Computational Electromagnetics
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Surface Integral Equations
Lecture – 6.5
Conclusions of surface integral equations

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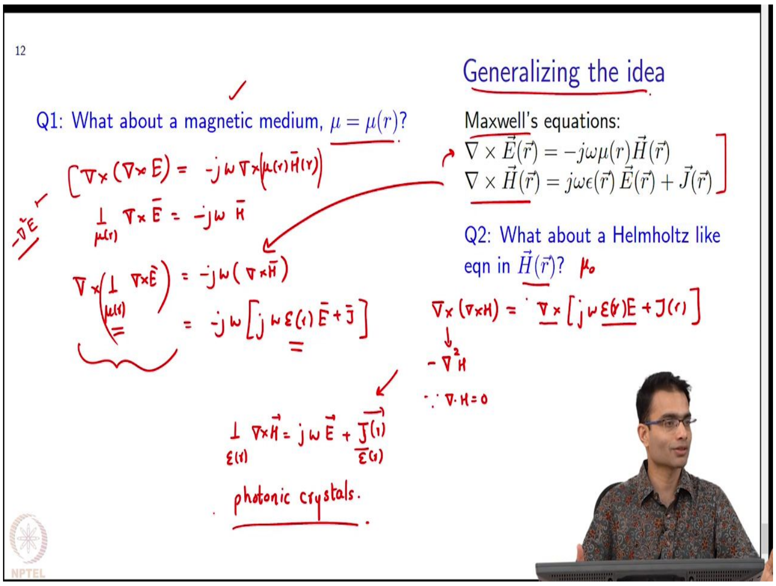

So, to conclude this module I just have one final set of observations to generalize what we have done so far. First question is what if the medium was magnetic? That is if μ was a function of space also. So, remember what we did was we took this first equation and what did we do it?

Student: Curl.

We took curl; so we did $\nabla \times \nabla \times E(\vec{r})$. And that gave that basically said that $j\omega\nabla \times \mu(r)\vec{H}(r)$ and that is a problem right because I know $\nabla \times \vec{H}(r)$; I do not know $\nabla \times \mu(r)\vec{H}(r)$. So, whatever process I did, I will not be able to follow it exactly in the same; so what do you suggest we do in this problem?

Student: Can you take mu are also into epsilon naught.

No. μ is in one equation and ε is in another equation. How do we observe the two?

Student: Actually there is a bright out the magnetic and.

How these are coupled equation how do we separate them out? The answer is actually much simpler just look at it.

Student: Solve for (Refer Time: 01:35).

μ is given to you, there is no need to solve for it.

Student: We called nu r H and H dash.

But I still do not know curl of that, I know only $\nabla \times \vec{H}(r)$, I cannot use a second equation. The second equation is the clue I want to do whatever you do I want to only take $\nabla \times \vec{H}(r)$. So, what should I do to the should I straight away take the curl of the first equation or do something before that?

Supposing I will take μ to the other side. So, supposing I do $1/\mu_r \nabla \times \nabla \times \vec{E} = -j\omega\mu_0\vec{H}$. Can I take a curl now? On both sides, so I will get

$$\nabla \times (1/\mu_r \nabla \times E(\vec{r})) = -j\omega\mu_0 \nabla \times \vec{H}$$

and then I can substitute over here. So, by doing this, would I have eliminated $\vec{H}$ yes right, because the next step itself I write minus j omega and I substitute over here $j\omega\varepsilon_r\vec{E} + \vec{J}$.

So, I have got one equation in electric field which has all the given information about the problem, the given information is μ_r and ε_r. What is the price I have to pay?

Student: (Refer Time: 03:05).

This is not a very pretty expression right. It is curl of some funny looking object and then we have to use rules of the vector calculus to open this up. So, you the price you pay is that the analytical formulation before I discretization solve is a little bit more complicated, but it is nothing that we cannot deal with you just more terms; earlier this thing at simplified very nicely into only a $-\nabla^2\vec{E}$. But now I will have many more terms that is fine. So, that is how we will deal with a magnetic medium.

Now, what we did was we took these equations, let us assume that we do not have a magnetic medium let us assume that I have just mu naught, we formed an equation the Helmholtz of the wave equation only in $\vec{E}$. Can I do the same thing in $\vec{H}$? Can I form a $\nabla^2\vec{H}$? I have one answer that says yes. Anyone else? So, if I do I have to do this right $\nabla \times \vec{H}$ that is what I have to do. And since mu naught is just constant then this would give me $-\nabla^2\vec{H}$ because $\nabla \cdot \vec{H} = 0$, but what do I have on the other side? I have a $j\omega\nabla \times \varepsilon_r\vec{E}$. Sorry, I wrote the whole thing again let us write this.

So, I have $\nabla \times \nabla \times \vec{H}$. So it is $j\omega\nabla \times \varepsilon_r\vec{E} + \vec{J}(r)$. Can I proceed with this? Though I know $\nabla \times \vec{E}$, $\nabla \times \varepsilon_r\vec{E}$ I cannot actually get a Helmholtz equation in $\vec{H}$; because I do not know $\nabla \times \varepsilon_r\vec{E}$ I only know $\nabla \times \vec{E}$. So, I cannot simplify this, so what would you in this case? If you wanted to get an equation in $\vec{H}$. Would you do the same trick as this? So, what I will do is I will get a $1/\varepsilon_r$ over here and $\nabla \times \vec{H}$.

And then you have a j by that is I do not need I mean all I want is some way of simplifying. Now I can take the curl. I will eliminate $\vec{E}$ from this equation I will get so, I am paying a price now I am my I have a j/ε everywhere fine. But those are the ways in which you will get around this form if you wanted to solve if you wanted to get a Helmholtz equation in $\vec{H}$.

So, this is actually done a lot in the case of the photonic crystals community. If there is time later on the course will have a look at it, but it's common I am not just you know cooking these up. People in different parts of physics use these different formulations depending on what is convenient.

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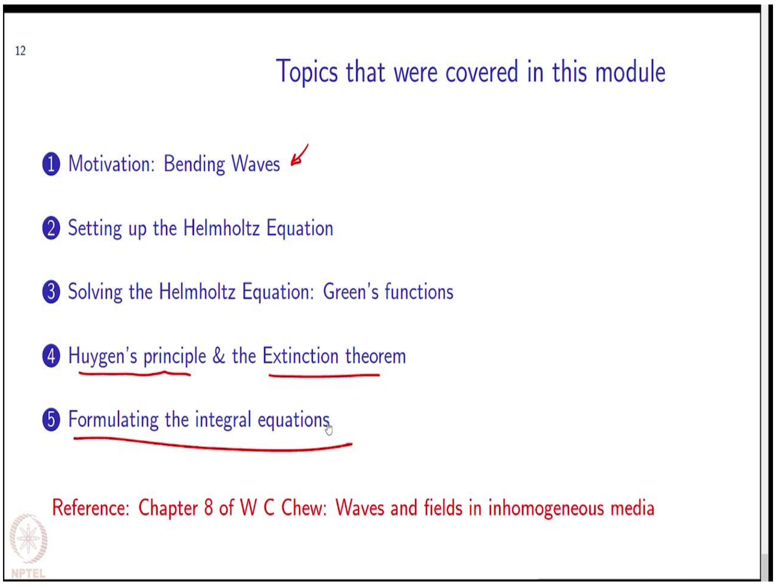

So, that brings us to an end of this discussion on surface or boundary integral equations and where we started with the motivation of wave spending, and found out this mathematical expression for Huygens principle. We use the extinction theorem to formulate the integral equations; then we just qualitatively discuss how to solve them? And we look at further details in upcoming modules.

So, the reference for this is this is a very nice book by Chew and book is waves in fields in inhomogeneous media chapter 8 right. So, any questions? Well it's grungy. I know I know the definition of $\nabla \times$ any vector and I have to calculate it, if not analytically numerically.

Student: So, in the (Refer Time: 07:19) exception to take those two equations right. From there which is the apply boundary condition.

Yeah.

Student: So, what was the (Refer Time: 07:25).

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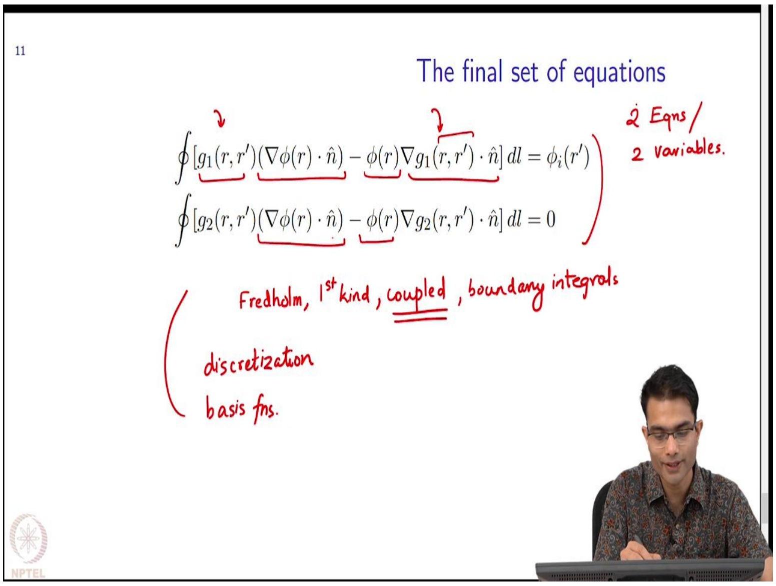

So, once we get these, once we solve these using this is the Extinction theorem right.

Student: Yeah.

We solve it, we get $\nabla\phi\cdot\hat{n}$.

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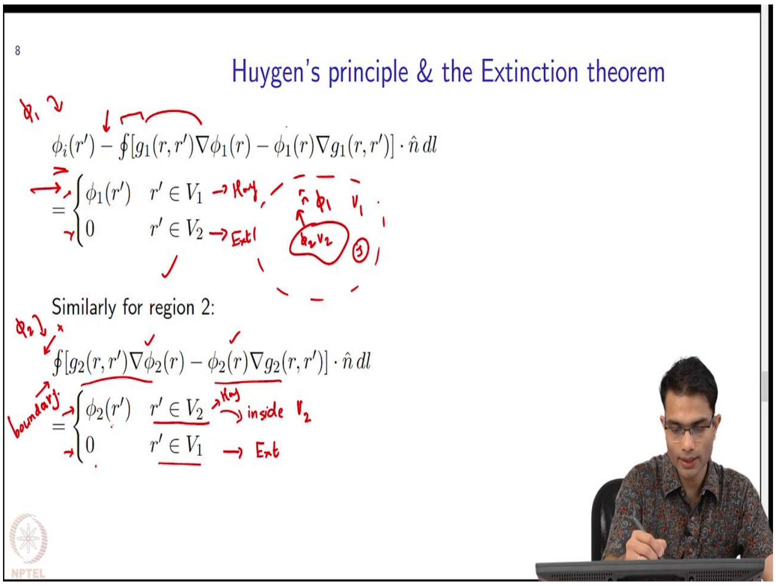

Once we have got it then we go back to Huygens principles and substituted into this one. So, I know this now, I know this now, I will get field anywhere so the two step process alright.