Computational Electromagnetics
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Method of Moments
Lecture – 8.8
Volume Integral Equations: Setting Up

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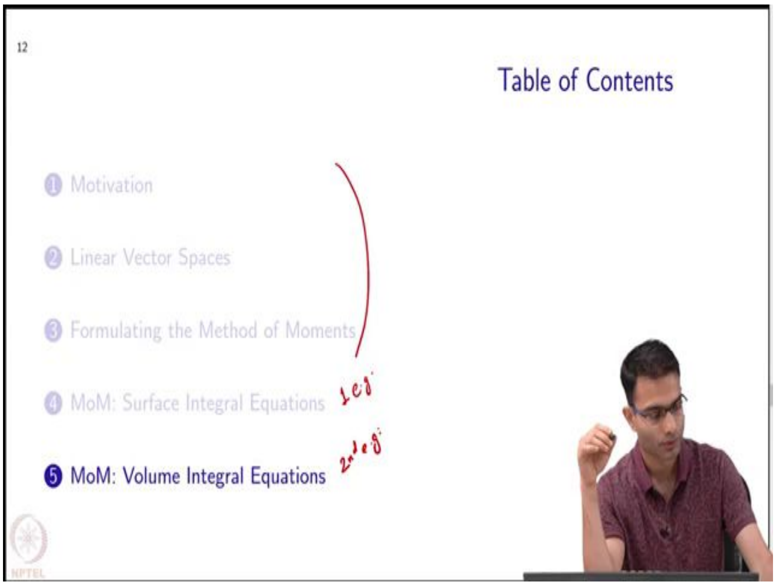

What we have seen so far is when we started with the mathematical formulation of method of moments, the first example we took was the surface integral formulation. The 2nd example is going to be related complementary method which is called volume integral ok.

So, we I will explain when 1 is needed and when one when the other is needed ok.

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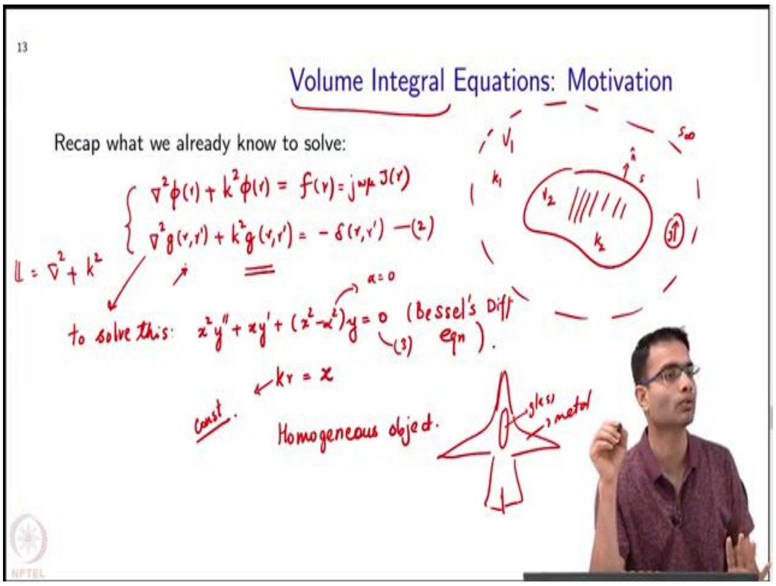

So, let us sort of start by summarizing what we already know ok, what are we already solved. So, what we have I am going to a draw our favourite diagram once again right. This was our V_1 this is our V_2 and some current source over here J right that is our problem I have call this as infinity and this was source s ok.

So, what kind of equation did we write for these 2 regions there were the wave equation right. So, after I simplified everything I had something like in either region and the where did the current source appear? There was a current source J over here. In these equations where did the current source find an appearance which of the terms contains the current source?

Student: $f(r)$.

$f(r)$ right. So, $f(r) = j\omega\mu J(r)$ right. So, we just absorbed it into $f(r)$. So, this equation we have seen it many times when now we I will call this it is like a Helmholtz equation and then we said how do we solve this. So, to solve this we said we invented one new function we called it the Green's function right.

So, I call I said. So, I said if I can find out this Green function I can solve this problem right and that was the impulse response idea. But notice one thing that was required in both of these equations is that the operator that I have on the left hand side that for both the equations is the same right and that operator is what it is like $\nabla^2 + k^2$. When the operator is same I can use the Green's function ok. Now when we went to solve for this Green's function, what did we do the steps briefly?

We converted it first to 2D polar coordinates I wrote down the Laplace in the polar coordinates and after some amount of algebraic manipulation, what did I end up with? I ended up with the differential equation right. So, and that differential equation looked like a well known differential equation which is? Bessel's differential equation right. So, when I went to solve this I got something that looked like $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$ right that was what I called Bessel's differential equation right. So far so good and in our case turned out α was what.

Student: 0.

$\alpha = 0$ that is why our solutions was $H_0^{(1)}$ and $H_0^{(2)}$ so, so far this just a revision. Now this equation over here let us call as equation 3 and equation 2 right. Equation 3 did not have any k term over here correct there is there is only x, y and α which is 0, but our original equation 2 had a k. So, we had to do one more manipulation and that was.

Student: $x = kr$.

Right. So, I had the substitution that $x = kr$ right the x is absorbed into it what is implicit over here was is a k is a constant right. So, that is why I could do the substitution easily it was a very simple change of variables right. So, here what we did without really being very explicit about is that k is a constant right. So, if I ask you what is the physical meaning of k. So, there is some value of k over here we called it k_1 and some value of k over here I called it k_2. For this approach to work what must be true about k? We have seen that to get Bessel's solution out of this it should be a constant right.

So, k_1 let us say a vacuum, k_2 whatever material, but implicit because I want the same solution strategy to work is that k should be constant therefore, this in what we have done so far everywhere over here in this region k_2 is constant. So, what is what kind of an object is that? Homogenous object right. So, right now real life objects will they always be homogenous; obviously not right think of your things that your calculating the RCS of some you know aircraft over here right this is some aircraft there is some cockpit over here.

So, you will have some you know glass over here, some metal over here right different different components are there. So, this is all realistic objects are going to be heterogeneous. So, can I apply this approach that I have discovered so far this surface integral approach can I use it to this kind of a problem? Well answer is yes and no yes, if I can break it up into small sub regions each of which is constant k. So, I can say as long. So, I can take all the metal parts and say this is.

So, instead of volume 1 and 2 I may have volume 1 2 3 4 up to n then I will have. So, many different surface integrals each of those I will have to solve put boundary conditions and solve it that is one approach, but that will break down if for example, my my material has some continuous variation in refractive in refractive index permittivity whatever you want to call it right. So, for an object is truly heterogeneous then this approach we can see its doomed to fail that is where this other approach comes which is volume integral. So, as the word suggests surface integral formulates the equations around the surface volume integral goes into the volume and solves it.

So, advantages I can model more difficult objects which have heterogeneity and what what do you think with the with the disadvantage be or the down side?

Student: More computation.

More computations because.

Student: (Refer Time: 07:17).

When will we will talk about Green's function? The computation number of computation as I take a surface area will have more number of elements to calculate then just the line around it. So, computation is going to increase, but that is a price that I have to pay for solving this. So, the question about what about Green's function?

Student: (Refer Time: 07:40).

Right. So, we have a Green's function that where using over here for the surface integral formulation, we are keeping we are going to continue to assume that is the 2D problem that we are solving. So, we're not going to 3D ok. So, our difficulty is we do not know in closed form Green's function for a heterogeneous medium that is our problem. I am not saying its not possible to find it there will not be a closed form solution there will be a numerical solution ok.

So, we want to avoid that as much as possible and use what we already know and what we already know is a homogeneous Green's function. So, the question is can I still use that somehow and get some accurate way of calculating it ok. So, we will use the Green's function, but the how to get it is little bit tricky, so, we will get to it.

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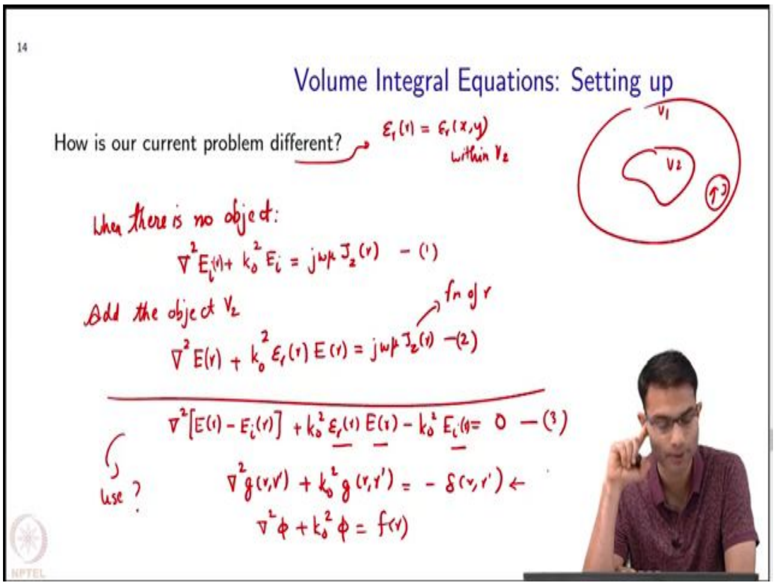

So, how the current problem is different as I have already said that $\varepsilon_r(r)$ is actually a function of space within V_2 right. So, the V_2 right.

So, within V_2 this is varying as a function of x, y. So, now how do we sort of how do we continue to use our approach over here? So, let us use what we already know ok. So, what we already know is the wave equation right. So, let us say when there is no object ok. So, I have the current source over here and there is no object over here. So, what will my Helmholtz equation be?

So, I will have a $\nabla^2 E_i(r)$ because there is nothing to scatter the field. So, the total field is just the incident field produced by the source right then next is I will write $\nabla^2 E_i(r) + k_0{}^2 E_i = j\omega\mu J_z(r)$. These guys E_i is also function of r I am just not writing it over here. So, this is our equation in the absence of object. Now when I add the object now let us add the object in this case it is V_2 so, what happens now?

I will say the same operator remains similar ok. So, $\nabla^2 E(r) + k_0{}^2 \varepsilon_r(r) E(r) = j\omega\mu J_z(r)$. The permittivity of the medium enters into this k term. So, it becomes the relative permittivity comes in a way. What about the right hand side does it change? No right the current source is still there ok.

Student: (Refer Time: 11:24).

In V_2 it is 0. So, even J is a function of r. So, in V_2 the current source is 0. So, whenever you are asked to evaluate this. So, take a value of r let us say r is in V_1 then what will happen? Then the relative permittivity will be 1 that relative permittivity of vacuum is 1 and current source is present. I evaluate take r inside V_2 what will happen? I will have relative permittivity to be different from 1 and current source will be 0 right. So, both are functions of r now is the most general thing the good thing is that I do not have to specially say this is valid for region 1 this is valid for region 2 its a general equation.

So, typically I do not know the current source and I do want to get into your details how the current is flowing in some antenna somewhere right. So, what can I do with these two equation to get rid of current source subtract right. So, if I do that what do I get is. So, $\nabla^2 (E(r) - E_i) + k_0{}^2 \varepsilon_r(r) E(r) - k_0{}^2 E_i = 0$. So, this is an equation which has it has pretty much everything we want it has the incident field.

It has the object information and it has the variable that I want to find out that I want to solve for it has everything and I have eliminated this current source. Actually the influence of the current source has been absorbed into the problem in which term? The influence of the current source has made its appearance in the incident field right. So, it is not that have completely removed the current source, so, this is the equation that we have. Now can we solve this equation using what we already know which is the Green's function over here can I use this what do you think?

So, in order to use this Green's function over here I should have another equation of the form what? Of the same form correct does the left hand side look like that of this equation over here equation 3 ok. So, $E - E_i = \phi$ right, but what about the second term the term that accompanies ${k_0}^2$ is it in that form yet its not right. So, I cannot immediately use this equation 4 to solve equation 3, but does it look impossible to do?

Student: No.

No right it is just some simple manipulation that I have to do.