**Computational Electromagnetics**
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**Introduction to the Finite Element Method**
**Lecture – 10.01**
**History & Overview of the FEM**

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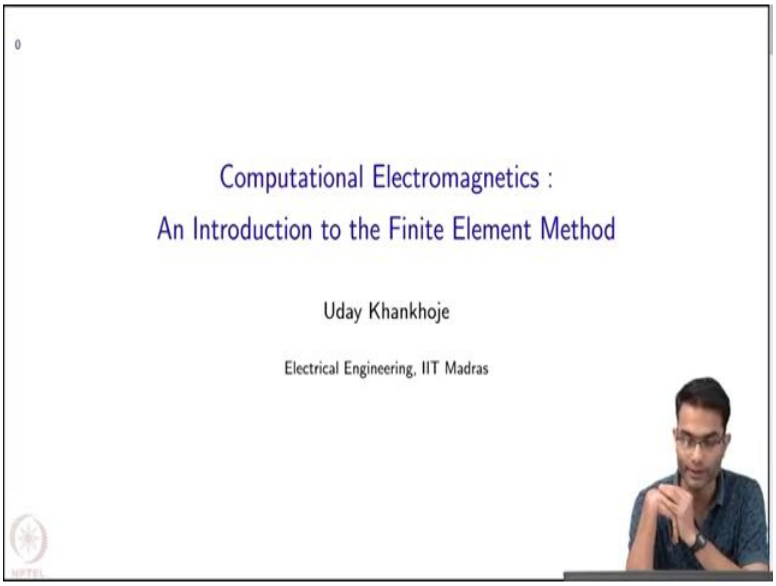


So the next module that we will talk about is the Finite Element Method ok. So, finite element method is; you've all heard the word right finite element method it is a standard method in many branches of engineering ok.

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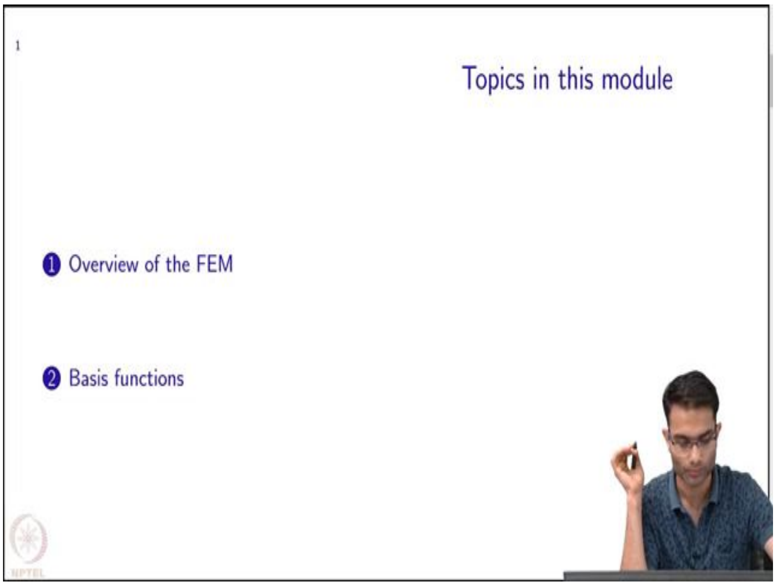


So, we will talk about the history of this field which comes under the overview right.

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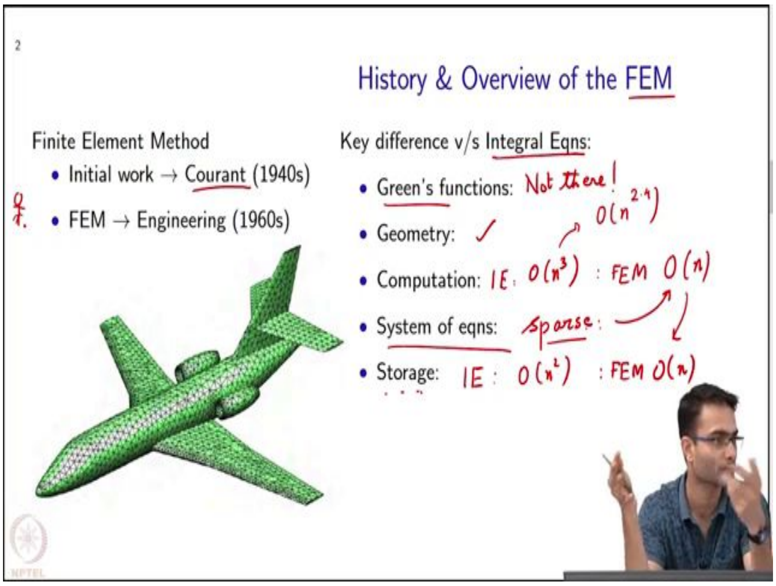


So, the short form for finite element method everywhere you will find is FEM and it is a relatively recent method. In the sense that it is a not very very old, partial differential equation of course, hundreds of years old. The famous mathematician Courant is the person who began to work on it in the 40s again for solving partial differential equations. And the engineering community began to take notice of it only by the 60s ok. So, that is what I mean when I say it is relatively new and finally, what is it? I mean it is a method of solving Maxwell's equations with prescribed boundary conditions, which is what the earlier model was also about integral equations was also the same thing.

So, naturally we want to find out what are the differences compared to what we already learnt ok. So, in integral equations which we looked at, one of the very difficult things to deal with mathematically was Green's function because, it has singularities and all of those things right. So, the good news about Green's functions in FEM is that not there, we do not need to know Green's functions for FEM in fact, FEM is formulated without Green's function.

In fact, it is not an integral equation ok. So, the starting point itself is different, from Maxwell's equation you do not convert all the way to an integral equation and then solve you stop at the differential form itself and then solve and we will discuss in detail about it. So, this is one big advantage because green functions are actually very difficult to calculate in heterogeneous objects and so, on. Then geometry this is one of the major plus point of FEM, it allows you to deal with many complex shaped objects. So, like this aircraft that has been shown over here.

So, these are actual simulations of you know if you want to find out the radar cross section of aircraft of something, this is how you would break up the domain. So, here they have made these triangles all over the surface and these are conformal to the object ok. So, takes the shape of the object. So, it allows you to use a standard CAD software to mesh whatever complex object you want and work with. So, it gives you a lot of real life functionality ok.

Then something that we will look into details I will as we go along the computational cost is very very low compared to integral equation methods. So, integral equation methods say you got a $n \times n$ system of equations, what was the complexity of solving integral equation methods? Those of you done linear algebra $n \times n$ system of equations what is the complexity of solving them?

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$n^3$ right. So, integral equation was $O(n^3)$ right. In FEM this turns out to be almost $O(n)$. So, how is this great feat achieved? :Linear you get a linear system of equation and, but you can solve it in $O(n)$ time. So, what might be the trick?

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There is a speciality of the A matrix or the system matrix and that is the next point the system of equation that you get is actually sparse ok. So, question is when I want to calculate the radar cross section of this aircraft, will you get a point for each. Student: Yes, the value for each.

A value for each triangle at; no; so, the radar cross section is something that is defined for a observer far away is an integral over every little-little triangle over here. So, every little little triangle is an unknown that you are solving for and that forms the system of equations. Once you get it finally, this guy absorbs a number one value. So, this sparse system of equations is what allows you to get this order n. And, this also tells you that if I wanted to simulate and solve for a real life very complicated aircraft or ship or something, integral equation methods they are just impossible to work with because they take so, much time $O(n^3)$ versus $O(n)$.

So, I must point out this $O(n^3)$ is for you know your text book LU decomposition or whatever. The state of the art for this is something more like 2.4 roughly. Research level thing yeah. So, if I mean $O(n^3)$ is textbook stuff, but you can do better, but not as good as $O(n)$. Then the storage of the system of equations in the case of integral equations was storing a $n \times n$ system you need how much memory? $n^2$ right. So, storage in integral equations was $O(n^2)$ whereas, in FEM because I have a sparse matrix n equations each equation is sparse means very few entries are non zero. So, the number of entries are FEM is again $O(n)$.

So, both in terms of storage and computation FEM is a big winner. So, I mean looking at this naturally you should ask a question if everything is so, great about FEM, why did we bother studying this integral equation method at all? So, the answer is that the accuracy that you get with integral equation methods is much better. So, to get the same level of accuracy you would need to discretize finer in FEM than integral equation method ok. And, we will see that we will see a very good conceptual reason for that as we go along that why is it that way.

So, that is the price you get for a sparse system of equations there has to be there is no free lunch and that is why you lose out. So, your a lot of your commercial software for example, the most popular commercial software is for example, ANSYS or HFSS which is called it is based on the finite element method. Now, they have some more modules which allow you to do integral equations here and there. But, the way you are able to really work with very large electrical objects is FEM because memory and speed are reasonable.