the disadvantage when I come to the radiation boundary condition the disadvantage is, that this boundary condition which I just derive radiation boundary condition it was obeyed by whom? Scattered field, not the total field. So, if my variable is total field like we are done in the case of 1D I have to subtract of the incident field and then impose the boundary condition right.

So, slightly harder. Not very hard, but we just have to keep taking care of it. So, let us let us look at the expression that I had. So, $\hat{n} \times (1/\varepsilon_r \nabla \times \vec{H})$ that was the term that I was struggling with right. So, what do I do now? So, first of all I have to write this as $\hat{n} \times (1/\varepsilon_r \nabla \times \vec{H})$ this is the slightly tricky part H incident. So, I add and subtract incident field fine. So, what happens as a result of that first term becomes $\hat{n} \times (1/\varepsilon_r \nabla \times \vec{H}_{inc})$ this term is known right and what happens over here?

So, $\hat{n} \times 1/\varepsilon_r$ let us it is write it once $\vec{H} - \vec{H}_{inc}$ right. This expression remains as it is and I am left over here with that approximate expressions.

(Refer Slide Time: 04:56)



So, that was what $\hat{n} \times 1/\varepsilon_r$ and this was replaced by let us have a look over here yeah. So, this whole expression over here so, $-jk/\varepsilon_r \, \hat{n} \times (\hat{n} \times (\vec{H} - \vec{H}_{inc}))$ the same expression must hold true right. Now, in these expressions there are terms that are known and there are terms that are not known. So, the known terms for example, I know $\hat{n}$, I know ε_r, I know H_{inc} incident. So, I can gather all of these known terms and they will become $jk/\varepsilon_r \, \hat{n} \times (\hat{n} \times \vec{H}_{inc})$. These are the known terms and I am left with $-jk/\varepsilon_r \, \hat{n} \times (\hat{n} \times \vec{H})$. This is just a fancy way of writing what we have already done in 1D; in 1D we had a term we wanted to impose a radiation boundary condition we wrote we and we could only impose it on the scattered field.

So, how to get the scattered field? Write it like this $\vec{H}_{inc} + \vec{H}_{scat}$ and since my variable is $\vec{H}_{total}$, I should not introduce a new H s over here I will just confusing things. So, this term over here was my H_{scat} right and the rest is just substitution ok. So, remember this term was going to coming into play only on the boundary right only on boundary. So, these this known term over here this is what is going to give me a non zero right hand side. This H over here which has the unknown term I will move it back to when I get $Ax = b$. I will move this guy back to the left hand side and keep this on the right hand side right.

So, in $Ax = b$ this term will go back into A and this term will go into b that is what will happen right. So, that is the way with the total field formulation that is how we will work. So, it is not tricky just taking care of on which field I am imposing the boundary condition itself.

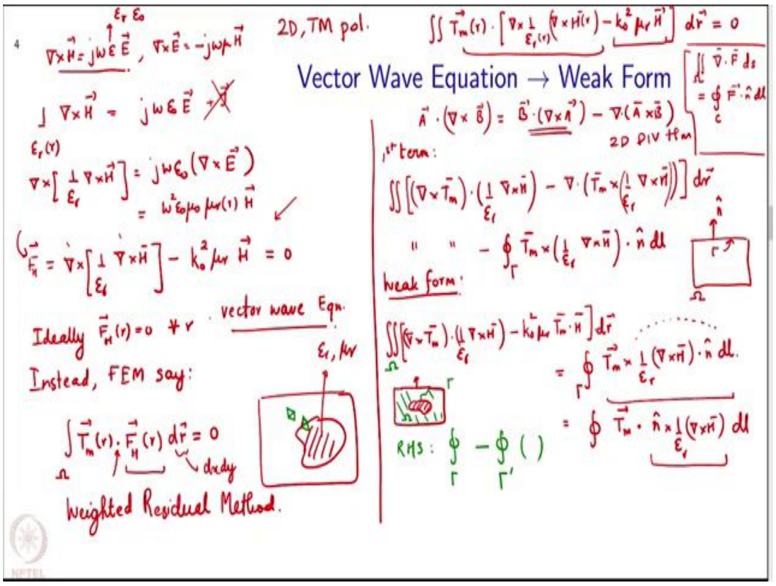
So, I will yeah H_{inc} is known. So, I will just substituted over there and H is the unknown which in which I will replace the basis functions. So, in subsequent modules we will go and take a concrete case and see how do I get the system of equations once I substitute the basis function.

Student: If we take the gross total (Refer Time: 08:24).

Can you say again?

Well I would not worry about it. So, right now what I have written over here this expression just whole thing needs to be dotted with $\vec{T}_m$.

(Refer Slide Time: 08:41)



If you look over here is it $\vec{T}_m$ dot waiting for you right. So, Tm dot this is going to be a scalar. So, I have so far I have left the $\vec{T}_m$ out because that is always there like a constant. So, it is going to be a scalar over here.