Computational Electromagnetics
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Method of Moments
Lecture – 8.11
Volume Integral Equations: Summary

So, the system of equations comes from.

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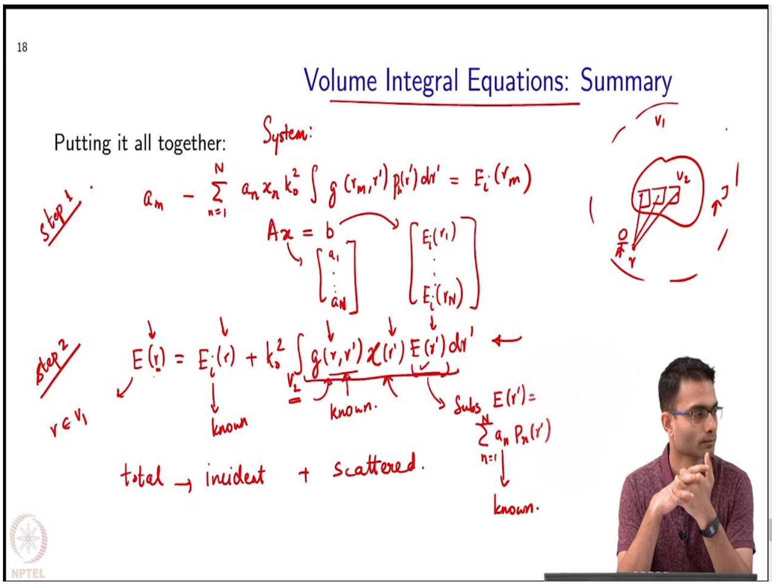

So, I have $a_m - \sum_{n=1}^{N} a_n \chi_n k_0^{(2)} \int g(r_m, r') p_n(r') dr' = F_i(r_m)$. So, this gives us full system of equations of the form $Ax = b$ right.

The x is the column vector of a_1 up to a_N and b is a column vector of the incident field at various points $E_i(r_1), \ldots, E_i(r_N)$ ok. So, now, we know pretty much everything if I want to find out now this was my original problem over here; V_1 and this is the current source over here I am standing over here this is my location r what is the field that I want to measure. I will write $E(r) = E_i(r) + k_0^2 \int_{V_2} g(r, r') \chi(r') E(r') dr'$.

Now, I know this right these integrals I know now; I know everything because I was in the previous step, I have solved and I have got my E inside the domain right this integration was all V_2. So, this whole thing is also known. So, what I have got is incident plus scattered is giving me total.

Student: Now V_1.

Yes, in this expression what this fellow over here is $r \in V_1$ and r' of course, the region of integration is shown to be V_2 ok. So, this is the second step right. So, this was step 2 and what we did over here was step 1.

Student: Then we will ask the integration E over e c e r W of r prime right.

Yeah $r' \in V_2$ that does not change.

Student: E 1.

So, the confusion is that the equation is actually the same I have the same equation for step 1 and step 2; the only difference in these equation I mean the equation is the same what I am changing is r, r is appearing here, here and here this never changes when I wanted to solve for $E(r')$ I need it. So, here $r' \in V_2$.

So, I have to make sure that r over here also belong to V_2. So, in step 1 I made r belong to V_2 in step 2 I make r belong to V_1 because that is my region of interest. But inside this never changes, r' is always belonging to V_2. So, now, I know everything and in fact, in evaluating step 2, there is there is no problem of singularities because.

r is always outside over here. So, you are always looking at the distance from this pulse then this pulse then this then this pulse, this is the region of integration no singularities.

Student: Even if 0.

Well yeah if you put r inside V_2 you will get back one of the equations you are.

Student: Yeah.

Already used. So, it is a redundant to do that ok. So, this was a very quick run through of volume integral equations I went a little fast because we know all the building blocks. We knew the Helmholtz equation we knew Green's function right and when you how to deal with singularities go over the three building blocks which we have buy now very comfortable with ok. So, these volume integral formulations as I said in the very beginning, when my problem is actually heterogeneous this is what I will use; the incident field is going to be ah. So, the equations are the same step 1 step 2 the equations are the same.

So, wherever you put r you calculate $E_i(r)$ whether it is in V_1 or V_2 it is a numerical value right. So, does not matter. So, it will be different in step 1 and step 2 because in step 1 r is belonging to V_2. So, you will be finding out E_i inside V_2. In step 2 you will be finding out E_i at the location of the observer.

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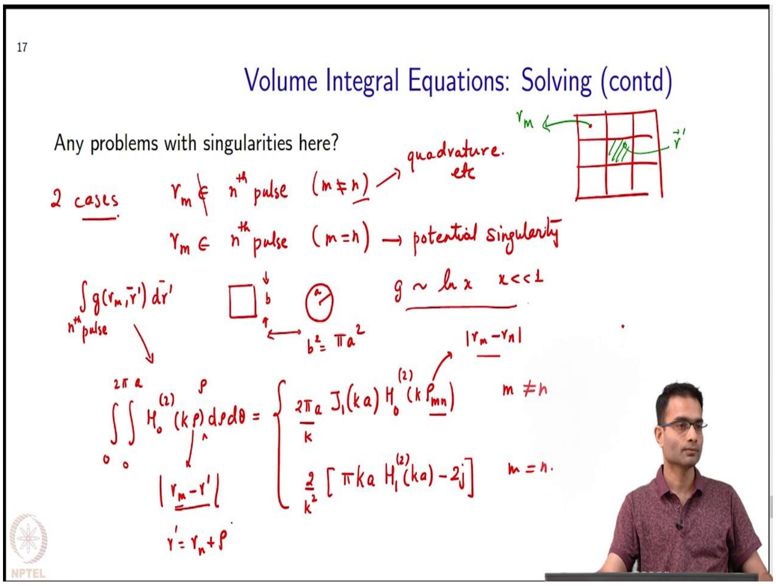

Student: So.

So, question about the grid over here.

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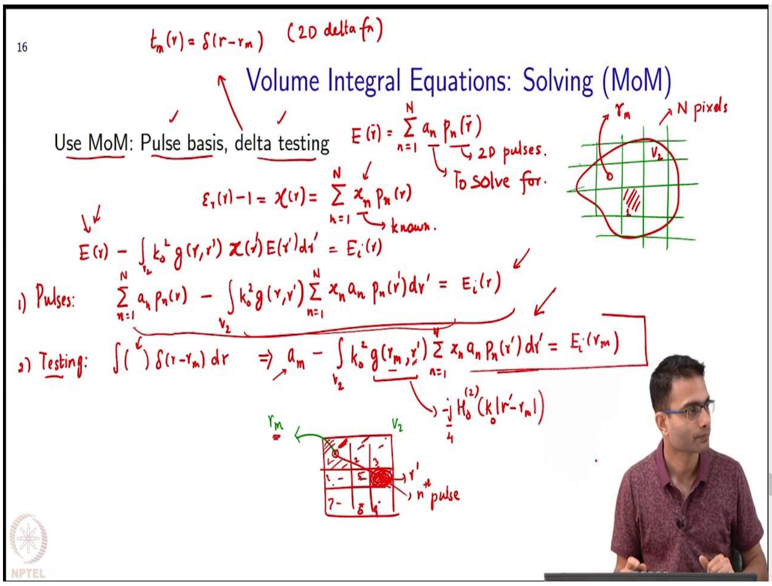

So, quick clarification about this grid over here what I my do so, let us look at this equation over here and in particular we are looking at this integral. So, $g(r,r')$, when I do the testing it becomes this expression right this you agree. So, here in this Green's function r_m is being held constant because the region of integration is I mean the variable of integration is my prime coordinate.

So, r_m is fixed over here, r' is allowed to run over breakup the summation for N pulses. So, 1 pulse after the other. So, when I take let us say the 6th pulse over here r' varies over this pulse. So, what is there is no singularity to worry about. So, $g(r_m, r'), r_m$ is fixed r' is varying over this pulse. No problem I calculate this, but this summation goes over all the pulses. So, at some point it will come back to pulse number 1 where r_m or r_1 lives.

Student: So, r' is only varying or a particular pulse.

r' is varying over the entire V_2, but entire V_2, were split up into pulses. So, I have to go pulse by pulse that is how I split this up yeah this is the last slide yeah so in step 2.

Student: Yeah then when you calculate the integral.

Yeah.

Student: So.

So, here exactly here is where we will substitute $E(r') = \sum_{n=1}^{N} a_n p_n(r')$ now this is known.

Student: So, after the basically you can know at the newer part a taking (Refer Time: 06:47)

At any point after this you can know $E(r)$ at any point.

Student: So, we assembly say square integrated approximation (Refer Time: 06:55)

Yeah. So, you use the same square grid, square to circle grid approximation, you do not change the where you would evaluate this integral right. The only thing that changes as the observer location changes is this Green's function, that is all let you have to change.

Student: Yeah and that fine.

The chi they will not change, the chi belongs to the domain. So, the χ we will remain the same its a its nonzero only in V_2. So, that we will remain the same the only thing that is changing the only thing that depends on r in this expression is this guy.

Student: That ρ I mean when comes.

Yes that is why the ρ I mean comes in.

Student: Comma r in.

No so here this $\rho = |r_m - r'|$ after I have evaluated the integral, it turns out to be ρ_{mn} is equal to this. This is the integration here right now r' is varying. Right and I intermediate into a new scalar variable, I called it ρ. So, this became this.

Student: So, here χ_n is known then a.

χ_n is known.

It yeah you; so, I can write this in terms of $r'dr'd\theta$ and then gets converted to $\rho d\rho d\theta$. Remember here this is a vector. So, 2 dimensional integration I have broken it down into 2 scalar terms ρ and θ in polar coordinates no the limits of integration has a same regardless of which pulse of and because have to integrate over the whole pulse.

Student: (Refer Time: 08:44).

No. So, when see over here r_m is being held constant right; so, this.

Student: So, r' we have writing it.

I see yeah. So, r' yeah. So, over here it looks like I am integrating over the same pulse yeah, but there will be an offset in this depending on which pulse I am in right.

Student: It should be offset inside the Bessel function.

It should be offset inside the Bessel function itself.

So, it will get I mean so its all in here.

Student: Yeah.

Right. So, you can think of $r' = r_n + \rho.....$ something like that.

Yeah that is yeah. So, those intermediate steps we have skipped in between.

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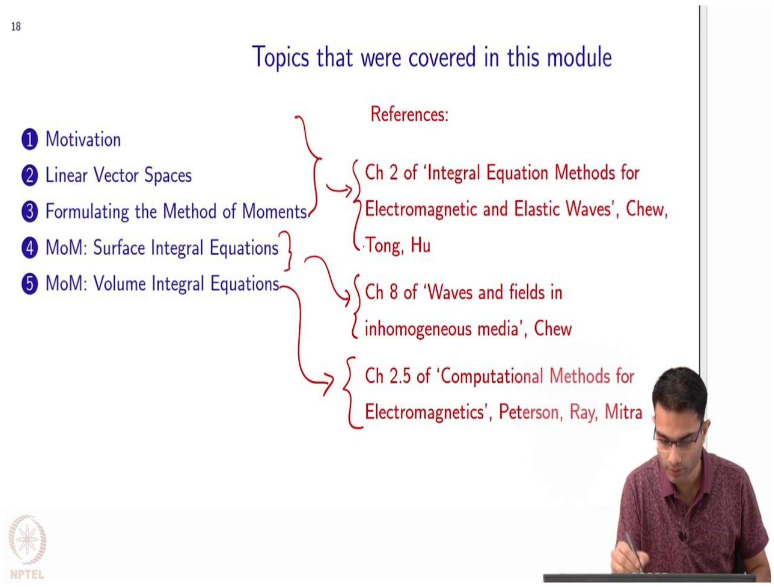

So, let us these are the references that we have this is particularly chapter 2.5 of Peterson's book is a very nice book, surface integral we had chapter 8 and for the basic theory we are referring to this cases ok.