Computational Electromagnetics
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Applications of Computational Electromagnetics
Lecture - 14.20
Finite Element-Boundary Integral - Part 2

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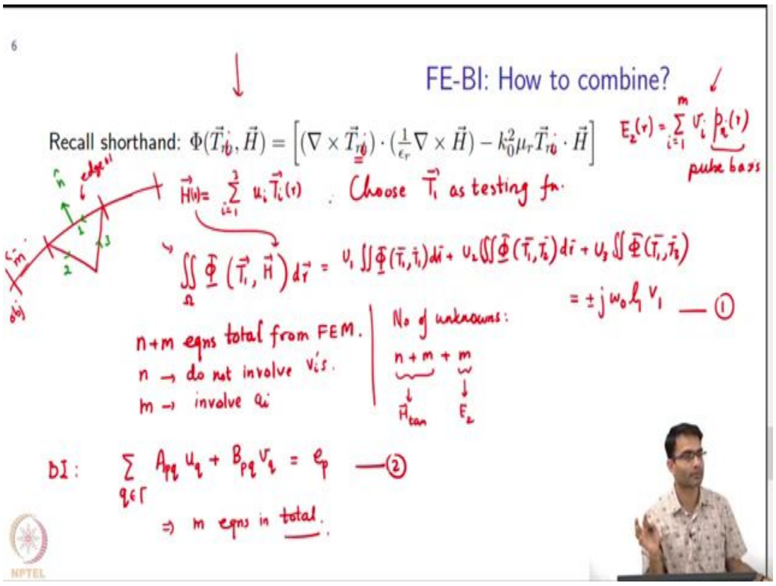

There is one short hand notation which I will ask you to recall.

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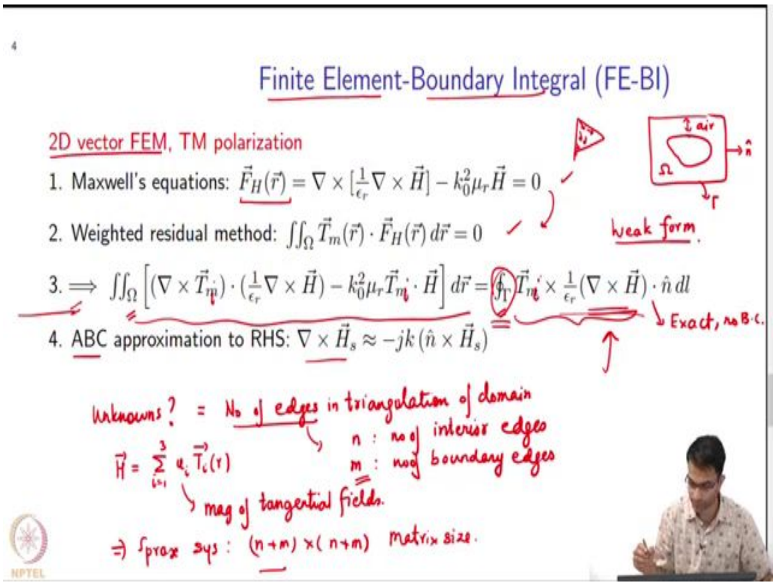

This if you notice this FEM this whole term over here becomes very tedious to write again and again. So, I am I have called it this Φ we had done this earlier also ok. So, this Φ you see is just this integrand ok. So, now what we are going to do is we let us take a object is over here let us take my one triangle on the boundary ok. So, this is my outward normal and I am just going to number these edges as 1, 2 and 3 for simplicity.

So, in the FEM what did I do? In the traditional FEM, I basically cycled through all different possible values of these testing functions, I took the testing function to be each one of these edges and I wrote down my system of equations and since there are n+m edges there are n+m testing functions. So, we are familiar with all of that, but now let us look at that equation which will get altered by what we have done and that will happen when my T_m is on the boundary ok.

A boundary edge is what is going to invoke this right hand side that is otherwise T_m is 0 on the boundary right; if T_m is not on the boundary if this m is not on the boundary this term is 0 bad choice of variables m here is suppose to be variable, but we made it equal to number of boundary edges, but hopefully its clear you can make this all into i if you want.

So, what is the magnetic field expressed as you already know this is going to be written as $\vec{H} = \sum_{i=1}^{3} u_i \vec{T}_i$. So, my FEM equations are going to give me I am going to choose a testing function right. So, I am going to choose T_{m1} as testing function and generate one row of my matrix equation from FEM only one row ok. So, what will happen; so, integral over this triangle is going to be $\int_\Omega \int \Phi(\vec{T}_1, \vec{H}) d\vec{r} = u_1 \int_\Omega \int \Phi(\vec{T}_1, \vec{T}_1) d\vec{r} + u_2 \int_\Omega \int \Phi(\vec{T}_1, \vec{T}_2) d\vec{r} + u_3 \int_\Omega \int \Phi(\vec{T}_1, \vec{T}_3) d\vec{r}$

You notice this guy it is it is linear in H if I replace $\vec{H}$ by $\vec{H}_1 + \vec{H}_2$ it will just become capital phi of the sum of those two terms right. So, this is where I am getting my this row will only involve u_1, u_2, u_3 and no other edges of the entire formulation that is why it is fast right and now this is going to be equal to what?

Student: Boundary.

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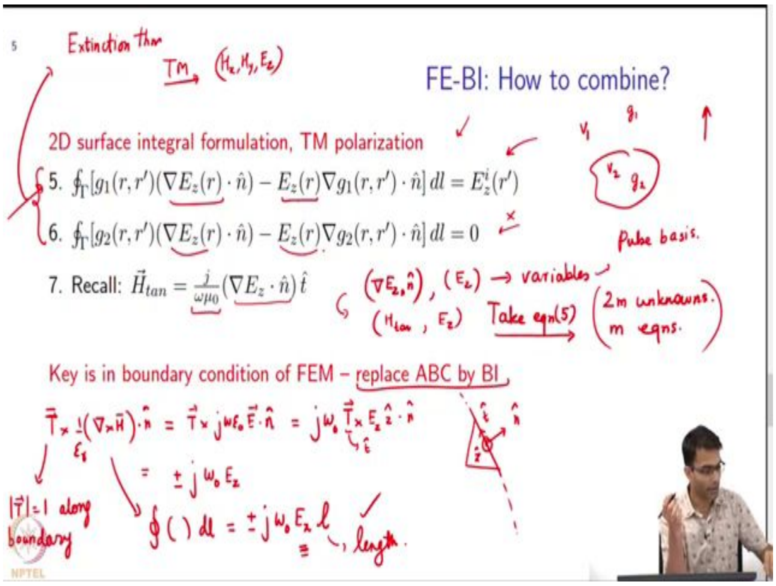

The boundary term which by our careful thing over here is going to be this term right. So, it is going to be $\pm j\omega_0 l E_z = \pm j\omega_0 l a_1$ where $E_z(r) = \sum_{i=1}^{m} a_i p_i(r)$.

Remember this is one boundary one edge along the boundary right and then there is going to be another edge and then there is going to be another edge like this there are how many such edges?

Student: m

m edges right and the pulse basis function is one along each edge right. So, over here this is going to be basically a_1 if the numbering begins from here if this is also edge 1 right. So, let us called this edge 1.

So, you can imagine there is some book keeping to be little bit careful about how you are numbering things fine. So, what kind of a sort of system of equations have I got now? So, if I just do the FEM system of equations I will have how many equations; I can test along each of the boundaries I mean each of the edges. So, n+m equations right, but in m of those equations right. So, I will have n+m equations total from.

Student: But, you cannot apply the boundary conditions (Refer Time: 07:05).

Yeah we are coming to the boundary conditions n of them are interior. So, they do not.

Student: (Refer Time: 07:130).

Involves the a_i's; obviously, m of them involves a_i ok, but our number of variables in this problem has become how much now. If I am going to use this equation 5 over here my number of unknowns has now become.

Student: (Refer Time: 07:42).

n+2m so, it is going to be n+m+m. So, what are these? These are H tangential and what is this?

Student: (Refer Time: 08:00).

E_z so, I am making my system of equations a little bit larger right and the FEM equations are going to involve this E_z fellows right and what happens to I mean there are not enough equations right now, I only have n+m equations whereas a number of unknowns is n+2m

But I know that if I use equation 5 over here. I am going to get another m equations. So, what will the BI equations look like? The BI equations the boundary integral equations involves what and what? So, for example, when I take equation 5 there is E_z and there is.

H tangential, H tangential is related to?

Student: E z.

No, H tangential is related to u's.

Student: (Refer Time: 08:57).

H tan is u's and E z is.

Student: a.

a's right. So, if I write this out into a matrix equation what will I get I am going to get something of the following form.

$$\sum_{q\in\Gamma} A_{pq}u_q + B_{pq}v_q = e_p$$

Actually you know what instead of calling is a let us just call it v. So, that the there is no confusion between lower case a and upper case a ok. So, the coefficients in the pulse basis for the electric field I am calling v fine let us just change it from small a.

So, this is the equation and what is the right hand side going to be equal to? Is going to be equal to this guy incident field; So, this is m equations in total and most importantly what has happened is the information about the electric field the incident electric field has nicely appeared over here. So, now, do I have enough equations? If I combine so, combine this 1 and this 2, how many unknowns do I have? n+m+m.

Student: No, there will be a there will be (Refer Time: 10:58).

In the total problem, the total problem has a interior discretization as well as boundary discretization; interior discretization.

Student: (Refer Time: 11:07).

Has n edges and those unknowns continue to be unknown they are not determined. So, those are those n, on the boundary I have my E_z and H tangential both are undetermined. So, how many unknowns are there? n+2m. How many equations you have in total with me now?

Student: m plus (Refer Time: 11:29).

m+n from FEM n from boundary integral. So, I have actually manage to merge these two in a very clever way.

Student: Second thing (Refer Time: 11:40) one.