Computational Electromagnetics
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Introduction to the Finite Element Method
Lecture – 10.11
1D Wave Equation: Matrix assembly

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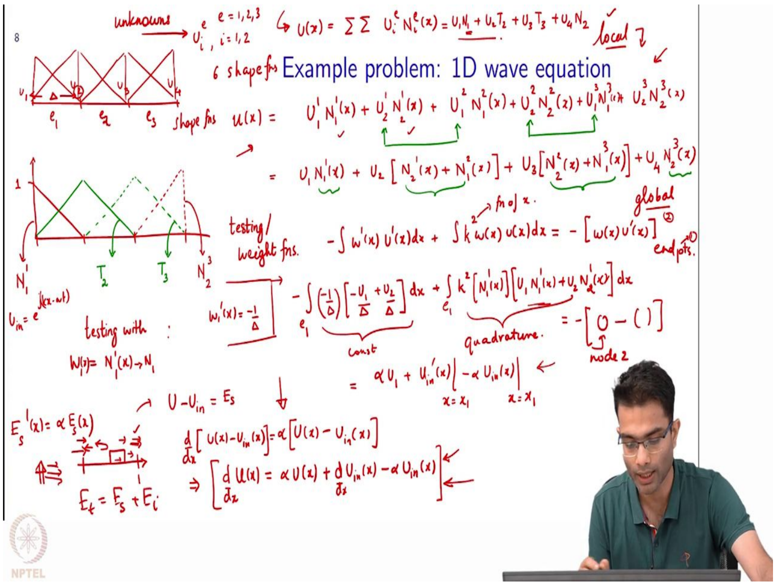

So, this is just this is the only complication that appears at the boundary points. You notice that the contribution from node 2 was 0 so; similarly you will find that the contribution from interior nodes becomes 0 ok. So, what you are left with is just this contribution from the end point.

So now, let us go back to this equation that we have over here. First term $U_1 U_2$, second term $U_1 U_2$, right hand side is going to give me one more U_1 and some constants right. So, in the equation form what will I write it as, I will maybe I can write it over here now.

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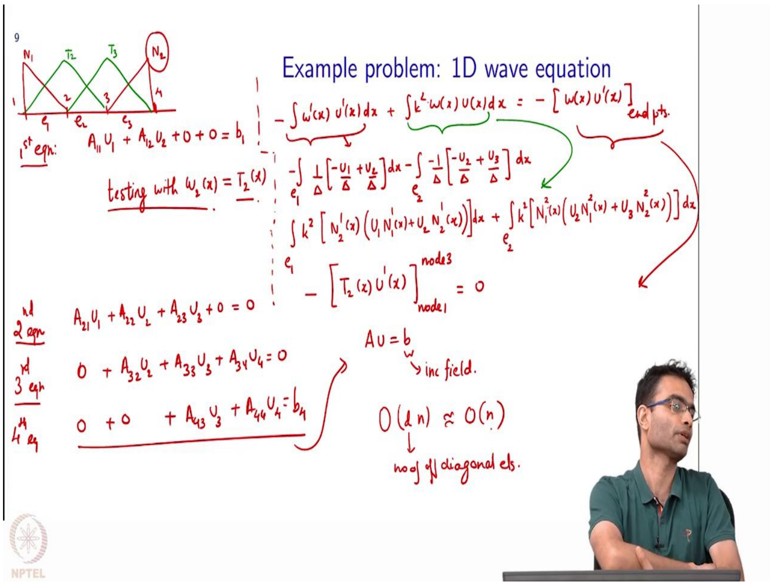

So, the first equation, it gave me some term $A_{11}U_1 + A_{12}U_2$, did I have any U_3 and U_4 terms? No, and did I have a constant term on the right hand side? That is my first equation ok. Now let us do testing with T_2 ok. So, we have to test with let us say weighting function 2, we did with weighting function 1 now we have to do with weighting function 2 which is $T_2(x)$ and that will give us the general idea.

So now, we will look at the first term $-W'(x)U'(x)dx$ then I had a $k^2W(x)U(x)dx$ is equal to something endpoints ok. So, minus let us write it properly all right. So, now, this is what we are setting. So, now I am going to put; so, now, T_2. What is the domain of interest for T_2 where it is non-zero?

Student: e_1.

e_1 and e_2 right. So, this is e_1 and e_2 so, w prime is going to be what W' in this segment e_1e_2.

Student: e_1 is (Refer Time: 03:21).

So, it is going to be plus and then.

Student: Minus.

Minus right; so, this will split into two terms. So, this there will be an integral minus over e_1, the derivative here is $1/\Delta\ W'U'$ which terms will appear go back over here U_1 which two terms will appear? We are integrating over e_1 this is your expression for u

Student: U and U.

U_1 will appear and.

Student: If it is same as before.

Same as before right; so, what will I have?

Student: Minus (Refer Time: 03:57).

$-U_1/\Delta$.

Student: Delta.

Right.

Student: (Refer Time: 04:00).

$-U_1/\Delta + U_2/\Delta$ by delta this is the first term plus sorry minus because the integration is continued over the second segment, I am only writing the first term of the integral what is the derivative now.

Student: Minus 1.

$-1/\Delta$ that is w' ok, what appears now?

Student: Minus U 2.

$-U_3/\Delta + U_4/\Delta\, dx$ ok, now let us take the next term. So, what happens over here; again, I have to do the integration over $e_1\ k^2$ right now this $w(x)$ is as I have written it before over here, it is going to be $N_{12}(x)$ and $U(x)$ over e_1. So, both of these terms will come from here as before right $U_1 N_1{}^1(x) + U_2 N_2{}^1(x)$ first term and another term which is coming from integration over e_2.

Student: e_2.

k^2, what term will come now this is segment 2 element 1; I mean shape function 1, what will I come now so, this is 1 2 3 4. So, now, I am in segment e_2. So, what are the two things that will come? $U_2 N_1{}^2(x) + U_3 N_2{}^2(x)$.

Student: (Refer Time: 06:13).

Right one, this is a second term ok. And then this third term over here what happens, w x u x so it will be a $-[T_2(x)\, u'(x)]$ at node 2 minus node sorry what will it be?

Student: node (Refer Time: 06:53).

Node.

Student: 3 and 1.

Node 3 and 1, yeah node 1 minus node 3 fine, what will this be; what is the value of $T_2(x)$ at node 3 at node 1 let us say? 0; right, this triangle this green triangle is starting from node 1 to node 2 and then going to 0 at node 3. So, at both the node points what is its value.

Student: 0.

0 so, very easy; so, we notice this with this right hand side term comes into play only at the end points because it gets cancelled like this is a clever choice of basis of testing function also right very good. So, now, having done this the second equation becomes, so notice this 1st equation over here what all variables does it have $U_1 U_2 U_3$ and they are all constants that are a function of x integrating them is trivial.

Student: Hm.

Second thing quadratics with k ok; so, it can be done by quadrature what variables does it have; $U_1 U_2 U_3$ is there any U_4 negative right. So, this 2nd equation over here becomes 0×0.

Student: 2 1.

$A_{21}U_1 + A_{22}U_2 + A_{23}U_3 + 0 = 0$

Student: (Refer Time: 08:36).

So, exactly. Now let us see the pattern 3rd equation.

Student: 0.

So, it will be 0 plus.

Student: 3.

$A_{32}U_2 + A_{33}U_3 + A_{34}U_4 = 0$ and 4th equation will give me $A_{43}U_3 + A_{44}U_4 = b_4$.

Student: plus 0.

Now the right hand side when I choose my N_2 over here in the final equation its value at the end point node is 1, so that is why I get a non- zero contribution over here. So, all of this put together is going to give me $Au = b$ and I have a non- zero b therefore, I will get a non- zero u also right. So, this in a nutshell is the method by which we solve this FEM in one-dimension ok. So, is it clear, what we did.

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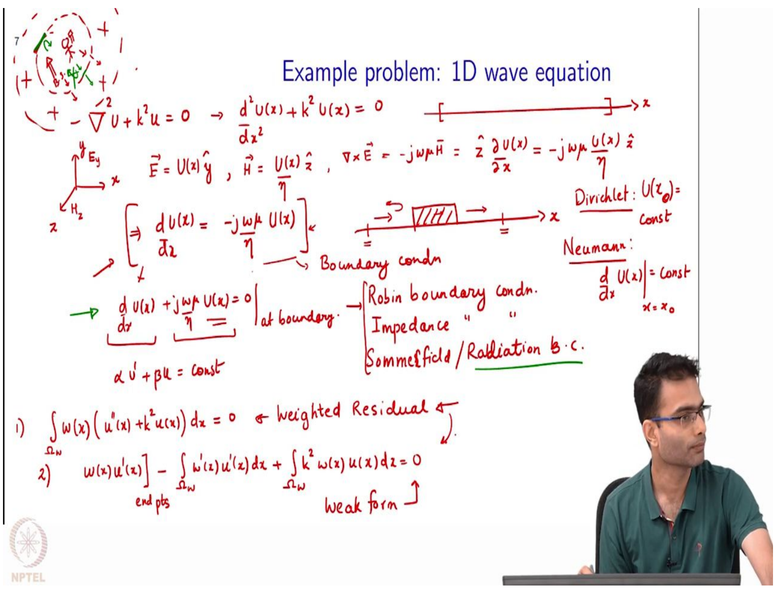

So, remember as starting point was the weighted residual form we converted that to the weak form, after converting to weak form we substituted the form of the unknown that I want to find out in terms of shape functions. The only little bit of cleverness I did is I chose my testing functions nicely. So, I got enough equations in variables. So, the endpoints shape functions I left them as it is and then whatever had a common node I combine those shape functions that gave me T_2 and T_3 these triangles. And that is one concept, the other concept was the boundary condition. I have the radiation boundary condition make sense only for something which is truly supposed to be outgoing and what is truly supposed to be outgoing?

Student: Scatter field.

Scatter field because E_{inc} will enter from one place and exit from another place. So, we should not apply any boundary condition on that, let's do its own thing. We applied on the scatter field and how do we get this scatter field? $U - U_{inc}$ right, so that is what this is the sort of crucial concept over here right. So, that gave me a condition for u' basically, that is what I wanted and that u' is what I substituted over here and that gives me a non-zero right hand side right. So, you can see all the incident field terms are going to appear here. So, trivial you can see if there was no incident field there is no scattering b is 0, u is 0 as you expect.

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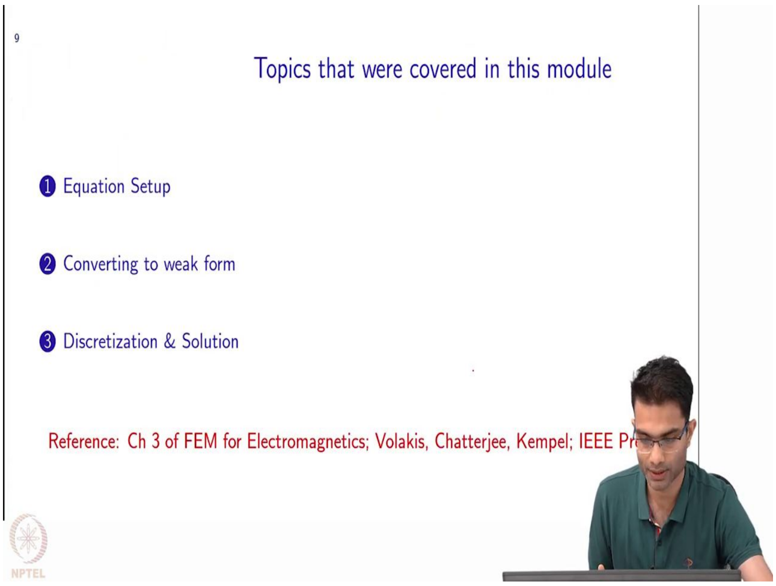

So, that brings us to an end of the 1D FEM equation.

Student: So, instead of these basis functions (Refer Time: 11:36).

Student: So, how do we can (Refer Time: 11:38).

You cannot use pulse basis functions because they do not satisfy the boundary conditions that we have.

Student: (Refer Time: 11:44).

Yeah, the first equation is e_1.

Student: Second.

Second is e_2 and e_3 sorry.

Student: Second is.

e_1 and e_2, the next is e_2 and e_3 and the next is e_3 alone, but I mean that came because of the domain of T_2; T_2 existed over 2 elements that is why I had to choose between those two elements.

Student: (Refer Time: 12:02).

So, one exercise which you should do is go back and try to use the 6th shape functions and derive this. You will get 6 equations and try to see how you actually they are basically only 4 equations, so very simple exercise. It will give you a little bit more clarity on the thing ok; I have given you the direct shortcut. And the reference book for example, which is chapter 3 of this book by Volakis, they do not do the shortcut they go through get a larger system of equations then do some algebra to reduce it further.

Student: (Refer Time: 12:37).

What is the complexity of solving this, so in this case it happens to be tridiagonal. In general it is a sparse system of equations. So, if you were to do LU decomposition to solve this complexity is simply order $w \times n$.

Student: (Refer Time: 12:54).

Oh, I should not use w what are the symbol will I use.

Student: (Refer Time: 12:58).

a b c d d; d has not been used d, d is the number of off diagonal elements ok. So, those of you have done linear algebra they you know that when you have only d number of off diagonal elements when you start doing LU decomposition, you do not have to do row eliminations many many times you start getting 0's very quickly only d times you have to do. So, that is why its orders dn. Typically, d is very small, d is 2 3 whatever and compared to n which is much large 1000's and 1000's. So, you will not find this order of dn written anywhere you will only find it as.

Student: Order n.

Order n right; so, compare this with MoM which is order n^3 that is why this is fast.