Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering (EE)
Indian Institute of Technology, Madras

Applications of Computational Electromagnetics
Lecture -14.18
Hybrid Methods - Motivation

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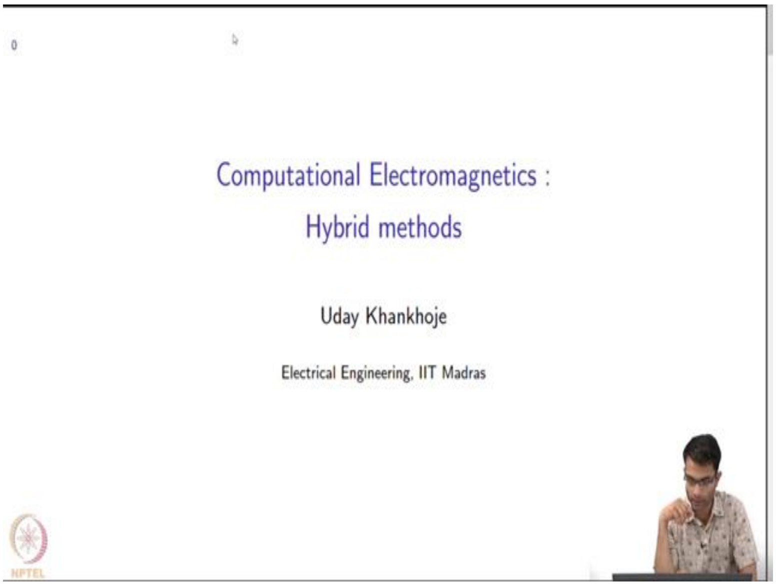

So, the final module that we are going to talk about are what are called Hybrid methods ok.

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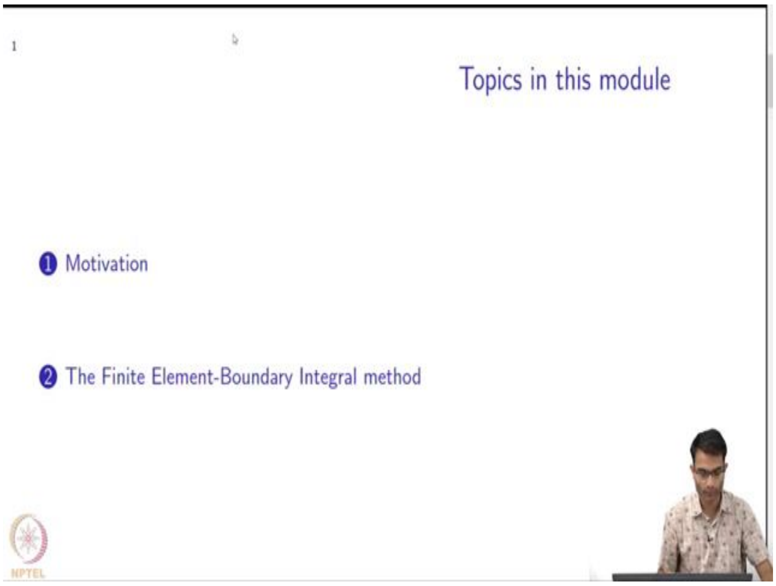

So, of course, we need to motivate what hybrid methods are and give a particular example of a hybrid method. So, what does a word hybrid imply? So, far we have studied individual methods we have looked at we started with integral equations and we looked at finite element then we looked at finite difference time domain right each method has their plus and minus points.

Hybrid methods are methods where we combine one or combine at least two methods. So, that we can take advantages of both methods into one sort of a formulation; in particular this module we are going to see how to integrate finite element method with boundary integral method. So, let us see let us try to motivate the most specific reasoning behind this ok.

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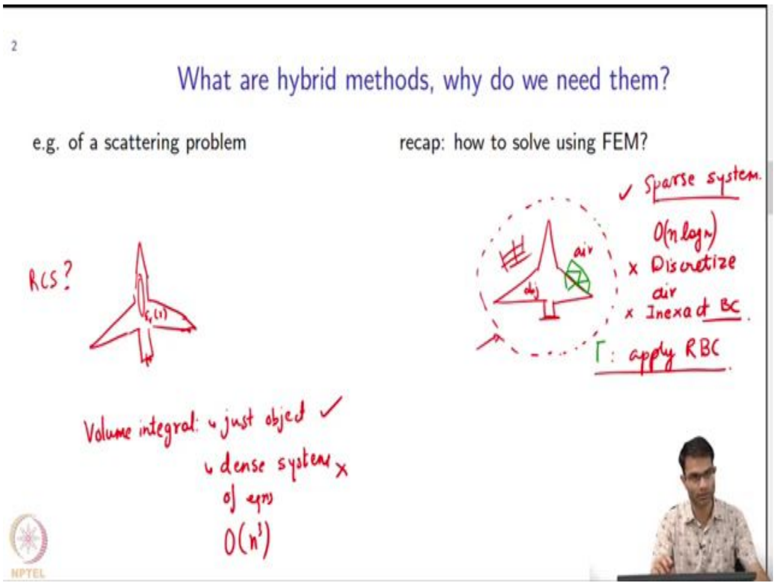

So, what are these methods and why do we need them? So, we can take any example a particular example that I will take is that of a scattering problem ok. So, scattering problem a typical problem is for example, there is a aircraft right, this is supposed to be an aircraft and it has some various components inside it and what are we trying to do, we are trying to find out the RCS of this object now, if I were to solve this.

So, let us take a realistic case where, I should not consider the entire object to be made up of one material right aircraft it will it will have a metallic frame, it will have a glass cockpit, it will have a some plastic components, the rubber tire, all of them they are different-different components. So, realistically I cannot just discretize the boundary of this object and solve the problem, I need some volume method either so, what are the volume methods that we know of?

Something that can handle you know ε_r as a function of space, what are the methods that we know of? There is volume integral method and then there is finite element method, where I go inside and discretize the interior of an object right. And these boundary integral method, supposing I want use a like a surface integral method for this problem, what will be the problem? In integral equations right, I can say that I will make my unknowns using Huygens principle to be the outermost boundary of this, what is the technical difficulty in this, if I use surface integral formulation? Boundary may be complicated, what is the more difficult challenge?

Student: Green’s function.

Green’s function for that medium will not be easy to calculate, I know Green’s function for free space that is easy to calculate, but Green’s function for a heterogeneous medium is not possible. So, that is the technical challenge that is why I cannot do surface integral formulation. So, my two options are FEM and volume integral formulation ok. So, let us take and whatever the main differences between FEM and volume integral in terms of computation.

Student: Response.

Right, FEM gave me a sparse system of equations volume integral give me a dense system of equations. So, in terms of speed FEM was very very fast and volume integral is much more time consuming. So, at least for engineering problems very commonly you would use FEM and many commercial software use FEM ok.

So, if I wanted to solve this problem right. So, this is I am asking what is RCS of this problem. So, the problem over here this is the problem that I want to calculate. Now, if I wanted to solve this problem using FEM, how would I do it? You do not have to list all the steps, but what is the main step? So, let us say that you know this is let us just make our aircraft once again ok.

Student: Break it into triangles.

Break it into triangles ok. So, break, but first of all what is, before I break something into triangle, what is the domain of interest now? Is it the boundary of the aircraft or little bit more?

Student: Little bit more.

Little bit more, why?

Student: (Refer Time: 04:35).

Because I need to apply the boundary condition and there the simplest boundary condition which we have learnt are absorbing boundary conditions and for that we have seen that there needs to be a little bit of air gap between the object and where I terminate the boundary right. So, I may have like a boundary like this just for simplicity I am showing it like a circle, but it can take conform to the shape of the thing right.

So, this is air, this is the object and now this whole domain has to be fractured into triangles and so on in the case of finite element, and this is how I solve it, this is the sort of a outermost boundary on which apply RBC and this gives me a sparse system and remember the problem with volume integral was. So, in volume integral, what would be the boundary that I have to choose?

Student: Aircraft (Refer Time: 05:46).

Just the aircraft right; so, it is just the object, but the problem is dense.

Student: Dense.

Dense system of equations right; so, this is a plus point and this is a negative point dense system of equations right. So, you will find that this becomes impractically difficult to solve because to solve a dense system of equations what was the complexity if you remember.

Student: n cube.

Roughly it is little bit less, but its order of n^3 and for the sparse system of equation it is.

Student: (Refer Time: 06:23).

It is more like nlog(n) ok, but still I mean log(n) is a small number. So, as n becomes larger and larger you are going to a 3-D aircraft or 3-D ship or whatever n^3 just becomes too much and the storage requirements also dense meter it has to store n squared elements, here I have to store like a scalar times n because only sparse entries are there right.

So, that is why you are going ahead with FEM and, but this was the plus side and the minus side is I have to discretize air. So, that seems like a bit of a disadvantage.

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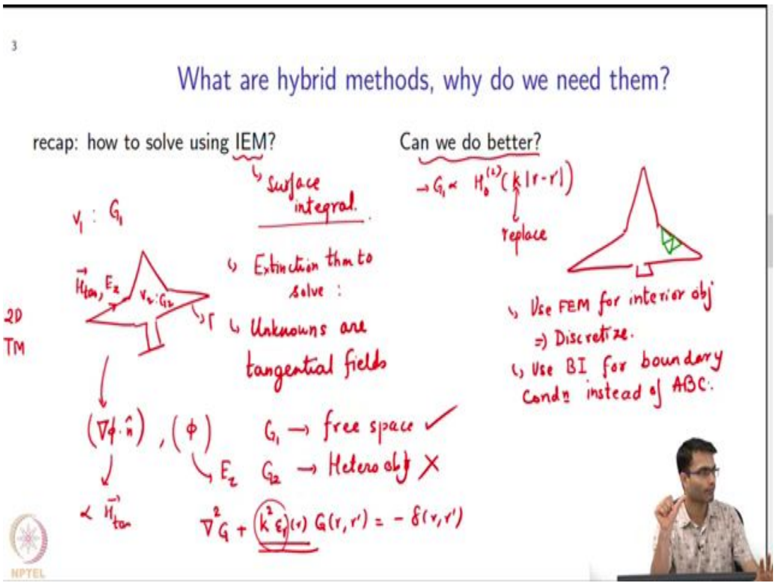

So, when we come to integral equation methods ok. So, by integral equation methods, I am particularly talking about surface integral methods ok. So, surface integral methods. In surface integral method so, let us say this is my a object over here with some with this boundary is defined to be some Γ. So, what was my approach in this case? So, let us say this is volume 1 and volume 2 sorry inside let us call it volume 2 and inside, I mean outside its volume 1, inside its volume 2. So, we had come up with couple of equations what was the name of those equations?

Student: (Refer Time: 07:55) Extinction theorem.

Extinction theorem right; so, we use the extinction theorem; extinction theorem was used to solve for the unknowns right, what were the unknowns?

Student: (Refer Time: 08:11).

Tangential fields, where?

Student: Boundaries.

One the boundary right; so, unknowns were tangential fields. So, in a 2-D problem let us take 2-D. So and lets make it TM. So, what were my unknowns? There was a H tangential which is a vector and a E_z, these are the two unknowns, remember if you if you recall the surface integral equations, my variables back then were $\nabla\phi.\hat{n}$ and ϕ these were my two unknowns right ϕ was my E_z and the other one was proportional to H tangential we had worked it out these were my two equations. In addition to this what equation, I mean what else I needed was the Green's functions G_1 and some G_2, G_1 was free space. So, therefore, easy and G_2; however, for a heterogeneous object not easy right; so, hetero object.

Student: (Refer Time: 09:29) value of k (Refer Time: 09:31).

No, there your differential equation, if you remember $\nabla^2 G + k^2\varepsilon_r(r)G(r,r') =- \delta(r,r')$

Student: What you discretizing it?

Even if you discretize it you cannot get a you can definitely not get a analytical expression for this G at best you can get numerical values for this. So, you if you want to use this approach, you will have to numerically calculate this G_2 every time. So, in order to solve your problem, you have to first solve a at every point you have to go and discretize your epsilon r.

Student: Sir (Refer Time: 10:13) method you (Refer Time: 10:14) discretizing the boundary.

Discretizing the boundary, correct.

Student: And then using G_1 except.

No, G_1 and G_2 the.

Student: Yes, G_2 means you change the k_2 whichever the value.

Right so, I should clarify this, in the surface integral formulation which we have studied in the previous modules, the object the second object that was volume 2 was a homogeneous object ok. So, for free space what was my Hankel my Green's function G_1 was of the form Hankel function of $k|r-r'|$ right some constants were there and let us call it. So, when the medium changed, but it remained homogenous then I could replace this k by the k of that medium.

But if the medium is heterogeneous not homogenous then k has no meaning it is actually this; so, then it is not going to be this Hankel function. In fact, it you cannot write it in close form.

Student: Sir, then probably you are approaching the surface integral.

So, surface integral that is what surface integral equations are very useful when that objects are homogenous, which can happen in some cases, if they do not happen then you will go to some volume discretization method like FEM or volume integral because calculating this G_2 it can be done, but it will it can be done numerically only which is expensive.

So, you do not choose that right. So, G_2 cannot be used ok. So, now, question is if you look back FEM had a very good advantage that gave me a sparse system can quickly solve it ok, but this extra air region had to be discretized, apart from discretizing air it was the any other problem with it.

Student: Boundary.

With the boundary yeah, what about the boundary?

Yeah, what is the problem with a radiation boundary condition?

Student: Because they are not always the normal.

Correct right; so, a boundary condition which we derive was exactly true when the wave hits it head on, but if a wave hits it at some angle, we have seen that the error is non zero right. So, apart from discretizing air I have the problem of inexact boundary condition whereas when we look at surface integral formulation, we did not have any problem of boundary because the extinction theorem was the exact relation obeyed by tangential fields, we did not make any approximations, we did not have to include any air, it is exactly the relation right. So, when I say can we do better, what would be your suggestion? Is I want to make use of the advantages of FEM and the advantages of surface integral formulation.

So, if you are thinking like an engineer who knows CEM how, what would you do? If I if you wanted to remove the disadvantage of both of them, the edges of the triangle do form the boundary, yes now so, you are saying that we start with this domain this entire domain.

Student: No.

Student: You just need the.

Just so, your suggestion is start with the boundary like this then.

Student: And then discretizing into triangles.

Discretizing to triangles, alright then.

Student: Then, I still do not know how to get G_2, but I think it goes on the converted (Refer Time: 13:34).

Ok so, I think we have a some sort of building blocks are correct. So, we want to do use FEM for interior of object.

Student: Sir, do we have to like travel along of the edges of the triangle (Refer Time: 13:52).

Yeah in the case of FEM your, I mean the boundary condition applies to every element on the boundary obviously.

Student: You know yeah in the boundary.

Yeah.

Student: But I am talking do you have to go inside the (Refer Time: 14:05).

Of course you have to go yeah inside yeah you have to discretize because I mean how do you, FEM system of equations the entire computational domain is broken up into triangles.

Student: Sir (Refer Time: 14:14) right.

Yeah.

Student: So, you can subtract each (Refer Time: 14:17) of a triangle.

So, your suggestion is to use yeah that is. So, the suggestion is to use boundary integral only, but to make it into piecewise homogeneous regions. And do a multiply corrected region and unknown tangential fields on everything right yeah that can be done that is going to be very

very tedious ok. So, what we will do is I mean, you are on the right track and then you deviated, use FEM for the interior objects.

That means, I discretize the interior boundary left is what, I have decided I do not want to pay the price of discretizing air and I do not want to pay the price of inexact boundary condition; so, the part of the FEM, where I impose the boundary condition. Is the part which instead of applying a ABC I apply, I somehow use the boundary integral equations, because they are exact and they are along the boundary. So, this is the trick of I mean this is the conceptual idea ok. So, use FEM for the interior objects and use.

So, I am going to start instead of calling it surface integral the literature calls it BI boundary integral, use boundary integral for boundary conditions instead of ABC that is going to be our approach the at the high level ok. So, now, we will go a little bit more into how can we actually accomplish this, but the motivation is clear, if we are somehow able to do this then we would have not discretized air and we would be taking advantage of exactness of boundary condition.