Computational Electromagnetics:
Introduction to greens Function
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Introduction to greens Function
Lecture – 7.3
1-D Example: Alternate Representation

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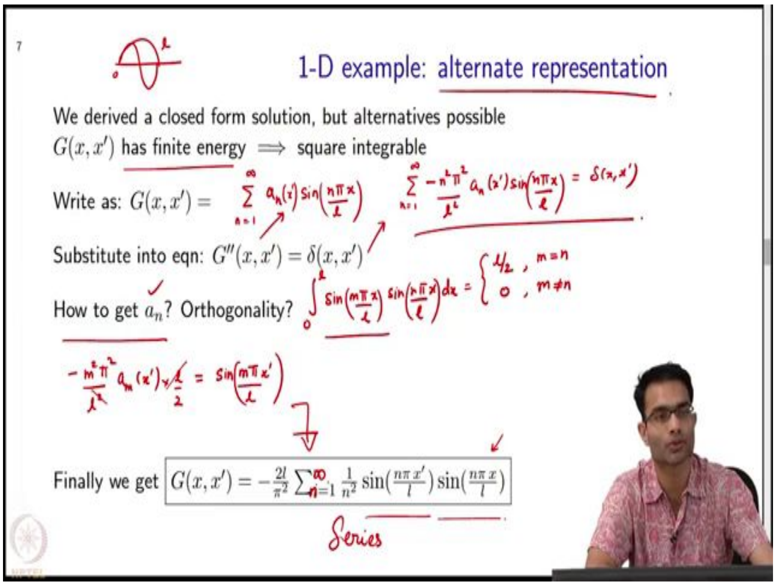

So, now turns out this is an interesting thing about impulse responses and there are Alternate Representations also possible ok. So, what we derived? We derived a closed form solution right.

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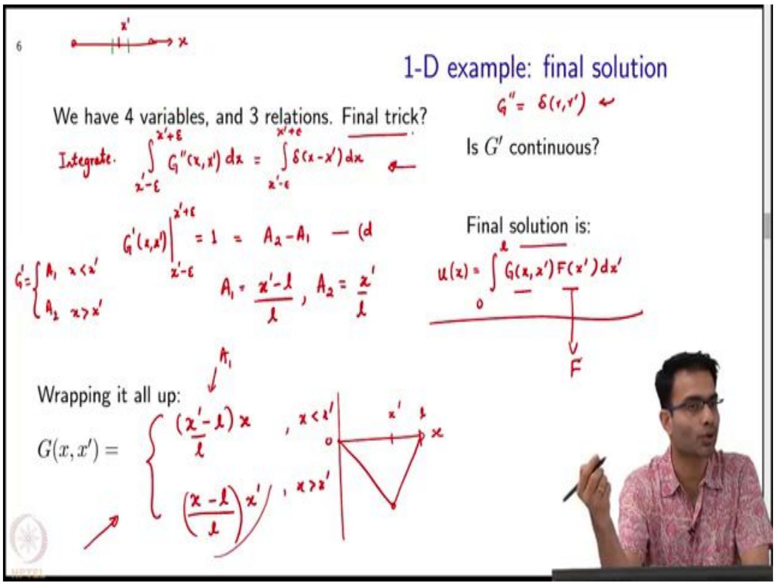

And moreover I just want to show you one thing. If I want to plot this function over here what will it look like? So, let us say is plot it over here right, so this is my x axis and let us say this is x', this is my 0. So, for $x < x'$ what does that look like, both are straight lines? The slope is negative or positive?

Student: Negative.

Negative because $x' < l$, so it is a line that is.

Student: (Refer Time: 00: 57).

Going down like this right, the other term.

Student: (Refer Time: 01:05) 1 by l.

So, the other term x' and $x - l$ also negative $x < l$, but it is straight line again and the slope of the straight line is.

Student: $l/1$.

Right, so it is, but it is going to be positive right, so it is going to be a increasing function over here is that look right everyone agrees.

Student: Sir.

So, you can do a check put $x = x'$ what do you get? You get the same relation on both sides and it is a negative quantity right, so that is what you get over here alright. So, this is I got a continuous looking function out of this right.

Student: Does that mean that G has a finite value and $x = x'$ there is no singularity.

There is no singularity in G, but look at G'; G on both sides it is continuous, but look at the derivative of G on the left hand side and the derivative of G on the right hand side; the discontinuity is in the derivative right which we sort of saw in this relation, the derivative was discontinuous ok. So, looking at this looking at the solution that I have got over here you can see it is continuous can you say therefore, that it has finite energy in this way; there are no kinks running off to infinity finite energy right.

So, finite energy signals also they are square integrable. So, what is the nice property of square integrable signals? Again we study this in signals and systems. It is over a closed domain 0 to l right, so what would you what is one possible way of writing this G? Can I write it as the Fourier series? Right, over the domain 0 to l, the signal has finite energy it should be representable by Fourier series.

What is Fourier series? Linear combination of sines and cosines of different frequencies right. Now in this case frequency is a spatial frequency right, the string can be like this, it can be like this or you know integer multiples like this right. So, this is my 0 and this is my l just by looking at the boundary conditions can you say something about you know which terms will be 0.

Student: Yes sir.

Non-zero.

Student: 0.

Exactly they will only be sine terms because cosine terms will have non-zero amplitude at 0 and l. So, I can straight away say that this is going to fit within 0 to l it should be a sine function right. So, I can write this as in the Fourier series form $\sum_{n=1}^{\infty} a_n sin(n\pi x/l)$ that is fine. So now, on the left hand side I have a function of x and x', right hand side seems to be only a function of x. So, where will the x' dependence come from?

Student: Coefficients.

Coefficients; coefficients will be x' dependent, so let us write it like this $a_n(x')$ ok. This is the most general possible way of representing it because it is I mean square integrable signal that I have ok. Now the next step is to find out what are these a's right, so I have a differential equation. So, what can I do? Substitute it. So, what will be the second derivative? So, first derivative of sine will give me.

Student: cos.

Cos one more derivative will give me a sine right as I can apply to each of the terms one by one. So, the left hand side will become summation will remain n is equal to 1 to infinity and what should I get?

Student: $n^2 +$.

$\sum_{n=1}^{\infty}(-n^2\pi^2/l^2)a_n(x')sin(n\pi x/l) = \delta(x,x')$ right. All of the tricks we learnt in Fourier series can be used over here to find out a_n what should I do?

Student: $x = x'$.

$x = x'$ would not help me.

Student: You can multiply.

Multiply by what?

Student: The $\delta(x)$.

No.

Student: Delta sin.

No, Fourier series yeah he has write answer.

Student: $sin(n\pi)$.

Exactly, right so orthogonality between sine's right. So, the relation that I have is at the; if I take this, so $sin(m\pi x/l)$ and $sin(n\pi x/l)$. These two are orthogonal functions on this interval. So, orthogonality will say that this is going to be equal to, so I have I have done the calculation you will get l/2 if m=n and 0 if $m \neq n$.

Student: Ok.

So, I can multiply both sides by $sin(m\pi x/l)$ and then integrate 0 to l right. So, what will the right hand side become? So, let us start with the left hand side. What happens is the left hand side, which term will survive? I am multiplying by $sin(m\pi x/l)$ on both sides of this equation and integrating 0 to l, which term survives?

Student: (Refer Time: 07:00).

m; a_m right so the left hand side will be $-(m^2\pi^2/l^2)a_m(x') \times \frac{1}{2}$ right and the right hand side is going to give me what?

Student: $sin(\pi)$.

$sin(m\pi)$.

Student: x'.

x'/l, so I have got my a_m right. So, I have got my a_m and then what do I do? I plug it back into this relation right. In this equation the only thing unknown was a_n s which I have evaluated now. So, what do I get? This is the final relation right. So, here you can see whether one of the l got cancelled off, so that is why this is $-\frac{2l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}sin(\frac{n\pi x'}{l})sin(\frac{n\pi x}{l})$ both terms are over here ok. So, now, you might I have asked that why should I have chose this form this looks so much more complicated, than my previous form over here did not look this much simpler right this look much simpler than this ok.

So, how do you know and this we will end this module over here with this discussion how do you know which form to choose? So, this is the series form and that was a closed form which form should we chose.

Student: What form of x?

Exactly so, remember the final solution is given here the integral of G and F, now depending on the kind of form of f one or the other will be easier to evaluate. For example, if capital F where also of the form $sin(k\pi x/l)$, then that integration is very simple I think it is orthogonality and this integration will be simpler all the terms will drop out I will be left with one term it will be exact calculation.

On the other hand if I have if use the previous form then I have to do the integration of x multiplied by sin x and do all that integration both will give me the same answer because is the same function ok. But, sort of the take home message from this final slide over here is, that Green's functions can have multiple representations closed form or series you should choose that form depending on the problem at hand basic idea is you want to reduce the calculations.

So, whatever helps us to achieve that, well yeah because this string is in the x coordinates only right. So, it has to be Fourier series in the x term only with some coefficients it should not depend on x. So, the only thing remaining is x'.

Student: (Refer Time: 10:10). So, in this summation n equal to 1 to infinity.

Yeah.

Student: (Refer Time: 10:13).

Yes, correct so this should be infinity yeah thanks for pointing out there and summation over n very good $\sum_{n=1}^{\infty}$ yeah there is no i over here.

Student: Sir.

Yeah.

Student: Now does that series solution converges to the (Refer Time: 10:31).

It is exactly equal to that it converges to that.

Student: Or what when do you?

For no value for it is a the sum of the infinite series is that yeah.

Student: Fourier series.

So, Fourier series, so if you keep finite number of terms it is not actually equal to that, but you can see these terms they begin to decay there is a $1/n^2$ as you go to higher and higher and they begin to decay.

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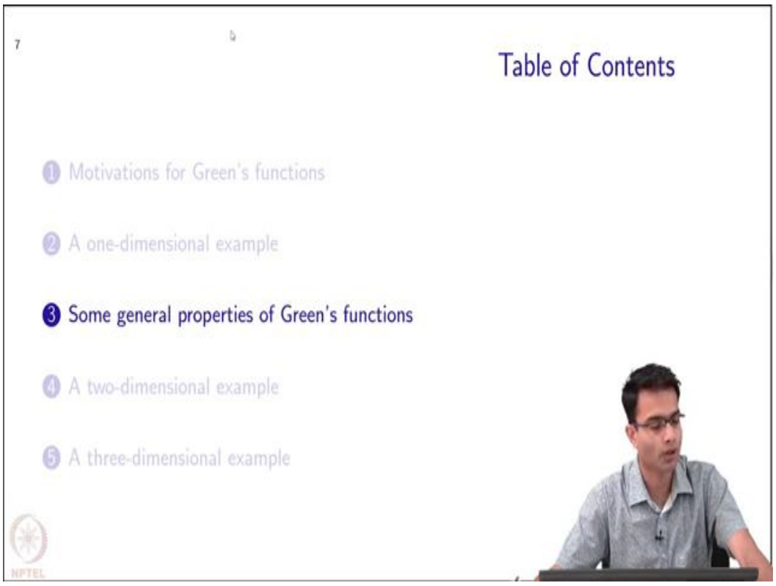

We will continue our discussion about Green's function; we have looked at 1 D Green's functions ok.

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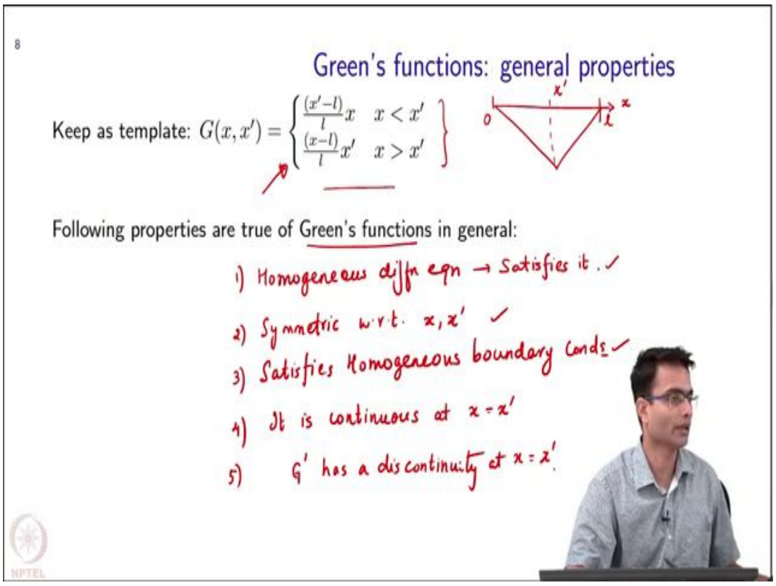

So, what I will do now is before we go to 2 D Green's functions I want to summarize a few general properties that Green's functions obey in general ok. So, this is the Green's function that we had from yesterday's class right. So, this was the you know when we plot it looks like something like this x prime and this is my x prime and so there are some general properties that you will observe. So, the first is the Green's function we are talking about properties of the Green's function. So, if I look at the homogeneous differential equation ok. Homogeneous differential equation means the right hand side is.

Student: 0.

0 right. So, this Green's function it satisfies it right, can we say that the way we derived this function was by looking at basically we ignore the point $x = x'$ and we derived this right. So, the homogeneous differential equation is satisfied by the Green's function that in general is how you will construct it ok. looking at the form of the function can you say anything about it is symmetry properties with respect to $x = x'$, if I interchange x and x' everywhere in this will I get the same function yes right because case 1 will become case 2; case 2 will become case 1 and I will get the same thing. So, it is symmetric with respect to x and x' right.

The other thing that we said is that it satisfied in this particular case, what kind of boundary conditions that we have? The boundary conditions were also homogeneous in some sense we said the string is clamped at both ends. So, it is like a homogeneous boundary condition where the field is displacement is 0 at both ends. So, it we can say that it satisfies homogeneous boundary conditions ok, so three conditions. The other condition is, the fine we had four variables and the fourth trick that we used to eliminate all four variables was that of.

Student: Continuity.

Continuity right; so, we can say that this it is continuous and $x = x'$. Is it differentiable?

Student: Satisfies the differential equation and all points other than x (Refer Time: 14:08).

That is the first point yeah.

Student: It is not everywhere the (Refer Time: 14:13).

I mean at the point of the; at the point of the singularity there is a bit of a problems, hard to say if the right hand side is infinity what do you say about the left hand side right. But, we will see in the next example that you will in some cases even construct a Green's function which has a singularity in it and it obeys the rest of the properties also it will be problem dependent. And yeah this is the other part right it is continuous, but not differentiable right, so we can say that for example, G' has a ok.

I mean these are just sort of simple observation you can get by looking at the form of the Green's function that we got and how we derived it ok, it is more like a review or summary of what you've done so far ok. So, this was the sort of simple case of one dimensional Green's function; we did not have much trouble in finding out any of the constants involved.