**Computational Electromagnetics**
**CEM:An Overview**
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**CEM: An Overview**
**Lecture – 2.1**
**Mathematical History**

So today's class is going to give you an overview of the field of Computational Electromagnetics. And what are the, what is the history of it? We need to know where is this field coming from, how old is it, how new is it and some of the places where this field will be applied to.

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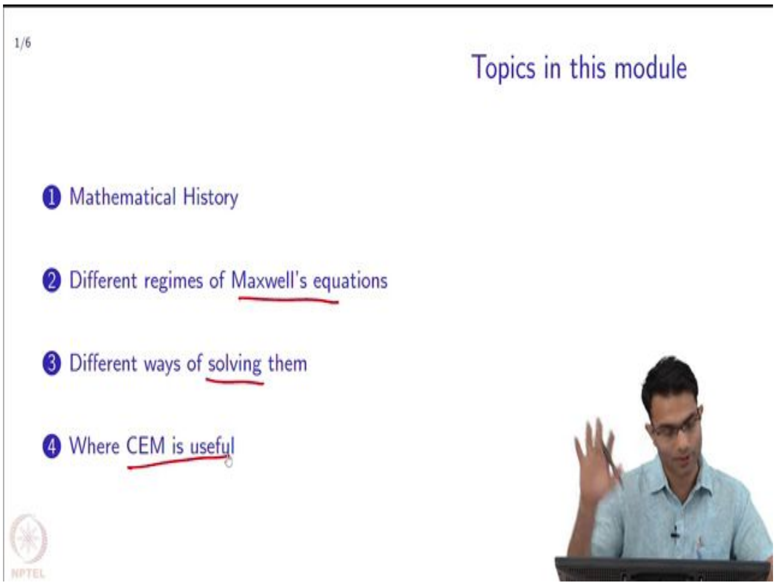


So, this is a broad outline, we will start with mathematical history of where this field has come from. Then the equations everyone knows that electromagnetic the equations are of are Maxwell's equations. So, there are several ways of solving Maxwell's equations that you would have heard of various different methods. So, when should one use which method right?

So, we will find out what are these different kinds of regimes of physics in which different methods get applied; you heard of finite element, you heard of a finite difference time domain, finite element, when should use which. So, that comes from some very simple concepts which we will talk about and also then how do we solve them. And finally, this is probably why you are in the class, why is computational electromagnetics useful.

So, some simple applications I will tell you about at the end of this course you should be able to at least read the papers in this area or look at a software in this area and understand what is happening. Many times you will I have seen that students they use commercial software, but they do not know what are the tick box over options that they are clicking for, what does it mean, what is physically happening, what is mathematically happening.

And that makes a huge difference. You know a simulation may take 1 day or 2 and it may take 3 days to run. If you did not pick your options properly; so at the very least you should know that and at the very most well you can think of there is a lot of scope for startup companies in this area, if you are able to write your own commercial software or your own software for solving these problems. So, that is the other end of the spectrum.

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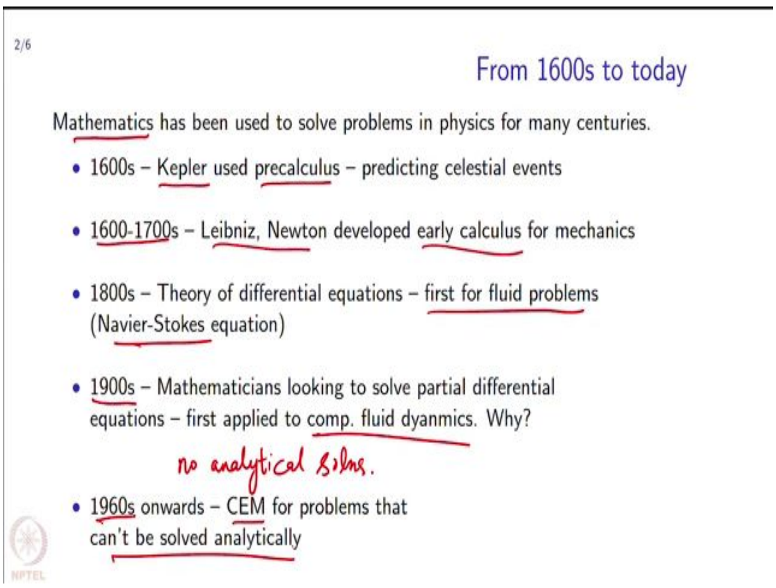


So, coming to the mathematical history of the field of computational electromagnetic; so I mean it is hardly necessary to say that, math has been used for solving physics problems for a long time. But at least in the recent modern history, you start with the Kepler who everyone is heard of and he used a sort of not calculus, but something predating calculus a very elementary sort of math to predict how planets moved. So, he was the guy who told us that planets move not in circular orbit, but elliptical orbits.

So, we all studied these Kepler's law in school right. So, that was pre calculus, then you know people began to make more detailed observations and not every observation match the existing theory. So, that in the next 100 years about 1600 or 1700s it was the birth of calculus right. So, that is jointly by Leibniz and Newton. So, that is early calculus and mostly for mechanics problems. So, again the motion of planets and such objects was the main interest which led to all of these laws being formulated.

So, that is where calculus comes in and simultaneously a lot of work was a lot of people were interested in how do fluids flow you know whether. Let us say in water in a river or in a pipe or whatever how is fluid flow understood. So, they needed the theory of differential equations for it and so the theory of differential equations came about first for fluid flow problems. And many of you will know that these equations are called the Navier Stokes equations. Now what is interesting about this Navier Stokes equation is that they are non-linear and they do not have analytical solutions.

So, you can imagine that that would have led to a lot of work starting from the 1900s. If you are if I tell you that a method does not have a closed form solution, there is no analytical solution for it, then what will you do? You have to solve it numerically right. So, the first effort in trying to numerically solve a differential equation; it came about in this field and that field became to became to be called computational fluid dynamics or CFD, that is what a lot of students in aerospace and mechanical engineering study. And it is sort of like the old saying that necessity is the mother of invention so they had no analytical solution. So, they had no choice, but to figure out how do we solve this numerically so.

So, Let us just write that down over here we know analytical solutions and then, the situation and electromagnetic theory was a little bit better. So, we look at electromagnetic equations they are actually linear, there is no square for electric field or something for the most 99 percent of the cases. So, when I have linear system of equations; it turns out there is a lot of good theory already built up for how to solve these analytically in some cases.

So for the most part what people do this, let us use these analytical solutions for most cases and make approximations were required to get back. Then around the 1960s is where the field of computational Electromagnetics get started, because now people get ambitious computers are there. Now I want to calculate more complicated problems for which there is no analytical solution. So, therefore, the need to now solid numerically comes about and you can see 1960s is also around the time when computers are becoming somewhat practical. It is not that a computer occupies a building a five story building, but it is getting that you can see that we can do this calculation.

Still it used to take lot of time they use to do things with punch cards and things which do our minds in 2018 seems pretty historic, but it is not this is 50 years ago that they were doing these things. And so again this is for problems that cannot be solved analytically, the problems that can be solved analytically in Electromagnetics are simple things like you know, you have a cylinder or a sphere or a plane how does a wave interact with this.

So, people have written analytical solutions, not that they are easy seems very complicated when you look at them lots of Bessel functions and this and that and we will see a lot of writing this course ah. So, that is 60s onwards is when this field as come over, so you can see it is not a very very old field. If you look at the early papers that mentality was still there that; let us get analytical solution. So, you will find papers were long equations are there, you will not find that in papers in the last say 20-30 years, so that is just something to keep in mind.

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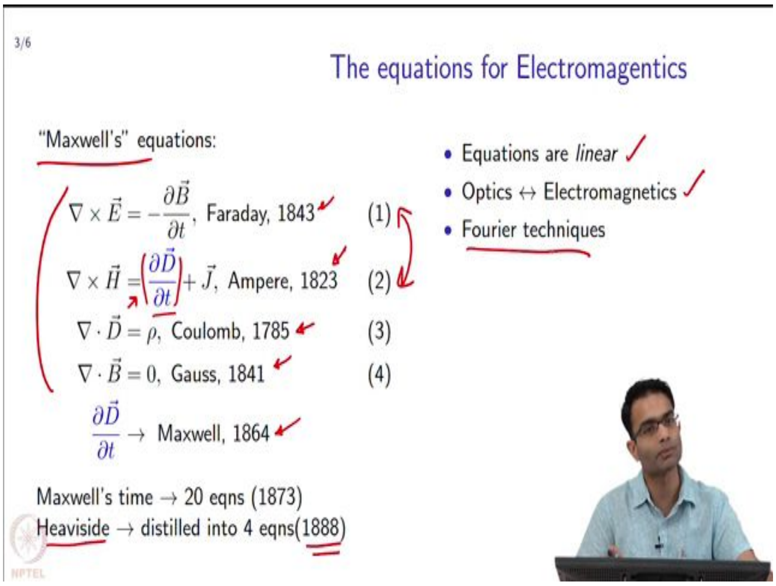


Then let us come to the equations of electromagnetics. So, we will have a separate review lecture for the equations of Maxwell, but this is again part of history, so which you know as any student of electromagnetics should know. So, what I have written over here? These are the four modern equations of electromagnetics, but the story was not always this way. So, when we started you the oldest law was actually by the Coulomb's law which we have all studied in high school right. I have a charge what is the field around it $q/4\pi\varepsilon r^2$ those kind of things. It comes from Coulomb's law which is 1785 and at that time electricity and magnetism. These were thought to be two different things right.

Then comes along the work of Ampere without the displacement current term so, Ampere had this $\nabla \times H$ and a $J$ term and that was Ampere circuital law right. So, for 1823 that is what was known. Then you move forward Gauss began to study magnetism. So, we had this $\nabla \cdot B = 0$ equation it came. And, then Faraday finally, had this an equation which related the electric field to the magnetic field, but still it was not these were at that time these equations were not just 4 equation, but they were written in some different format there were plenty of them. And it is actually Maxwell who came in 1864 who said that these equations; there is an inconsistency in them.

And that inconsistency actually is simple enough for us to see and we will have that as a problem in the homework, that inconsistency is fixed was fixed by him when he added this one term is, $dD/dt$ the displacement conundrum, when he adds this term to it all of these 4 equations they become consistent. And the unification of electricity and magnetism happens. So, even though none of these 4 equations has his name. It is because of his singular contribution of that displacement current term that all of these equations in today's they are called Maxwell's equations.

Again at the time of Maxwell's these were 20 separate equations. This is not how Maxwell wrote, Maxwell wrote them in some slightly you know elaborate form. Then there was this scientist by the name of Heaviside who condense these 20 equations finally, into 4 equations which is what we have been studying since 1888 right. And why have these equations been so successful any anyone wants to take a guess hm.

Student: (Refer Time: 09:45).

It explains light it survived one major test what happened in the early part of 1900?

Student: Relativity.

Relativity right; so, what these equations they turned out to be consistent with the theory of relativity as well.

So, that is why so these equations survive that test also and they have been very very successful. There are versions of this which you can use for quantum mechanics also. So, quantum optics is a very interesting field as well; those equations look a little bit different, but the idea is the same. So, as you can look at these 4 equations they are linear there is no E square there is no E into H right there all by themselves in a linear form. So, these equations are linear and that is why people were able to solve them analytically for a large part.

Adding this term over here, this d D by dt is what led to people realizing that optics and electromagnetics; these are not two different theories, but the same theory. So, we will do this again later on in the course if I take these first two equations and I solve them together outcomes the equation of a plane wave or a wave. And that explained why light is able to travel at the speed of light from the sun to us and things like that. And, keep in mind these equations when they were formulated nobody was thinking about a wave of traveling from you know sun to earth or from some pulsar to earth; these equations like Faraday's equation or amperes equation it was an equation built on circuit laws right.

Circuital law it was a current going I calculate how much current is threading through this. So, its quite interesting that things that are working with the resistors and capacitors, a law that is built on that somehow is able to explain how light travels from the sun to earth. And that was really the power of this displacement current term over here which Maxwell was able to do right. So, the linking of optics and electromagnetics was made possible by Maxwell. The other thing is that these equations being linear there is a large set of techniques which have already been built by Fourier right.

So, I can take Fourier transforms and Fourier transforms they give us a new set of techniques to solve equations right, at the end of the day they are techniques. So, Fourier techniques can be applied to solve Maxwell's equations which also explains why people did not need to solve them computationally for a long time because. So, many tools and techniques were available.

So, Fourier techniques we will study them what is the one great advantage of Fourier techniques is, if I know the response for a few frequencies I can do an inverse Fourier transform and find out the response in time. So, these kinds of playing between frequency and time become possible. As you see it these equations do not have frequency directly anyway right. So, that is why Fourier techniques become useful to go into the frequency domain. So, we will look at those when it is relevant.