Computational Electromagnetics
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Introduction to Integral Equations
Lecture - 5.3
Basis Functions

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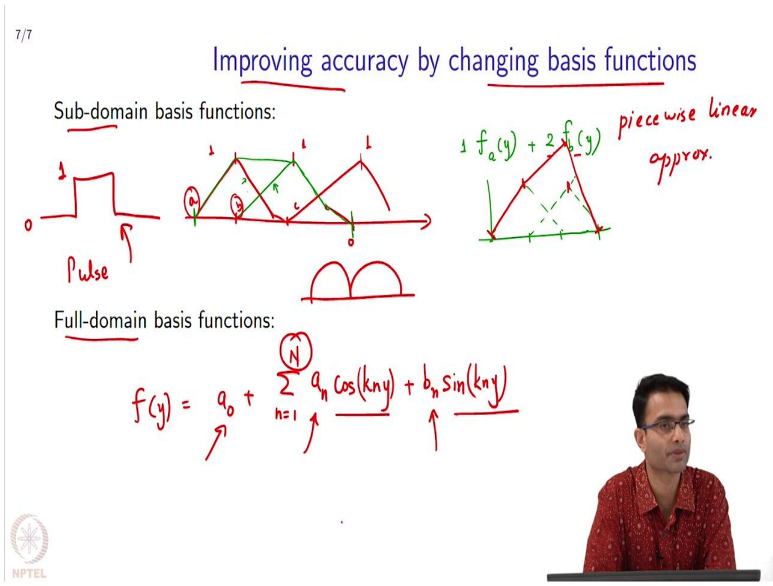

So, that brings us to the final discussion on improving accuracy by changing the Basis Functions. So, we will talk about basis functions. There are broadly two classes one are called sub domain and the other are called full domain. So, what does this mean? The pulse function which you already saw right so the pulse was right it is 0 over here and 1 over here.

So, this basis function is non zero only in some domain of the entire domain it is not nonzero everywhere right that is why the word is self explanatory that is why it is called a sub domain basis function.

So, instead of a pulse you can also do better you can do. So, these are like the Taylor series this is the first term of the trailer series constant the next would be linear right. So, you can have these triangular basis functions. So for example, this is one basis function the other basis function can be sometimes like this and so on.

So, these are another class so this is so, this is basis function a, basis function b, basis function c. So, the way these are constructed is very clever again the place where this there is 1, the other basis function has a value 0. Similarly you can see this function is one over here my drawing is little bad, but this 0 supposed to come over here and similarly for this should match.

Student: Sir (Refer Time: 01:52).

Yeah. So, a is the first basis function, b is the second basis function, c is the third basis function and so on. So, if I for example, just let us just take basis function a and basis function d. Supposing I add these two basis functions what will the result look like? So, let us start from here and go all the way up to here. So, I am adding a and b right.

So, when I start from here what will this function look like b is 0 right. So, this guy will go like this when I come to this point b is still 0. And now by the time I come over here it is going to be 1 again. So, this is going to be I am adding this guy and this guy I am going to actually get a line. You can see that these two curves they are going to add to a constant and then, as I go down it is going to follow this all the way to 0. Because basis function a is 0 by now.

Right. I can extend this idea supposing now I do something like basis function a as a function of y. Let us not choose b choose basis function a as the function of y. Let us give it some value 1 and 2 times the basis function b. So, what will happen? How will this guy look? So, this is where right so basis function a is like this, basis function b is like this.

Now, I am combining these two basis functions, but with a different height. So, what will it look like now? When I start from here the value will follow along a right. So, it is going to be read like this and this height is being forced to be 2. So how will it go from here to here? Straight line.

Student: (Refer Time: 03:57).

Yeah. So, it is going to be the value at this point of the function will be 2 because at this point fa is 0 and fb is 2 right so I mean 2 times fb is there. So, this guy is going to go up and then come down over here. So, by doing this, what are what will you call these three segments? They are linear. So, what I have done is I have replaced this function by piece wise linear approximation.

So, my function that I am approximating now is continuous. Earlier my function that I was doing expressing using pulse basis function was discontinuous. So, I have now moved to a continuous basis function, but still not differentiable right it is not smooth, but it is it is better than pulse. Now only price you have to pay now is calculating a mn will require a little bit more work earlier my you know in the numerator my g was just 1. So, it was very simple to integrate now it will be a linear function right.

So, you can extend this idea you dont have to deal with lines you can deal with you know half cosines these can be your basis function. So, those are about sub domain basis function finally, there are full domain basis functions which as the name suggest they are nonzero all through the line. Any guesses for one very popular full domain cos or cosh you all studied in first year of engineering.

Student: Full domain.

Full domain means it is nonzero everywhere in the domain the basis function sink is a more complicated something simpler.

Student: Cosh.

What about combination of sine and cos what are they called?

Student: (Refer Time: 05:53).

No more complicated what are the combination of sine and cos called you studied in first year singles and systems.

Student: Fourier.

Fourier series right, so if I have some I can write down my basis function as like this. So, I can see $f(y) = a_0 + \Sigma^N{}_{n=1} a_n cos(kny) + b_n sin(kny)$. You have all studied why Furrier series are very nice they can approximate a function a periodic function very well blah, blah right.

So, your basis functions now have become this cos and sin cos and sin are nonzero everywhere and now I am representing them by coefficients a_n and b_n and there is of course, a dc term that I have to take care. So, if I know beforehand that my solution is going to look oscillatory then why choose pulse basis function or piecewise up linear.

Next we will choose the function we choose the basis function which I intuitively now is going to satisfy the I mean it is going to be close to the basis function. By doing that I will get away with a small value of capital N, if the actual solution is sinusoidal

Student: (Refer Time: 07:08).

Yeah.

Student: (Refer Time: 07:11) divided by root (Refer Time: 07:13).

Yeah they are divided by root I mean the r term will remain as it is now your numerator will have one function of y.

Student: (Refer Time: 07:20).

Gauss quadrature Gaussian Gauss Legendre any quadrature rule. I do not care what the function is I just need to evaluate at one point. So, more and more you will appreciate the power of a quadrature rule you don't care about the function. Just evaluate it. sin and cos are easy to evaluate what is there right. So, that is the strategy that we will use. So, this is also bit of a art choosing which basis function depending on the problem. So, with this we can bring this particular introduction to integral equations to end.