Computational Electromagnetics
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Method of Moments
Lecture – 8.2
Linear Vector Spaces

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Let us put this in the language of math we need linear algebra now ok.

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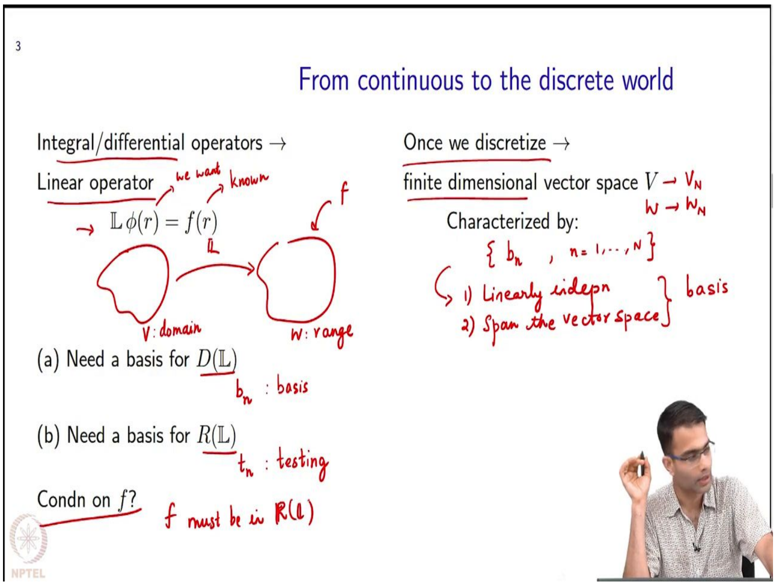

So, here is where your knowledge of linear algebra will come into play. So, let us start from the top, those of you who have done linear algebra. They know that if an operator has a differentiation or an integration then that operator is.

Student: Linear operator.

It is a linear operator right. So, anyone who has taken a basic course in linear algebra knows that the operator is a linear operator. We have seen the kind of operators that we have had so far right, $\nabla^2 + k^2$ differentiation operators are there. Then when we went to integration there was an integral sign inside it.

So, there was only there is differentiation there is integration, there is nothing more fancy like exponential or something like that right. So, the operators that you get in Computational Electromagnetics are linear operators. And the great relief or the great sort of saving grace about it is that linear operators, we can talk about in the language of linear algebra.

So, all the tools of linear algebra can be applied to these problems to solve them ok. So, that is what really save this we could not do this if our operator was not linear ok. So, let us take a very general problem over here. So, L over here this is some operator acting on

ϕ and giving f. So, the usual interpretation is what we will use that f is known, and this ψ over here or ϕ rather is something we want ok.

Now, in terms of an operator right now we are not discretizing anything it is just an operator ok. So, what does any operator do? Is going to take one space to another space alright; so I can say that say ϕ lives over here and the operator takes it let us say call this some space V and the operator over here this L takes it to another space over here W, at least we hope that f is in this space right . So, what I am describing is that these are so let me not write f and ϕ yet over here. So, L over here is the operator that takes one space to another space.

So, the left hand side space is what would be called, what would be called the this the space on the left hand side? The domain right. So, I will call this a domain and the domain gets mapped to the range right; so this is the range ok. So, the domain the symbol for that is $D(L)$ and the range symbol is $R(L)$ right. Now is when we go to the discrete world. So when we discretize this continuous space over here V and W; it gets discretize also. So, it becomes a finite dimensional vector space ok.

So, it becomes V_N let us call it and similarly W becomes W_N ok. So, these are finite dimensional vector spaces, to characterize vector space, what does the most sort of basic building block?

Student: Basis functions.

Basis functions right. So, basis function so it is characterized by set of basis function. So, let us call them say $b_n(r)$ alright n is equal to 1 to N ok. You do not even need this over here. This N vectors alright these N vectors will characterize a vector space and for these n vectors to characterize the vector space what is the requirement on these vectors.

Student: (Refer Time: 04:23).

Right; so first is that they should be linearly independent right. Second.

Student: Orthogonal.

Need not be orthogonal, I can have non orthogonal basis also. So, when will I call the set of vectors a basis for a vector space?

Student: Vector they span.

They should span the whole vector space right. So for example, if I give you three dimensional space x vector and y vector they are linearly independent, but they do not span three dimensional space. So, these vectors put together should span the vector space ok. So, these are the two requirements to be called a basis fairly simple stuff, you know I mean we already sort of know this, but we are just formally put into the language of math. So, therefore, the for the domain I need a basis and for the range also I need a basis ok.

So, I am going to call the set of basis vectors for the domain I am going to give the symbol b ok. So, b_n and for the range I am going to use a slightly different symbol I am going to call them t n ok. So, the way to remember it is sort of what will come later on, for the domain I am going to call this basis functions and for the range I am going to call this testing functions that is why letters b and t ok. So, it will become clear as we go what why these letters or words basis and testing.

In order for us to solve this equation if I just go back to the original equation over here and I say give me a solution to this right. So, very beginning of linear algebra what do you say should be a condition on f for you to be able to solve this equation?

Student: That should be in the range.

The range right; so if I am going to be able to solve this equation f should be inside this range, otherwise there is no way that L will map anything and get me f alright. So, f must be in the range of L that is kind of obvious, but I am just stating it anyway ok. So, this is sort of a brief review of linear algebra that we will use ok; because the whole method of moments is a very very nice tool from linear algebra only, it uses basis vectors and it uses inner products nothing much nothing more complicated in that is this fine so far?