Computational Electromagnetics
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Method of Moments
Lecture – 8.4
Surface Integral Equations: Recap

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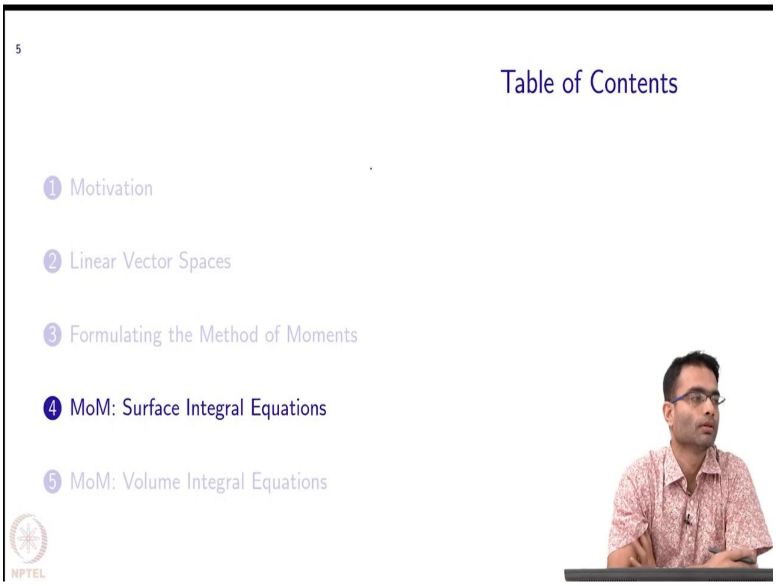

So, we will continue with this module on the method of moments and look at the next section which is applying this method of moments to Surface Integral Equations ok.

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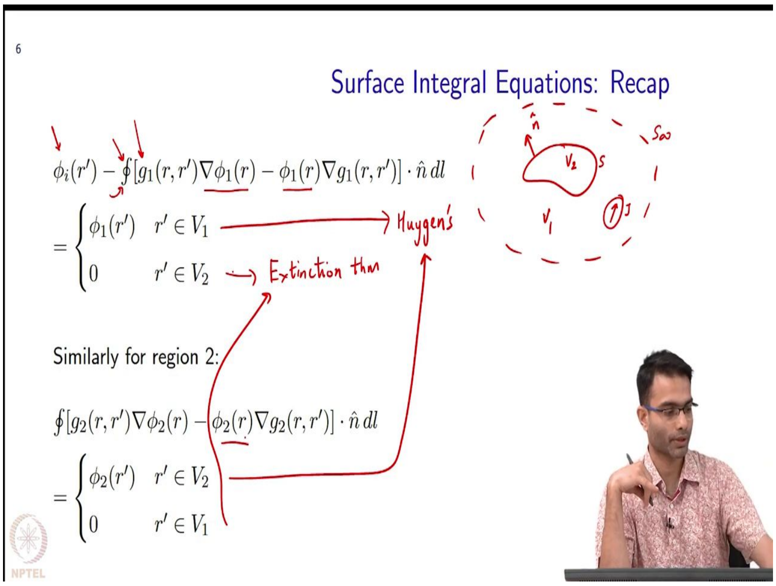

So again we will do a little bit of revision. If you remember this was the two body 2 volume problem. I had some scattering object. So, I will indicate it like this and this was surrounded in a bigger region I call this boundary S_∞ the source S and there was some current source over here J. This was my volume 1; this was my volume 2 and this volume 2 is some object which I know.

So, these were the two equations that we had got right, everyone remembers these equations. So, what just to refresh your memory what changed to produce either this or this the first or the second equation position of r' yeah, but why do I get two different right hand sides?

Student: Because delta.

Because of location of the delta function; if the delta function was not in the region of integration then I got a 0 which happened when $r' \in V_2$ right and when $r' \in V_1$ then the delta function acted with ϕ_1 and I got $\phi_1(r')$ right. Similarly for region 2 again just to refresh your memory; why is there a minus sign in the 1st equation and a plus sign in the 2nd equation?

Student: $\hat{n}$.

$\hat{n}$ right I had fixed the direction of $\hat{n}$ to be this way. So, for the outward the surface the outward normal being fixed this way for V_1 and V_2 because of minus sign difference; any other difference that we notice the incident field there is a incident field only in?

Student: Region 1.

In region 1 in V_1 right that is why it is there here, but in the 2nd equation it is not there ok. So, everyone remember this we had even given names to these equations. So, the first equation on both sides, what did we call it?

Student: Huygens principle.

Huygens principle ok, similarly for here and this one we called.

Student: Extinction.

Extinction theorem; let us show right. So, this is just a revision for you and now the after we introduced these equations what did we do we went and had a long discussion how do we calculate g's ok? And we also looked at integration because you can see there is an integration here we will have to worry about integration ok. So, that was the general setup.

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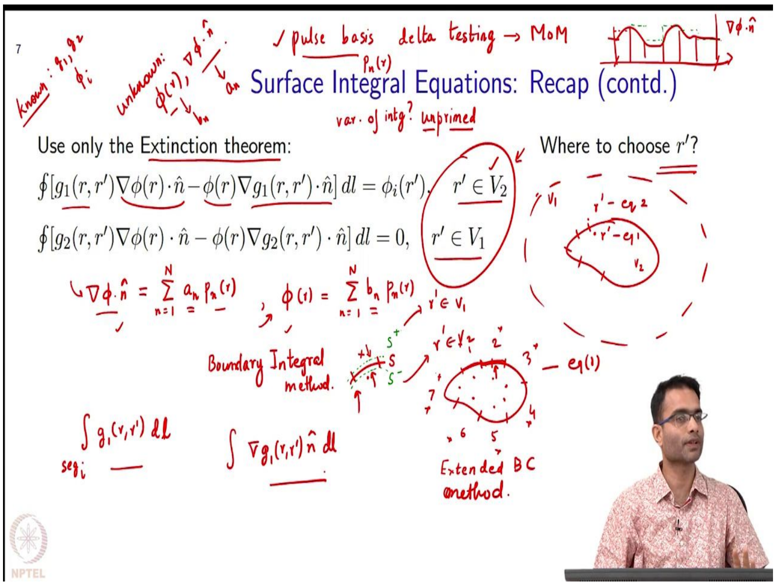

Moving further one thing we realized was that $\nabla \phi_1$ and ϕ_1 these are not being evaluated anywhere in space there only being evaluated on the look this is the line integral or a contour integral. So, these were only being evaluated on the surface S and then what relation did we use to say that ϕ_1 and ϕ_2 should be the same? So, there is a ϕ_1 here and there is a ϕ_2 here, the 2 on the boundary are the same because continuity of tangential equations right. So, continuity of tangential if will give me $\phi_1 = \phi_2$ and continuity of H_{tan} give me $\nabla \phi_1 = \nabla \phi_2$ is the same on both sides ok.

So, recognizing that I am writing a new symbol $\nabla \phi .\hat{n}$ that is the same in both equations and ϕ right. So, what I have done is I have chosen only the extinction theorem right. So, now, in these in these equations what is known if I ask you looking at these equations what all is known? I start from the left do I know g_1.

Student: Yes.

Yes its Greens function for medium for the volume 1 I know it, g_1 is known, g_2 therefore, is also known do I know the phi's? I do not know the phi's.

Student: Alright.

That is what I want to find out, but I know the right hand side this guy I know the incident field ϕ_i is known. And what is unknown now? Which I want to solve for? ϕ_1 is definitely unknown can we make the other unknown a little bit more precise.

$\nabla \phi .\hat{n}$ right because I mean $\hat{n}$ is known, but what is easier to solve for a scalar or a vector?

Student: Scalar.

Scalar right so, if I know $\hat{n}$ instead of trying to find out $\nabla \phi$ which is a vector may as well make the new unknown $\nabla \phi .\hat{n}$ right. So, $\nabla \phi .\hat{n}$ right. So, these are my 2 unknowns. So, I have 2 sets of equations in 2 variables and the goal is to solve these 2 equations. So, now let us use our method of moments approach to solve these problems ok. Now, one thing that you will notice is in these equations what is the variable of integration?

Student: r.

Is it r or r'?

Student: r.

r un-primed coordinates right that is that is indicated over here. So, its un-primed that is important. So, in these equation there is one thing that is yet not been specified and that is staring at us in the face which is I did not say anything about r' right. So, r' unless I specify r prime I cannot actually evaluate this first or second integral was where is r prime all I have said is all that extinction theorem says is $r' \in V_2$ and $r' \in V_1$ in the second part right. So, let us look at this let us draw this again.

So, for the 1st equation where can a first 1st equation says $r' \in V_2$ right. So, this is where r' should be right for equation 1 and this is where r' should be for equation?

Student: 2.

2 ok, it is very its a very common mistake is to forget about this thing, if you forget about this if we put r' in the wrong place then the right hand side will change right. So, these are the 2 constraints that have to be kept in mind, but other than that r' is in my hands. So, now the question is, fine r prime must belong to V 2 in the first case and V 1 in the next case, but where exactly should I put them? I want to solve this by something like method of moments and will take the simplest kind of method of moments; what is the simplest kind?

Student: Pulse basis.

Pulse basis.

Student: Delta testing.

Delta testing; we'll keep this as the prototype alright. So, this having being specified now for the testing location I have to testing location is corresponding to where I choose my r'. So, where can I choose my r'? Let us start let us take one step back let us start with the basis functions we will come to testing like, say let us start with the basis functions. So, what are my unknown functions are ϕ and $\nabla\phi.\hat{n}$. So, for $\nabla\phi.\hat{n}$ I can say that this is now my unknown function which I will write in terms of a known basis; known basis I can let me call the pulse basis as $p_n(r)$ ok. So, coefficient I can write as $\nabla\phi.\hat{n} = \sum_{n=1}^{N} a_n p_n(r)$ ok. Similarly for the other variable $\phi = \sum_{n=1}^{N} b_n p_n(r)$ ok.

So, what I have done? I have chopped up; I have chopped up this into n segments for each segment I have assumed 1 pulse with a coefficient that is going to be determined ok. So, the new so the unknown so, the ϕ I have made them into b_n s and the $\nabla\phi.\hat{n}$ I have made them into a_n s ok. So, the pulse basis the basis function is done clear next comes the location of the testing function, the testing point is a delta testing. So, I just have to pick up point where should I place my testing function.

So, what should we do? I mean it is not very complicated what would you do. So, look at what the constraint r' for the first equation can be anywhere in V_2 right. So, supposing let us take one case over here this is my V_2 and there are various n segments over here or like this and it is up to me where I choose this points can I choose these. So, so how many segments are there 1 2 3 4 5 6 7 segments. So, how many points should I tested? 7 points right.

The 1st equation will give me 7 in the next will also give me another 7. So, I will get 14; I will get all my 14 variables, there are 14 variables in this case right each segment has a a_n and a b_n that I need to find out right. So, I could choose these 7 points anywhere here, I could choose these points like this right for this is for equation 1. Can I choose my points like this? Yes, technically there is nothing wrong with this. Why because if I put these points in over here one by one if I go through them how many equations will I get? I will get 7 equations; in how many variables? The 1st equation how many variables are in it?

Student: 14.

14 variables. So, I got 7 equations because I tested at 7 different points I got another of 14 variables. So, I got 14 variables 7 equations. So, then, but I have used only half the information problem the other half is equation 2; equation 2 how many times should I test?

Again 7 times that I will get 14 equations in 14 variables. So, where can I choose these points? They have to belong to volume.

Student: 1.

1; so, I could choose some points like this right. So, 1 2 3 4 5 6 7; 7 points I have been chosen. So, that gives me when I test the 2nd equation with these 7 points I will get the remaining 7 equations and the same 14 variables right, because grad phi is appearing in both places phi is appearing in both equations right. So, I am going to get 14 equations in 14 variables I can solve it I will get my answer. Is there any problem with this approach? No there is no problem. Only thing is that the location of these testing points seems a little arbitrary right, I just pick them anywhere right. So, if I pick these points like this arbitrarily in V_1 and V_2 then I mean theoretically it is correct and doing so is called the extended boundary condition method; extended boundary condition method. Why extended? Because it is away from the boundary so extended boundary condition method.

But as it turns out. So, I am not going to derive this, but I will just give you information. If you pick these points at random like this away from the boundary the error in the solution is high. So, what might be another way of picking it? I led these points go very close to the surface, if you look thing back at the first integral equation that I did that wire where did I pick my testing points middle of each segment; can I do middle of each segment here? I get one no anyone saying yes.

Student: (Refer Time: 13:03).

I mean a segment like this; can I pick my?

Student: No but.

r' here.

Student: It will also gives.

Its known so, if the boundary is known to me right $\hat{n}$ is known to me. So, I can pick my location anywhere as long as I am obeying these constraints, $\hat{n}$ is given to me I know the boundary, can I pick my points very very can I move this guy over here. I can move it as long as it is not even touching the boundary is ok; The boundary is a sort of peculiar object like the boundary belongs to both V_1 and V_2.

So, as long as I do not go across the boundary I am not violating the condition of extinction theorem right. So, what I can do is let just take one zooming in the segment over here.

Student: It is a relatively what the a_n s of description.

a_n s ok so, the question is what are a_n s in this picture. So, let us take once, if you look at I have n segments over here. So, a_n represents on this boundary there are n segments. So, a_1 for example, will be the amplitude of the first pulse basis function a_2 will be the amplitude of the second pulse function that corresponds to $\nabla\phi.\hat{n}$; $\nabla\phi.\hat{n}$ is a scalar its a scalar function that varies on this boundary right, I am expressing this function by saying that it is made up of 7 pulses. So, a_1 is the amplitude of the first pulse. So, its a little bit like this I have some function let say that this is what $\nabla\phi.\hat{n}$ looks like and the starting and ending point is; obviously, the same because I am going on a circle ok. So, technically I should not draw it like this. So, let say something like this ok. So, this point and this point is the same.

So, what I am doing is I am I am chopping it up like this right, that is my $\nabla\phi.\hat{n}$ is the continuous curve I am saying I will approximated by something like, this these pulses ok. As I make more and more pulses I can approximate better and better right this is the old stuff we have already seen this ok. So, now, coming back to this one segment over here the first point over here I can move it in a limit very close to the midpoint of the segment and the testing point from the 2nd equation I can move it very close to the middle of the segment, again I can take a limit ok.

So, I can think of this being the surface S and one imaginary surface let me use green over here drawn like this and an imaginary surface drawn like this over here ok. So, the inside one I can call S_- and the outside one I can call S_+ right. So, the plus and minus is to make sure that we obey the conditions of extinction theorem ok. As long as I am in S_- this condition is valid because its in V_2 right.

So, what I am going to do is if I choose these points basically to be the midpoints of the segments, then I have a very clear unambiguous way of specifying how to solve this problem right. Pick these basis functions 1 and 2, pick the testing points to be the midpoints of each of these segments just like how we had done in the first example on integral equations right.

So, that is the strategy that we will use and when we solve it like that its called the. So, its simply called the boundary integral method ok. So, this is would be called the boundary integral method ok. So, S_- is for $r' \in V_2$ and S_+ is for $r' \in V_1$ that is what we have done. Now, why I need to define this S_+ and S_- will become clear as I go I may as well just have said pick the midpoint why to tell you about S_+ and S_- ok. So, that will become clear as we go along ah. So, what else is there in this equation to solve in the method of moments, what will happen? I will substitute this guy and this guy, ϕ and $\nabla\phi.\hat{n}$ into this equation.

So, what kind of terms do I have to evaluate? I will have to evaluate the integral of g_1 over some segment and the integral of $\nabla g_1.\hat{n}$ these are the 2 functions whose integral I have to evaluate right, when we do the method of moments. We looked at the A_{mn} the matrix coefficient A_{mn} in terms of the operator yesterday in the language of method of moments, that is the coupling coefficients so to speak that I have to evaluate.

So, let us so, these are the things that I have to worry about let us take a segment let us call it segment i, I have to evaluate $\int g_1(r,r')dl$; I just call it d l because it may be curved and I have to evaluate $\int \nabla g_1(r,r').\hat{n}dl$ that is what I have to evaluate.

Student: (Refer Time: 18:53) segment I mean.

A segment means a segment like this and some over all such segments I have broken up the whole thing into several segments, I have to evaluate this integral over each segment.

Student: Sir basically that we have not part (Refer Time: 19:06) segment.

Yes so because my basis functions are pulse functions. So, this pulse function is non zero only over one segment right. So, when I go to evaluate the integral I have to do this integration sort of each segment one by one ok, that is because I am using sub domain basis functions. So, they are nonzero only in one part of the segment that clear ok.

So, these integrals are the ones I have to evaluate this and this for both g_1 and g_2 ok. Do we have; have we learned anything already which helps us to evaluate these integrals? Numerical integration, we have already seen one module on numerical integration and the reason why numerical integration is necessary is because these functions Hankel functions and so on, they are often not analytical integrable.

So, I need to evaluate these numerically. So, I will use things like quadrature quadrature rules to evaluate this right quadrature remember what was the advantage? I do not need to know anything about the function except its values at some points, multiplied by precomputed weights I get the integral. So, that is going to be our strategy.