Computational Electromagnetics
CEM:An Overview
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CEM: An Overview
Lecture – 2.2
Different Regimes of Maxwell's Equations

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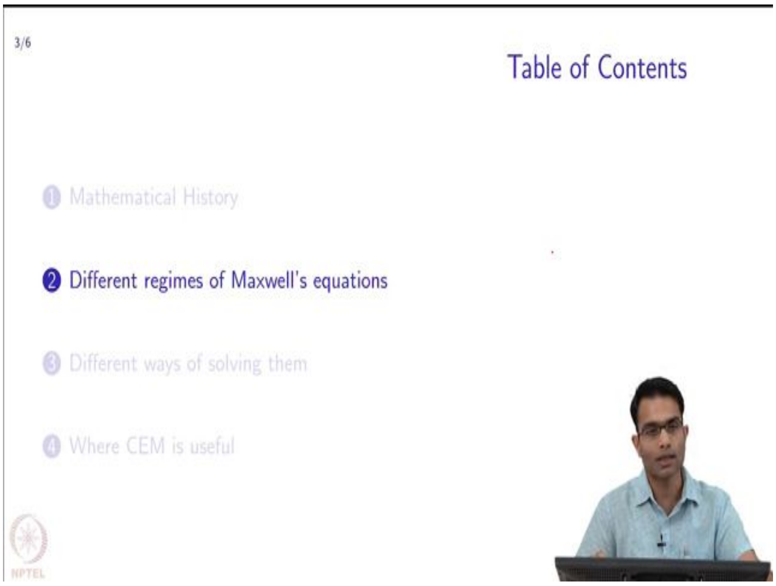

So, now let us come to the different so called regimes. So, what is regime mean? These different kind of different laws for different settings, the overall laws are the same which are Maxwell's equations. But I can specialize them depending on the situation. So, what are those special cases? So, those are called different regimes.

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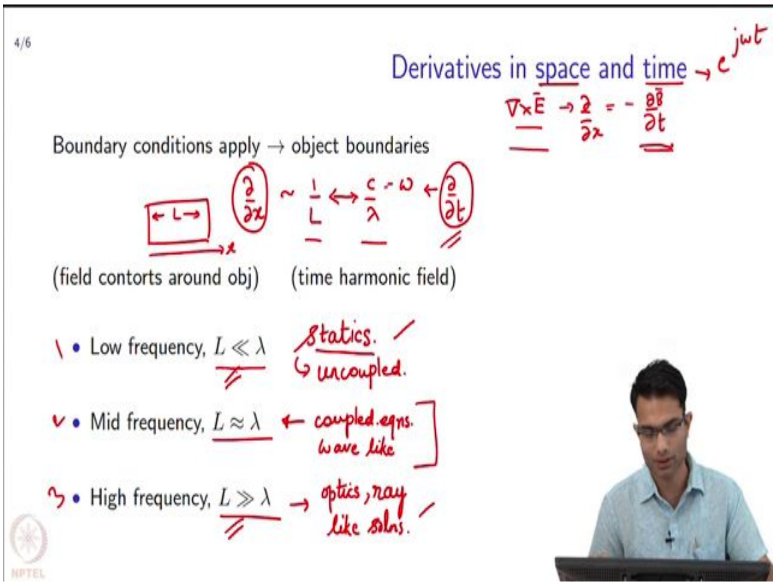

And so you notice one thing let us start from the top, Maxwell's equations had a derivative in space and a derivative in time right. They had terms like $\nabla \times E$. When I open this is spatial derivative right, I have terms like the d/dx and so on. And they also had terms like time derivative, so there is a derivative in space and there is a derivative in time; and so the simplest thing you can do is to find out what is the relative weight of these two derivatives. If one is much bigger than the other, I can ignore the smaller one. If they are comparable, I must keep both of them and this is the basic idea ok.

So, the other thing about Maxwell's equations is that you would have again heard this in your undergraduate course on electromagnetic says that, you all heard the word boundary conditions, boundary conditions right. Some boundary conditions have to be satisfied by the field on the boundary and we will look at those as we go. But again what is an intuitive idea behind boundary condition, is that if I have let us say an object over here, let us say size L the electromagnetic field that is flowing around this object, it has to sort of respect this boundary. You know it has to bend around it; twist around it cannot behave as though it is not there right.

So, let us for example, let us take this direction to be x, so again very intuitively if I asked you that for an object of size L how much do you think that this rate of change of the field, along that coordinate x? What would it be, how would you write its relation with respect to L? If you did not know you do not need to know Maxwell's equations intuitively, should it depend proportional to be proportional to L or inversely proportional to L? I am asking you rate of change of the field along x. What is your intuitive understanding? If you have two choices proportional to L proportional to 1/L what would you think?

Student: (Refer Time: 02:41)

Proportional to 1/L why because, so I say that this is proportional to 1/L. Think of the case where L is very large; however, let us say even infinite if it is very large, then there is no variation along the x direction, it is like the object is infinite. So, the field does not have anything to bend around L is so large. So, d/dx if I say $1/L$ will tend to 0. That means, there is no need for the field to change, because there is no change along that direction. On the other hand, if L was very small then the field will have to sort of bend around it.

So, the smaller the field is right so smaller the object; the larger will be the bending of this field ok. So, this is just intuitive right so we can say that, d/dx is proportional to $1/L$ that is one side of the story right that is the space derivative. The other thing that we are looking at is time derivative ok. And all of you have done the course on something like signals in systems. So, when I am looking at a linear system of equations then, what is the most convenient way of studying time of frequency? Fourier transform; before that represent everything in terms of a phasor right into the g omega t. Once I understand the response of the system to $e^{j\omega t}$, I basically understand everything right. So, here I can take $e^{j\omega t}$. So, when I take the time derivative of $e^{j\omega t}$ what do I get? So, if I say d/dt, what should I get?

Student: ω.

j ω; so forget j is a constant I am going to get ω; and ω in terms of wavelength is.

Student: (Refer Time: 04:28)

Inversely right so this is going to be.

Student: C.

c/λ right; forget all the constants right. So, all the action really is happening in the relation between these two terms. So, I have a space derivative which is proportional to 1/L, I have a time derivative which is proportional to $1/\lambda$ ok. So, these are the two completing terms. Remember this for example, this equation which I had $\nabla \times E$ is equal to $-dB/dt$ that was one of the equations right.

So, it is like a tug of war between these two terms and who is going to win right. So, based on the relative sort of ratio between L and λ, there are these three regimes of physics that happen. So, for example, when let us say this L is much smaller than λ so; that means, what can I say?

So λ is very large; if λ is very large, what happens to this term? $1/\lambda$ will become very small right. So, I can ignore the time part of these Maxwell's equations right. So, this part is going to become very small and this part will nominate right. So, if there is no time dependence, what are those kinds of equations called? We have studied in school.

Student: (Refer Time: 05:55).

Differential equations, but a little bit small special name for it.

Student: (Refer Time: 05:57).

Statics right, because there is no time dependence, we call them electrostatic magneto statics or whatever; in general we will just call them statics. And if I do not have the time terms, then I have the space derivative of electric field, space derivative of magnetic field and there is no coupling between them; because the coupling was happening via time derivative.

So, these statics they are uncoupled system of equations; electric field has its own story, magnetic field has its own story ok. So, we call them that is why you heard the words electrostatics and magnetostatics or you do not have electromagneto statics ok. So, these are uncoupled alright. Then comes the case obviously, where these two derivative terms are comparable right. So, that is what is called mid frequency ok.

So, in this mid frequency range there is no escape, I have to use both the space derivative and the time derivative ok. You can imagine that statics would have been easier because there is one term less. Mid frequency is the most difficult situation to handle because I have to take care of both these derivatives ok. And most of I mean this course will be focused mostly on the difficult part which is mid frequency range.

So, that is so here you get coupled equations ok. Another thing is that when these equations are coupled as you saw in the slides before when I showed you Maxwell's equations when I have coupled equations the equation of a wave comes out. So, the mid frequency range has wave like solutions ok. So, that is again something we will see. So, they are likely they are wave like and that is how you are able to your cell phone is able to send a signal to the base station and we are getting light from the sun, because these are all waves can travel. On the other hand, if you look at the solutions in statics they are not wave like they are smooth solutions know; Laplace equation, Poisson's equation they are not wave like they are smooth.

So, they are short range relative to at least the mid frequency range. And finally the third regime is where the object size is much larger than the wavelength. So, can anyone tell me where you have seen such a situation arise; it is very common light right?

Student: Yeah.

So, in light for example, the light with which we are seeing right now, the wavelength is how much? 500 nanometers, let us say what is the size of our day to day objects; on the order of centimeters meters right. We cannot see much less than I mean micron also we need a microscope right. So, there the wavelength is much smaller. So, the object I mean the field need not vary or change dramatically around an object because object size is so large.

So, what happens over here is you get this is a regime of optics; and you get ray like solutions right ok. So, we will be focusing in this course mostly on the mid frequency range for both the statics case and the ray optics case, there is a specialized theory which makes use of this assumption.

So, if you use these two assumptions you can simplify your mathematics a little bit more. You do not have to need, you do not have to use all the heavyweight machinery that mid frequency needs ok. On the other hand, if you understand mid frequency methods, it is much easier for you to understand the other methods because they are simpler version right. So, we will focus on this the mid frequency methods in this course any questions so far? Ok, it should be fairly clear till now.