Computational Electromagnetics
Introduction to Greens Function
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Introduction to Greens Function
Lecture – 7.5
2-D Wave Example : Boundary conds

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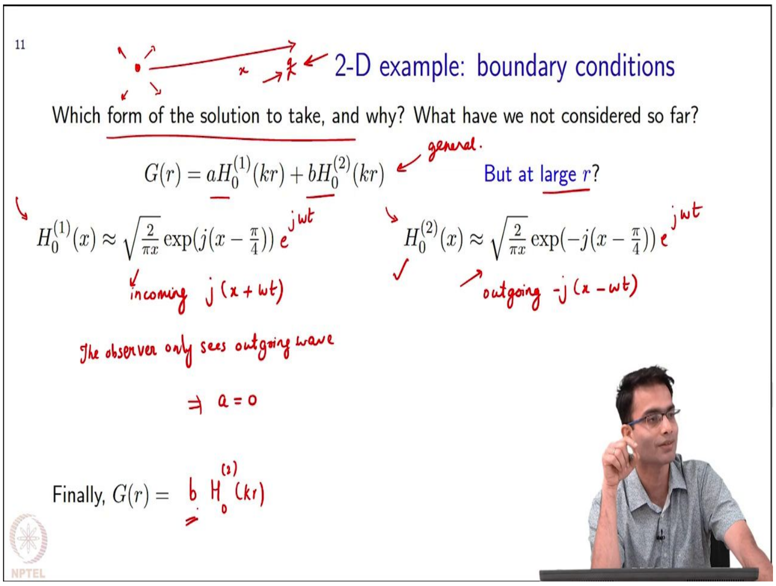

So, here comes the question of which form of the solution to take and why. So, I wrote it as $aH_0 + bH_0$ ok. And so far, we have not really figured out or put use the idea of boundary conditions at all ok. So, what are the boundary conditions forget what is on the board, what are the boundary conditions for this problem? I have a point source sitting at the origin alright and radiating a field. What is a boundary condition for this problem?

Student: 0 field that (Refer Time: 00:48).

As the field goes far away, I mean as I go far away the field should go to 0.

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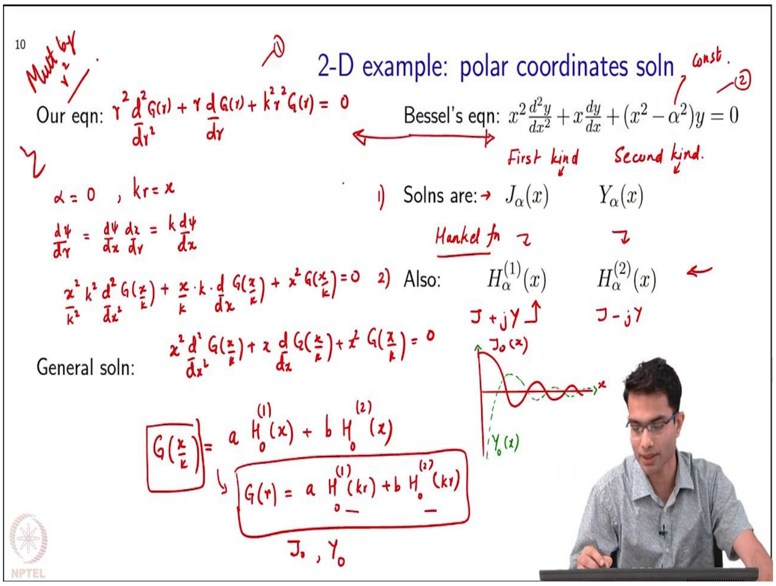

Now, when I look back at the plots that I have shown you of J_0 and K_0; if I take any linear combination of J_0 and K_0, will both of them go to 0 at large x? Yeah, individually each of them details. So, both $H_0^{(1)}$ and $H_0^{(2)}$ seem to satisfy at least the asymptotic limit ok. So, that is not sufficient; is there any other information that I can use physically? This will come from physics not math.

So, remember I am at the origin over here and it is a wave equation, so some wave it is going to go out from here that is where I have put my impulse. So, let me give you a hint, what is the time convention I have assumed throughout this thing. In the very beginning, there was a remember Maxwell's equation; there was a d/dt and I assumed a time harmonic case. I put $e^{j\omega t}$. So, that time derivative give me a minus omega square I mean $j\omega$ and the second time.

Student: $-\omega^2$.

$-\omega^2$, but my time convention was $e^{j\omega t}$.

Student: $j\omega$.

$+j\omega t$ ok; so, does that tell you anything? So, as to give you a hint let us look at what happens when I go far away at very large x from the location of the point source from the location of the impulse. If I go far away, I am giving you some extra information I am telling you that $H_0^{(1)}$ and $H_0^{(2)}$, they have these what are called asymptotic forms.

In general you cannot write Hankel functions or Bessel function in close form; they are special function, they do not have a closed form expression. But under some limits very small x very large x or whatever, they have some approximate form and that approximate form has been written over here for large values of r or x whatever ok. Now, looking at this; they you get an intuition of which form to which form is physically acceptable.

Student: Second one.

Second one because.

Student: Decaying.

Both are decaying

Student: Both are (Refer Time: 03:39).

What is the amplitude of both of these guys?

Student: $\sqrt{2/\pi x}$.

$\sqrt{2/\pi x}$. So, at large x both go to 0.

Student: $e^{j\omega t}$ groups.

Huh $e^{j\omega t}$.

Student: Has something has to counted at 0.

Why does have to counter act? It is a phaser which is constantly oscillating, but the amplitude is decaying no problem. So, let us write this term over here on both sides that would be the total solution in space and time right; no that is what we did with facer; solve it, put a $e^{j\omega t}$ and take real part.

Student: Take positive.

Wave is going in which direction?

Student: Positive r.

Is going outward?

Student: (Refer Time: 03:46).

Are both waves outward?

Student: No second one.

Only the second wave corresponds to a outgoing wave because, it is a $j(kx - \omega t)$ right and this one the other hand is.

Student: Inward.

Incoming wave because, it is a $j(kx + \omega t)$ right. So, for that reason because my impulse is at the origin; they can if I am stand going to large x over here this observer cannot possibly observe and incoming wave like this, that is what it would mean right. This fellow can only find observer wave that is out going from the origin which is matched only by this guy ok.

So, here is sort of a good juxtaposition of some physical intuition and mathematical formulation right. So, based on the observer only sees outgoing wave right, so implies $a = 0$ right, $a = 0$. So, therefore, I can write that the final form is $bH_0^{(2)}(kr)$ ok. So, we started with the general solution over here and we said physically that a is equal to 0 is the only meaningful thing and that gives me this. So, is my problem done?

Student: (Refer Time: 05:38).

Yeah exactly, so I mean it is a fine it is almost done except for the annoying factor b which I still do not know how what that value is, but I need to know it because otherwise I will have that one variable everywhere. My answers will be always off by a multiplicative constant.

So, what is the thing that we have been avoiding all the long? $r = 0$ right and we have good reason to avoid $r = 0$ because as we found out, let the Bessel Y function it plunges to minus infinity. But if I want to get the complete solution, there is no escape I have to go to the origin now ok. So, when dealing with singularities or infinities what is the one safe way of dealing with them?

Student: Limit.

Limit is one thing we already seen in the 1 D Green's function; when we had delta function, did we approach it directly or did we do something else to it?

Student: We did it.

We integrated it right. So, when you see a singularity, the best thing is to integrate, so that you get a finite quantity otherwise what do you do with an infinity setting there right.