Computational Electromagnetics
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Applications of Computational Electromagnetic
Lecture –14.21
Finite Element-Boundary Integral - Part 3

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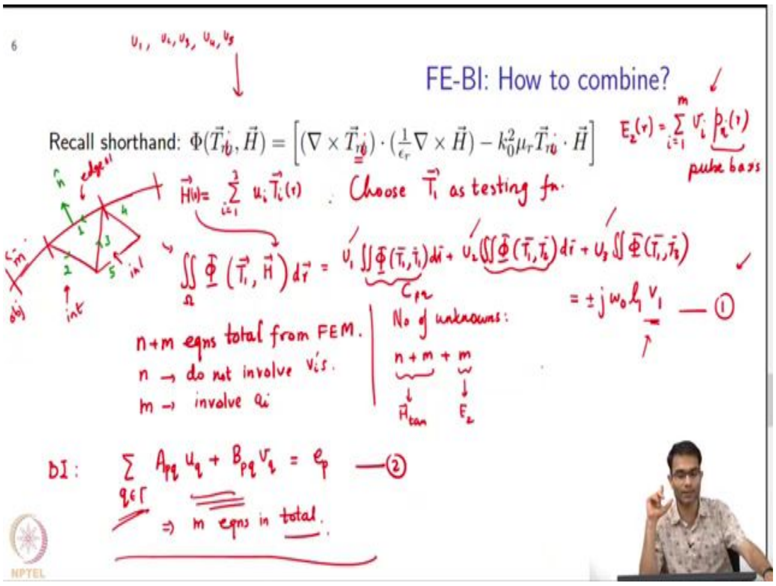

So, now, let us come to the overall matrix equation ok. So, this whole thing is just going to be some number right. So, this will be some matrix you call it C.

Student: (Refer Time: 00:28).

Pardon me ok. So, let us rewrite this so, equation 1 ok.

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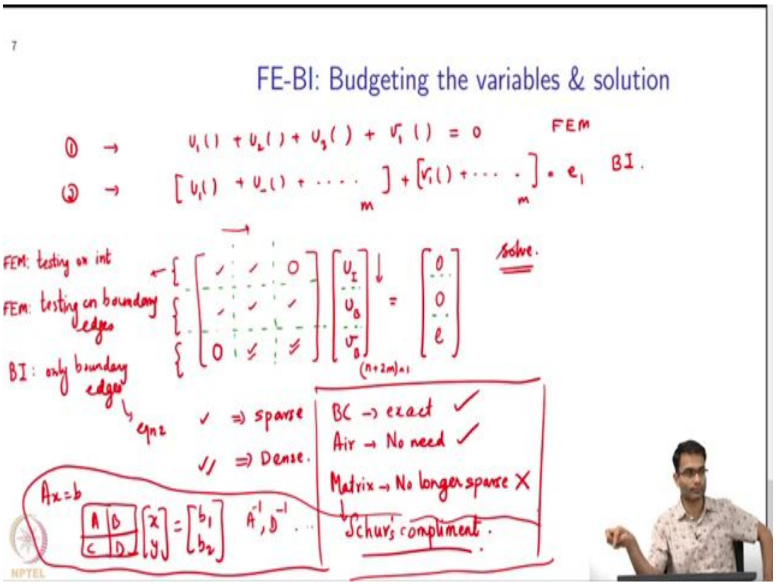

So, equation 1 is going to be of the form, there is going to be $u_1(\ )+u_2(\ )+u_3(\ )+v_1(\ )=0$ right. The second equation is so, this is from your FEM second equation is going to come from BI. So, what will this have for example?

So, again it will have $\sum_{i=1}^{m} u_i(\ )+v_i(\ )=e_1$

Student: (Refer Time: 01:13).

Yeah.

Student: We have the same equation (Refer Time: 01:18).

We will come that, we will come to that let us just do the book keeping of equation 2. Equation 2 will involve all the boundary us and the boundary vs right. So, this is my boundary integral.

Student: (Refer Time: 02:05) this is just equation that equation 5 (Refer Time: 02:07).

Yeah, this is just equation 5 written in terms of variables nothing different about it and equation 6 we did not touch ok. So, we have put all of these things together the question was what about interior triangles then supposing I have a triangle now like this ok. So, with edges

4 and 5 over here what happens when I write this equation? So, when I take this triangle over here my equation will have u_3 something. So, if I test along let us say edge 3 what all unknowns will appear if you remember from your FEM module?

Everything that is common to u_3 will come. So, I will get a u_1, u_2, u_3, u_4, u_5, but no others will be 0 right. So, that is I mean that is how it will; that is how it will appear. So, this the second part of interior edges there is no change at all it is as you do FEM before that is how you would do ok.

But when I combine all of these together what is the overall let me write down the unknowns now, unknowns are going to be. So, u in the interior, u on the boundary and v on the boundary these are my unknowns right. So, this is n+2 m length vector ok. Now I am going to partition this like these 3 parts ok.

So, the FEM equations yeah.

Student: The second equation will have m and m (Refer Time: 04:07).

Yes these are m of them and these are (Refer Time: 04:10).

Student: First one we (Refer Time: 04:14).

The first is us which are H tangential and the next are v s which are.

E_z ok. So, what do you think will be the structure of this matrix that I have shown you over here I have broken it up into sub parts. So, when you take. So, for example, the first these equations will correspond to testing on interior edges ok. Those are the n equations, but testing on the interior will it involve any. So, the example which was being I just now.

Supposing I take a interior triangle something like this and I test along an interior edge. So, example 3 is a good example of an interior edge. So, the equation for 3 involves u_1 to u_5 correct that is what I will get will there be any v s in that?

Student: (Refer Time: 05:23).

There will not be any v's in that right. So, when I look at this equation over here, I will have all the us I mean not all whatever us are close to it are associated with it, but the v's will be.

Student: (Refer Time: 05:37).

v's will be 0 correct. So, when I write this equation over here, I will have some u's and v's will be 0. So, I am going to write a 0 over here because we are multiply this guy in this fashion with the elements over here think of writing one row of an equation.

So, it will have the ui's and the ub's, but no v's because these are interior edges then I move to the FEM equations, but considering the boundary edges right. So, that is the example that we took over here for example, this equation 1 it involves the u's and the v's ok. So, it will it involves the u's and the v's. So, all of them are non zero in general finally, when I come to. So, this is testing on interior this is FEM, then I take this, this is FEM testing on boundary edges this involves as you have seen it involves if you look over here, it involves u_1 , u_2 , u_3 and it involves v_1 which is on the; obviously, on the boundary only.

So, it involves all of these 3 blocks will be non-zero ok. So, this is there this is there this is there this is there and this is there ok. Now we finally, come to the remaining m equations which are from the boundary integral only boundary edges, which of these blocks will be 0 which will be non zero; first block is going to be 0 because the equation 2 that you have seen over here only involves the u's and the v's on the boundary right. So, this will be non zero is equal to.

Student: (Refer Time: 07:56).

Yeah.

Student: (Refer Time: 08:00).

The last is the second equation right ok. Which of these entries are non-zero? When I test along the any of the FEM edges will I get any non-zero term on the right hand side. Look at for example, equation 1 is there any right hand side term? There is no right hand side term and I am left with just the e which is coming from BI. So, I have system of equations which is correct I have n+2m equations in n+2m variables, and now I can solve this.

Notice I have put a single tick on some boxes and a double tick on a another box, what might be the reason what is the single tick imply from FEM, but little bit more what will be the nature of this equations sparse right? So, single tick is sparse double tick is going to be dense because it involves every element every equation involves every in my equation 2, every edge is involved in everything that was the case in the boundary integral right surface integral every edge was involved that is why my system of equations was dense. So, this is dense. So, what was the let us summarize in terms of sort of a advantages and disadvantages, I have boundary condition is what happened I replaced ABC by an exact boundary condition right. So, it is exact. So, that is a great plus point ok.

Air; do I need to discretize air now? No need. So, my boundary condition I mean my domain of discretization has shrunk to the bare minimum required what is the disadvantage? System of equations is larger, but marginally. If it take a very large volume and take the ratio of interior edges to boundary edges it is not much of a price.

Student: (Refer Time: 10:33).

It is dense right. So, my system of my matrix is no longer sparse that is a big disadvantage because now I no longer have a nlog(n) solver, but it is not as bad as the if I were to I have made the whole thing into a for example, if I made the whole thing using volume integral method, then I will be discretizing the entire volume into one dense system of equations right so, it is partly sparse partly dense ok.

So, I need to come up with a special solver that can exploit the fact that part of the matrix is sparse and part of the matrix is dense ok. So, there are some ways of doing it one one example is using any guesses?

Student: (Refer Time: 11:34).

MATLAB is there; is the word Schur (Refer Time: 11:40).

Student: (Refer Time: 11:41).

Schur.

Student: (Refer Time: 11:43).

The decomposition. Is it Schur decomposition Schur ?

Student: (Refer Time: 11:48).

Schur decomposition was that decomposition we did of a matrix right. So, this is what we have what I am saying is not Schur decomposition, but Schur compliment ok. So, Schur compliments splits a matrix into.

Student: 4.

Into 4 parts right. So, for example, if I were to you can split this using Schur decomposition and inverses of sub matrix are inverse. So, you can further sparse part use the nlog(n) and solver for the dense part use a regular solver right and that is how it is done in the literature. So, if you just give this n+2m system of equations to matlab or any solver of your choice it is a bad idea.

Because, you are not telling the solver to exploit the structure of the problem. In any linear algebra problem if you can exploit the structure of the problem you will get big wins. If you did not tell it, it would treat this as a dense matrix of size n+2m and it will do the whole donkeys way of order n cube right the Schur's compliment is one way of leveraging this, I mean I will not cover it, but you can just look up Schur compliment on line.

Student: (Refer Time: 13:01).

It is not the same I mean I will when you when you see the Schur's compliment to you will realize very different from Schur's decomposition.

Student: That compliments the same as the (Refer Time: 13:16).

No anyway that is a separate part the main sort of summary over here is how we manage to combine the best of 2 worlds right. So, we did FEM and this is specific right I choose a vector FEM, when I choose a vector FEM my unknowns became the unknowns along the edges.

Student: (Refer Time: 13:33).

Yeah.

Student: (Refer Time: 13:39).

Yeah the other advantage, I get E_z and H on the boundary which is great because where does that directly plug into? When I want to calculate the RCS using Huygens principle, I need E and H. In the FEM what happen was I got H everywhere, but need E for calculating Huygens principle. So, then I will take a average over to elements or something like that here I get it.

Student: (Refer Time: 14:05).

In the interior we have only H everywhere which is good enough actually if I am solving a RCS problem Radar Cross Section problem, I do not care about these n of these values, I do not care about, but I still have to solve for them what do I do with it right. Finally, radar cross section depends on field far away field far away are right from Huygens principle, Huygens principle only needs the boundary. So, I need only 2m of those guys for the RCS not these n interior fellows.

But there is no way to decouple. So, I have to solve for everything. So, this comes out very nicely in this the only challenge is to write a good solver that exploits the structure, which is the solved problem in the literature you just have to look it up it has been done ok. Same principle could also be used if you wanted to write a mode solver to find the eigenmodes of some wave or whatever using this approach.

There also you will have the same problem, you have FEM for a wave guide, but then you need some air outside of it to apply boundary conditions. So, why do it apply boundary integral? So, the idea is what I want to convey what how to cleverly combine, change the boundary condition and get it from somewhere else.

So in fact, your commercial solvers many of your like HFSS and so, on they have nowadays these options inside either absorbing boundary condition or boundary integrals. So, now, you know what they are doing inside ok. So, I think that brings us to an end of hybrid methods here.

Student: Can we (Refer Time: 15:45).

Yes you can (Refer Time: 15:46) Huygen.

Student: (Refer Time: 15:47).

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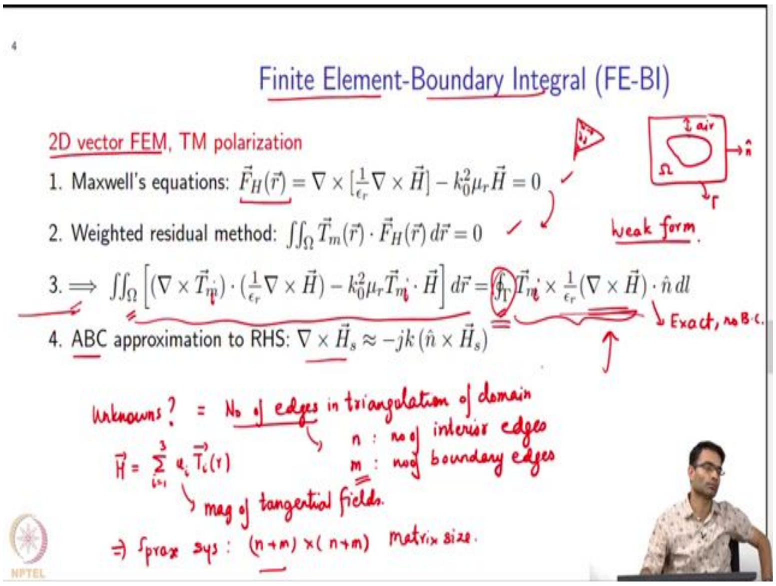

Yeah. So, to use it for Eigen an Eigenstructure calculation you would actually go back to step 1 itself, step 1 itself is where you will have to put in your H_{tan} have a $e^{j\beta z}$. So, you will follow the same idea, but you will have a an extra beta square term hanging out everywhere which we have to deal with carefully. If you deal with it carefully same recipe will hold because what we have finally, done is we just modify the boundary condition.

Student: (Refer Time: 16:22).

Yeah.

Student: (Refer Time: 16:36).

Yeah.

Student: There we had a (Refer Time: 16:28).

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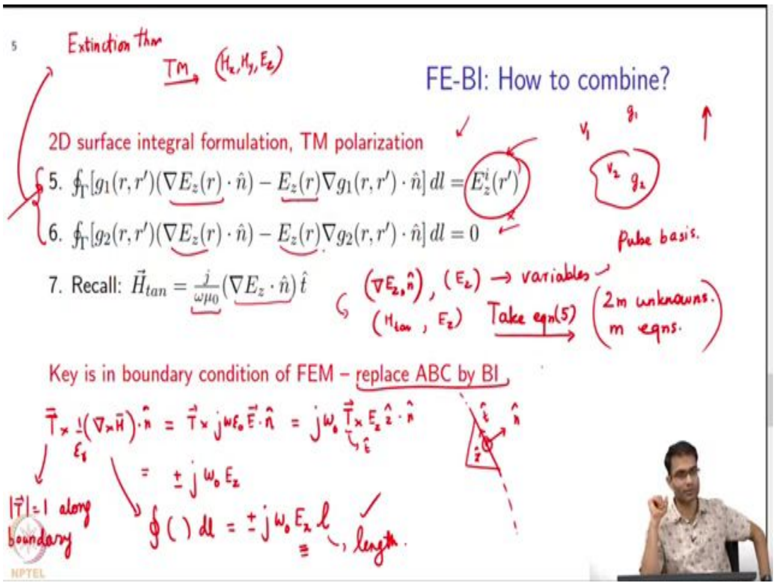

There we have. So, yeah everything has to be done carefully.

Student: (Refer Time: 16:31).

Of course, everything has to be changed if your problem changes right if you have a wave guide problem whose modes you want to calculate there is no e incident.

Student: This is basically (Refer Time: 16:42) calculate the incident field (Refer Time: 16:45).

Yeah, the total field in a computation domain is what we are able to calculate by doing all of this.

Student: (Refer Time: 16:52).

No. So, this is allowing the overall boundary we had actually worked it out when we write it for each element these terms cancelled off along the individual boundaries and you are only left with the outer most boundary that did not cancel. See at this point FEM has actually not come in. We had applied which theorem? We had applied your divergence theorem to convert one surface integral into a one area integral into a line integral which is why the right hand side became line integral.

So, this is true only on the boundary so.

Student: (Refer Time: 17:34).

So, T_i is going to be for a interior edge is going to be 0 on the right hand side. If i is a interior edge I am testing along that only the left hand side is 0 right hand is 0.

Student: (Refer Time: 17:47).

Third is say a again.

Student: Third is just a boundary condition.

Third is the boundary condition on the right hand side, left hand side is still the the LHS a matrix right that is going to give your a matrix Ax=b. So, I do not want to go more into how the Schur what the Schur compliment is, but it basically instead of right now what we know is given Ax=b how to solve it? Schur's compliment on the other hand breaks up A into let us say 4 part ok.

So, let us say A,B,C,D and then you write your split your variables also as xy and let us say b_1 b_2 and it there are certain formulas which allow you to write the final solution in terms of A^{-1} and D^{-1} and so, on. So, you do not have to take the inverse of the whole thing. So, for A^{-1} for example, A might be the sparse part. So, its inverse is easy to do right.

So, those are the kind of tricks that Schur's compliment does.

Student: (Refer Time: 18:48).

One of them will can be a 0 block and so, on right. So, this is I mean this is pure linear algebra ok.