Computational Electromagnetics
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Method of Moments
Lecture – 8.1
Motivation for MoM

So this module is going to be about the method of moments, which is the name given to the way in which we solve the integral equations. So far, we have seen one simple example of integral equations which is the, volume at the wire with the charge surface charge on it and we wanted to find out the electrostatic potential. The second thing that we formulated was the surface integral equation and we want to find out now how do we solve this ok.

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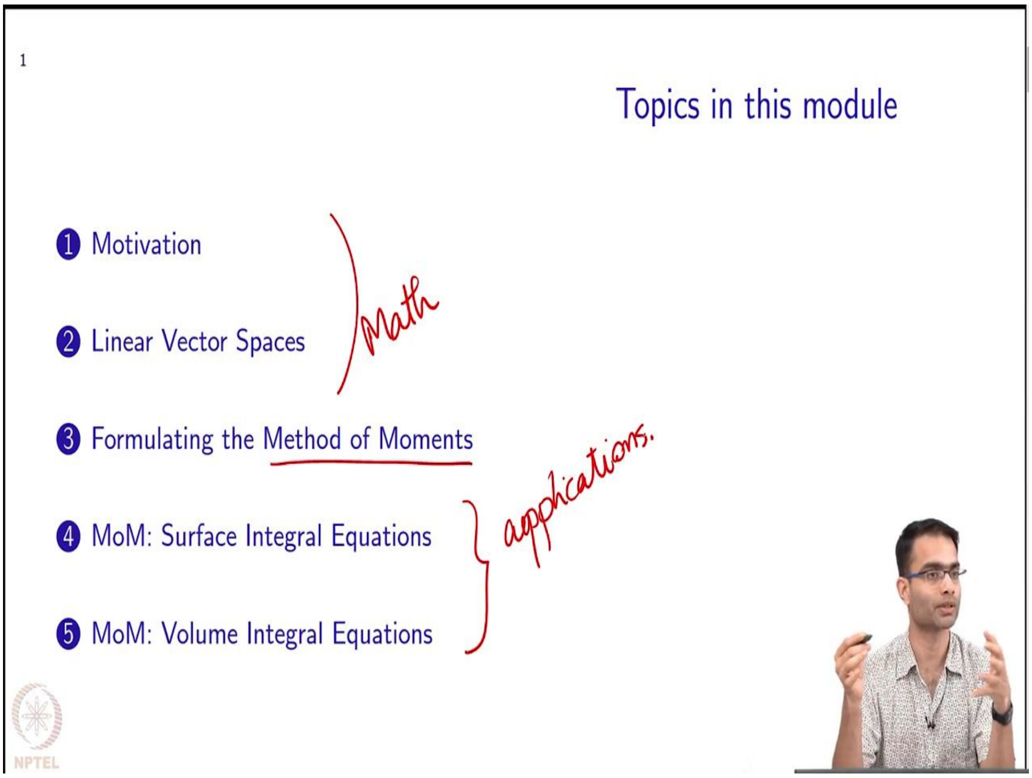

So roughly the layout that will follow, this is going to be a slightly lengthy module is before we get to actually before we get to the method of moments, we will look at some math motivation and generalization of the idea mathematically what it is. And then we will apply it to two these are applications ok. So, you learn the general theory and then we applied to two problems ok. So, that is going to be the approach and we will now when we do this, we will have to apply everything that we have learned so far ok. So, starting with integral equations, Green's functions numerical techniques of integration, everything comes together in this module alright.

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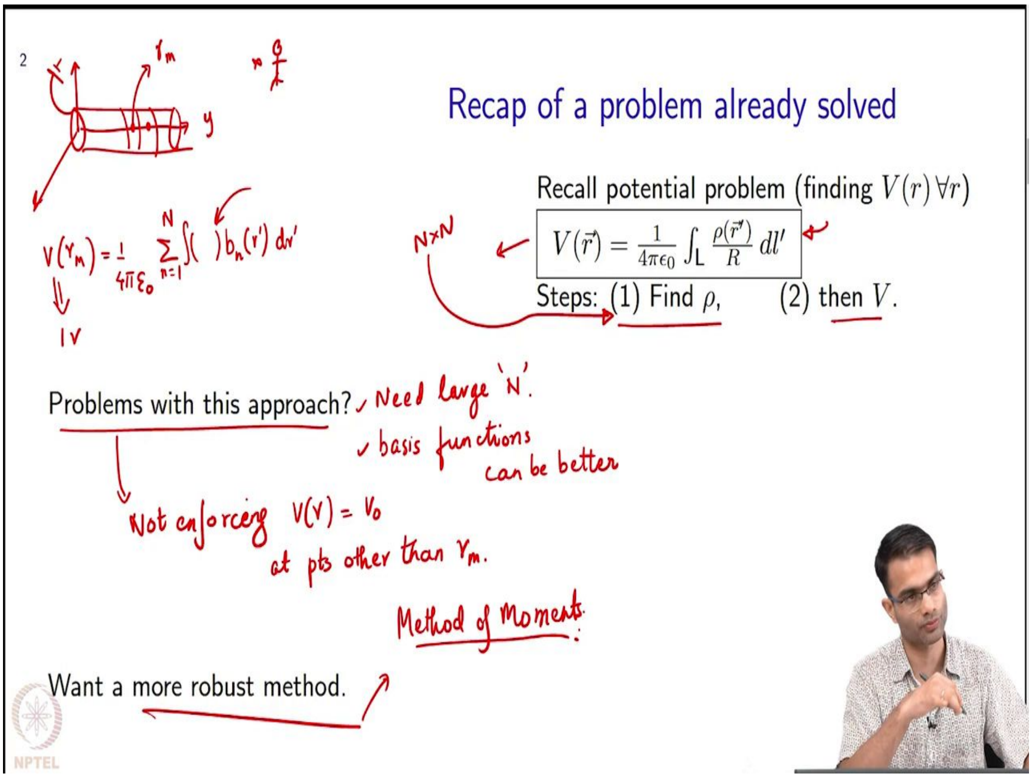

So, let us go ahead so let us look at the motivation over here. So, let us go back to the very original problem that we started with further for.

Student: (Refer Time: 01:43).

Motivating integral equations themselves right. So, everyone remembers this integral equation right. So, we had wire along the y direction right that is how it was. And we wanted to find out the potential at any point in space over here, that was what the problem was alright. And further what was given was that this cylinder was connected to a constant voltage right. So, everything was at a constant voltage and that is how we had set the problem up and in solving it we had two steps right. So, first of all in this problem what was unknown?

Student: Charge.

The charge distribution ρ right. So, ρ was the unknown and ah, so the first step was to find rho and to find rho what did we do? First step was to we discretize the like line segment and then we expressed it as in a.

Student: Basis.

Basis right in terms of basis function and we use a simple pulse basis function right. Once we knew this, then we knew everything inside the integrand and I could evaluate this and find out V anywhere right. That was the general approach that we did ok. So, looking at this formulation that we did so far right; so for example, if I wanted to take one equation so, remember this gave us a $N \times N$ system of equations, which we use to solve for ρ . So, let us look at just one of those equations ok.

So, let us take one of those equations that corresponded to let us say the mth segment and the midpoint of it right. So, this was let us say some r m. So, what did this equation read as the mth equation in these $N \times N$? So, the left hand side would be right so, that the $4\pi\varepsilon$, we can keep it on the right hand side. So, there is $V(r_m)$ right that is the point where I am enforcing my known condition is equal to $1/4\pi\varepsilon$.

And there is going to be a summation over small n is equal to 1 to m capital N sorry. And so there is there is going to be a bunch of things over here right and the pulse basis function if I call them as b_n right, b_n of let us call it r' right; it is going to be some integral over a dr' that is how we did it whatever is inside does not matter.

And in this equation the way we solved, it is that the left hand side this $V(r_m)$ we know it to be fixed at 1 volt that was given right. So, what we ended up finding out were the coefficients of these basis functions right. So, that allowed us to express ρ. So, it is just a review or a summary of what we have already done and seen so far ok.

So, now having seen this now that you have refreshed your memory about it, if I ask you what would be some of the problems with this approach in terms of accuracy. What do you think? The accuracy depended on the number of cells right, the larger the number of cells the better was the solution accuracy right. So, of course, that is enough approach need large number of N and we saw that in the graphical results also. Even if I fix a large number of N, is there any other problem conceptually that you can see?

Student: Basis.

Basis yeah so basis functions may be unrealistic like using a pulse basis function may be unrealistic, but using you know a triangular basis function may be better right. So, basis functions can be better. Anything else? So, let us say that I have got very good basis function I have chosen a large value of N anything else that could be a problem over here. So, the hint is the left hand side, what is known or rather what is given to you in the problem where is the voltage given?

Student: Surface.

Surface right of the surface of the cylinder this metallic perfect electric current is given to be this thing. So, every point on this cylinder has constant voltage. Where all are we enforcing this? We are, along the center, but at discrete points or how many discrete points?

Student: N.

N discrete points, by doing this are we doing anything to ensure that the voltage between r m and r_{m+1} is still 1 volt, we are enforcing the left hand side at r_m then the next; so we have one segment over here we have one more segment over here. So, first equation will take this point second equation will take this point and so on. But we are doing nothing we are just hoping that the solution turns out to be smooth and you know well behaved and all of that, but we are not doing anything to ensure that the solution is continuously one volt everywhere right.

Student: 1 volt I will just (Refer Time: 06:54).

1 volt you know the voltage that is given in that every point on the wire.

Student: But we are increasing N know.

As we increase n we are approaching that. So, my question is that keeping N fixed choosing good basis functions still is there something better that you could do right. So, the answer is yes and that is what the method of moments tries to do. So, what we did over here was, essentially we discretized the integral equation and solved it right.

So, the problem main problem with this approach is that we are not enforcing $V(r)$ is equal to V_0 at points other than r_m right. So, the more robust method is what is called the method of moments. And this method of moments is actually it is a very general term it includes integral equation methods; it also includes finite element methods ok. So, it is found throughout engineering.