Computational Electromagnetics
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Method of Moments
Lecture – 8.1.0
Volume Integral Equations: Solving – Part 2

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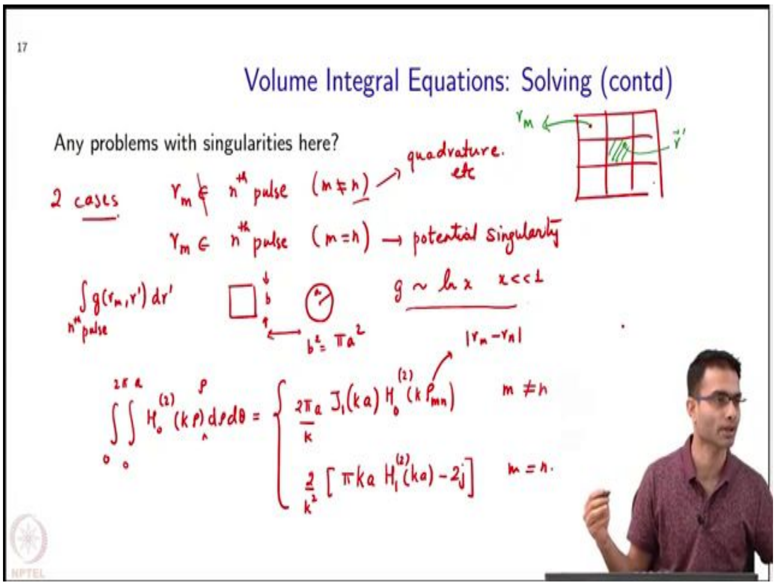

Exactly ok, so, we are coming exactly to that. So, the question is what happens when you integrate over a pulse where the Bessel's function goes to 0 right. So, this is my r_m and in general this could be some point over here which is r' ok. So, there are you know 2 cases possible. So, I can say that r_m does not belong to the n th pulse and r_m belongs to the n th pulse.

If you wanted very simply you can say $m \neq n$ and m=n. These are the 2 cases that we have to worry about. So, the first case what you think it is a finite integrand being integrated should there be any problem no right. So, we can even do quadrature rules or whatever. Second term is where there is a potential singularity potential right that is the keyword over here. Intuitively do you think this will be a problem what is, am I integrating g or ∇g?

Student: g.

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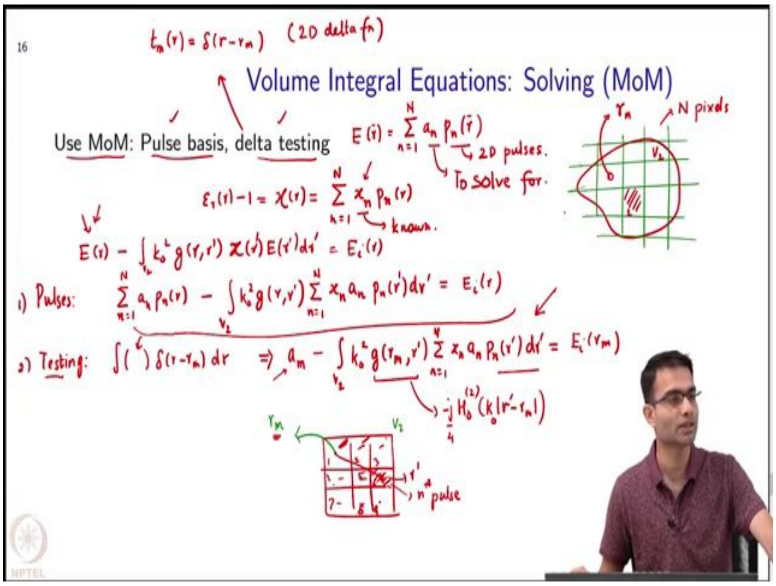

Right this integrand over here am I integrating g or ∇g?

Student: g.

g right and g is of the form for very small x it is of what form?

Student: $log(x)$.

1/x or $log(x)$?

Student: $log(x)$.

So, it is the of the form $log(x)$ for x very small.

Student: It is integrable.

So, it is an integrable singularity right. So, in this case I do not have to worry about.

Student: Singularity.

Some kind of a principal value integral and all of those complicated things which I had in the surface integral case. Remember the surface integral case the problem came from ∇g not g ok. So, intuitively we know that this is not going to be very difficult ok. So, that is one more advantage of volume integral equations is that your avoiding that the real singularity is being avoided. So, you have a integrable singularity over here. Now we can go ahead and do this integration and you can either use quadrature rules.

But in this case sort of nature has been even kinder to us the integral of. So, what I am integrating? I am integrating this Green's function $\int g(r_m, r')dr'$ essentially this is the integral right over the n th pulse this is the integration that I have to worry about. Now this the way I have drawn these are square boxes of let us say size b. Now it turns out I am just giving you information that the integral of this Hankel function over a square box there is no closed analytical form for it. Not surprising because its a Bessel's function.

However, if I do the same integral over a circle, it turns out that there is a closed analytical expression itself for the integral ok, but we discretized it into squares.

And we know the answer for a circle. So, the approximation made is approximate your circle sorry your square by a circle of the same area.

So, that you get a closed from expression. So, that you do not have the error of quadrature when I do quadrature integration there will be some error.

I get rid of that error and I replace it by a new error which is approximating a square by a circle. That error will go away as a make my discretization very very fine. So, that is the trick you do not have to do it, but we it is good to have closed form expressions. So, the area should be the same. So, b squared should be pi a squared ok. So, that is what I will do and once I know this it turns out I will just write down the final expression you can do this integration yourselves looking at the handbooks.

So, $\int_0^{2\pi}\int_0^{a} H_0^{(2)}(k\rho)\rho d\rho d\theta$.

So, I will write down the 2 expressions. So, the first expression is for $m \neq n$. So, this is $\frac{2\pi a}{k}J_1(ka)H_0^{(2)}(k\rho_{mn})$ .. So, you notice the radius of the cylinder is appearing ok, this is one expression. The second expression is where the singularity at the alleged singularity appears and the expression is again there is a closed form expression $\frac{2}{k^2}[\pi ka H_1^{(2)}(ka) - 2j]$.

This is just information we can we can actually go and show this integration its not very difficult ok. So, this turned out to be so much more simpler than the surface integral equations because no singularities to worry about really about.

$\rho_{mn} = |r_m - r|$

Rho m n is the distance.

Yeah. The distance between the centers of the squares or circles that is ρ_{mn} right ok. So, what are you doing we take we take 1 guy fixed integrate over integrate over all of these pulses right. So, I get 1 equation repeated for all 9 testing points I get 9 equations 9 variables ok.