Computational Electromagnetics
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Surface Integral Equations
Lecture – 6.3
Huygen’s principle & the Extinction theorem

Alright; so, let us review the, we were discussing the surface integral formulation. And so, let us just review what we have done so far.

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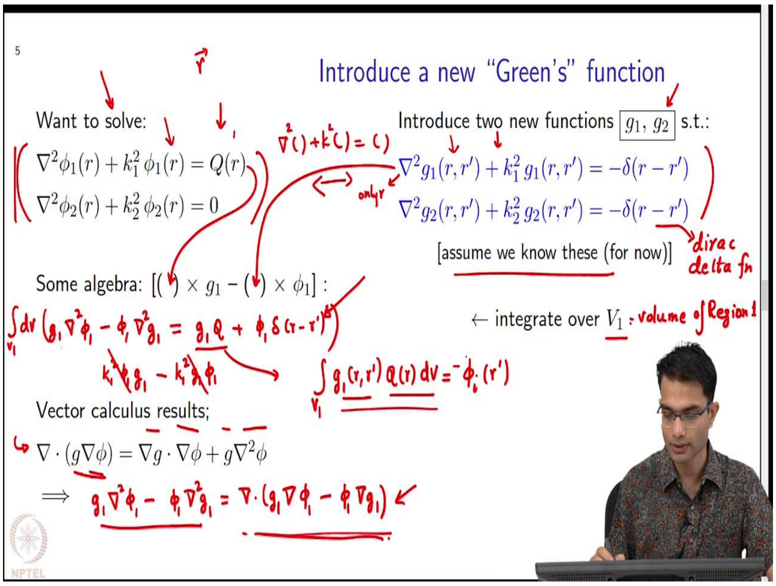

So, remember we had these two equations; yeah I had this equation and this equation, which at subtracted done some algebra and so on. So, let us just have a look at what we have done so far.

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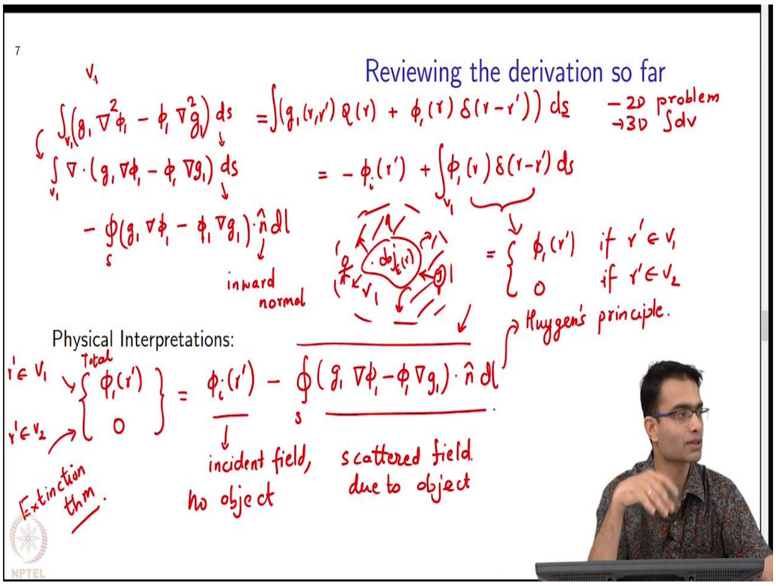

So, what we had, this was the left hand side right which we have integrated over this. So, 2D problem, so we have integrated over the whole surface right. And on the right hand side we had how many terms? Two terms right; the common terms of k_1^2 had got canceled off. So, what I was left with on the right hand side is so let us go back to this terms over here.

So, I had alright so this term over here so, g 1 that was the first term and then I had these were the terms that we had some right. So, this is just reviewing the terms that we had and of course, this whole thing was integrated both sides were integrated over S remember S because it is a 2D problem. In 3D it would be an integral dv that is the only difference, then we had to use one identity of vector calculus to convert this into a del dot something; and what was that something?

Student: g 1.

$g_1 \nabla \phi_1 - \phi_1 \nabla g_1$ and this remain as it is right this was over the entire this was over V_1. And then after that we use the divergence theorem in 2D and that came with a minus sign right. So, this became minus integral right over s. So, that was the left hand side that minus sign came because.

Student: (Refer Time: 02:54).

Because we choose $\hat{n}$ as the inward normal not outward normal, so this was due to convention. So, this was the left hand side right; what happened with the right hand side? So, one small change in what we did last time we had defined this integral $\int g_1(r,r')q(r)$ as $\phi_i(r')$. I am going to change that just a little bit by a minus sign that is just the convention.

So, we going to call this $-\phi_i$. So, this is going to become $-\phi_i(r')$, because the r coordinate has been integrated out and then I am left with this integral this over here right. So, $\int_{V_1} \phi(r')\delta(r-r')\, ds$ and this term over here they were two cases is equal to $\phi_1(r)$. Sorry this should be r.

Student: r.

So this should be a $\phi_1(r')$ if $r' \in V_1$ and 0 if $r' \in V_2$. So, what are the physical interpretations that we can draw from this? So, that is we will locate now. So, if I rearrange this I need not rearrange this. So, what am I saying what am I sort of seeing over here? If I just rewrite if I look at this these terms that I have so I have let me write on the left hand side now; so these are the two cases and is equal to what?

So, let me move everything else on to the other side. So, phi incident, I will take on the other side r' and this other term remains as it is right. So, I have a

$$\int_{V_1} g_1 \nabla \phi_1 - \phi_1 \nabla g_1 \cdot \hat{n}\, dl$$

that is where I was integrating. So, now, let us just try to give a physical interpretation to these equations. So, let us take the first case over here, when $r' \in V_1$.

So, what is it saying its saying. So, let us say that there is some observers standing over here in V_1 this observer and the source was over here right. So, that there is a field is emitted by this guy this field is going to get scattered by the object in all possible ways right. The observer here's supposing there was no object what would you observer see, there is no object only the current source and the observer.

Student: Source fields.

Only the source field there is no object. So, this is the incident field no object, now we introduce a new; now we introduce this object over here, what happens is I interpret this second term over here as so what should the interpretation of the second term.

Student: Scattered.

Scattered field we have defined this quantity earlier, then the difference between the fields in the presence and absence of an object is called the scattered field. So, this second term over here is called the scattered field, due to object right. So, that and on the left hand side this thing over here is what is called the total field. So, the total field is the sum of incident and scattered field intuitively they are always made sense, but now we are seeing it mathematically. This as I mentioned was is also known as the mathematical form of Huygens principle.

Now, let us see how that is? So what is this saying, the total field is now coming due to something that was originally there and where do all of these the second term over here; where is it non 0? It is purely a surface integral or in the 2D case a line integral. So, these are like secondary sources that are on the surface of the object right. So, in the very beginning when we started our discussion of this young's double slit in our we said that there are the secondary sources at the corners of the slit and they are emitting.

So, there are exactly like those secondary sources; because what is the location? The location is only on the boundary of the object and they are responsible for creating the scattered field. So, they may be anything happening inside this object this object can have any $\varepsilon(r)$ as a function of r; what we have done is we have some somehow converted that into this equivalent surface currents actually these are surface currents these are called impressed surface currents.

So, this is the mathematical form of Huygen's principle, what is the physical interpretation of the second term? This is when $r' \in V_2$. So, if I go inside the object what is the interpretation? The interpretation is that the incident field is exactly.

Student: Equal to.

Cancelled or equal to the scattered field inside the object that does not mean that the field inside is 0, it just tells you that this balance of these impressed currents as they called is such that outside the object it produces the correct field; inside the object it cancels the incident field. And this is the second term is as I mentioned extinction theorem, but do not worry about V_2 because V_2 will have its own equation right.

So, far everywhere you see the subscript 1 over here so, this was so, so V_1 when we go to V_2 remember what was a variable for the fields inside V_2.

Student: ϕ_2.

ϕ_2 right. We had said ϕ_2 is the field inside V_2 these equations do not talk about ϕ_2 they only talk about ϕ_1. So, and integration over volume 1 so this is telling me correctly what the fields are in V_1 no problem. So, now, let us sort of take all the now that we have done all the derivations so, we can just write keep the final form.

Student: phi 1 is the make field through volume V_1.

Yes, ϕ_1 is the net field that is what we will assume in the very beginning volume 1 ϕ_1 volume 2 ϕ_2. So, now, let us put all of these volume 1 and volume 2 together and see what happens.

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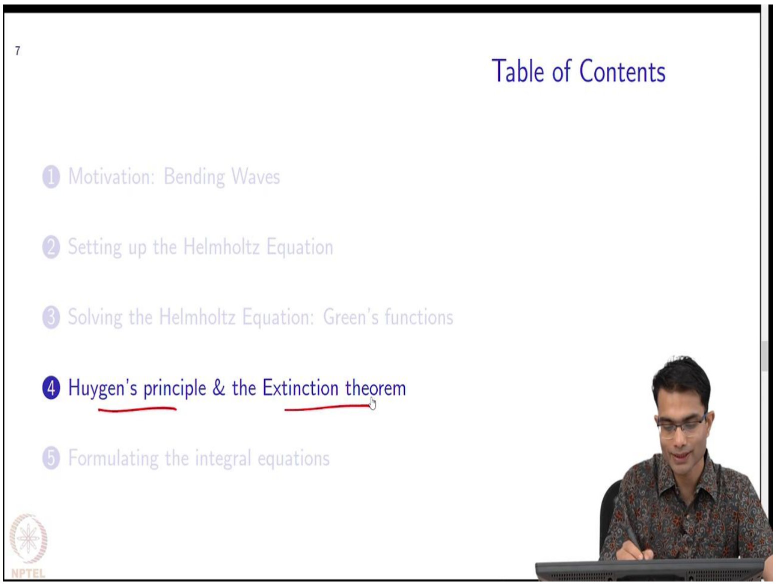

So, let us formally the Huygens principle and extinction theorem.

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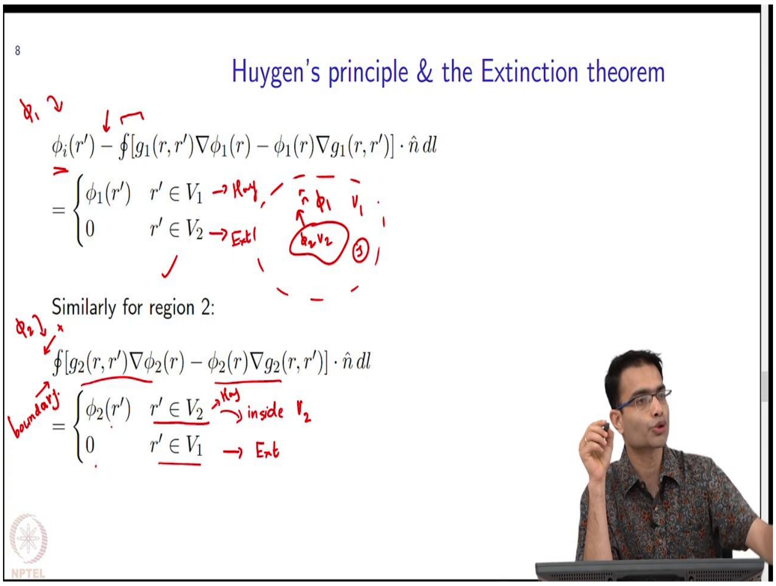

This is for ϕ_1; so, this for ϕ_1 over here the same equation that I had written before ϕ_i and the second term over here is the scattered field right. I can repeat the same process for volume 2 right. So, if in case you forgot this was my V_2 and this is my V_1 right. In volume 2, was there any current source? No.

So, how will this equation be different for ϕ_2? It will be identical accept that there is no incident field term that incident field term came as the integral of the current.

Student: Current.

But there is no current in volume 2 so when I repeat this derivation this is exactly the field that I will get; another important thing to note this scattered field term over here had a minus sign, because the normal was inward. In the second term over here what is the sign over here plus why because I am using the same $\hat{n}$ and for volume V_2 this is an outward normal so everything is consistent. Now evaluating the delta function in the case of region 2, again the situations reverse here when r prime belongs to V_2 the field is ϕ_2; and when r' is outside of V_2 that sits in V_1 then that delta function is not there in a region of integration, so, the answer is 0. So, what is the physical interpretation now? So, this one is we have already interpreted in this case what is it saying that if I am inside V_2, the field is simply the field that is produced by these surface currents right, ϕ_2 is equal to this. So, somehow what we have managed to do if you have managed to remove the volumes and talk only in terms of currents on the surface; that is why these what that is why the whole module is called surface integral equations.

Student: Sir which is surface (Refer Time: 12:17) boundary.

It is the boundary yeah, by surface I mean boundary in 3D surface is a surface in in 2D the surface will be just the line in closing it right. And to tell to make sure that its a surface or a boundary we are talking about the best indication is that this integral is a symbol of integrations the contour integration right. So, boundary integral actually also in the literature instead of surface integral you will find that these are called boundary integrals for the same reason remain the same thing. So, that is the story so far about these impressed surface currents. They are acting in some very strange way where in a strange way where they cancel the fields in an outside such that the field here is ϕ_1 and the field here is ϕ_2.

Now, even so far we have yet to encounter r boundary conditions so, but the good thing is that we have separated the physics into region 1 region 2 and one term on the boundary. So, that boundary term is where we will get to impose the boundary conditions, right now it does not seem very clear how we will do, but let us go towards it. So, let us formally these integral equations.

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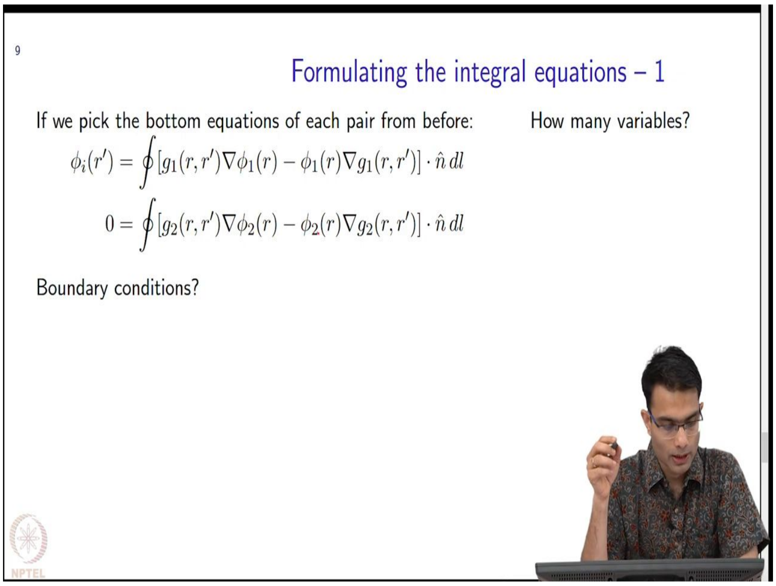

So, what I will do is now to solve these sets of equations, I am going to take these bottom equations these were the extinction theorems; both of these were extinction theorems and both the top ones were the Huygens theorems. So, I am going to take the bottom equation from each of these sets and just rewrite them.