Computational Electromagnetics
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Applications of Computational Electromagnetics
Lecture – 14.17
Antennas – Mutual Coupling – Part 2

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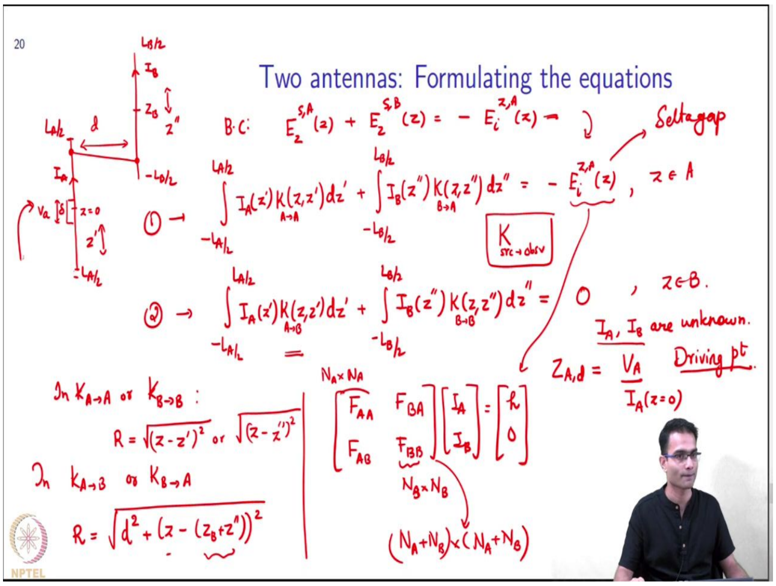

Let us draw are diagram again ok, so this is my V_A applied across delta; I am assuming a delta gap excitation ok. Let us call this z equal to 0 and second antenna element over here space apart by d, but its parallel to each other, this center is at Z_B and the length is L_B. Geometry is clear length L_A length L_B current is I_A, current is I_B ok.

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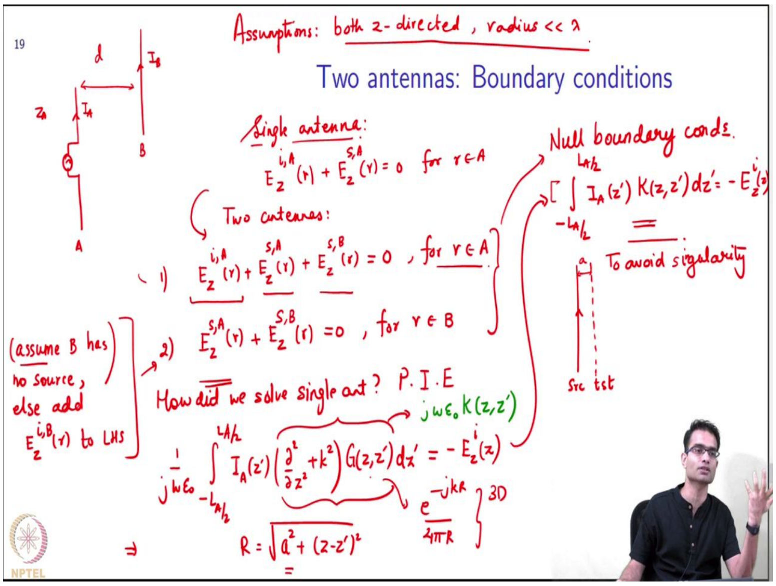

Now, in the single antenna case over here, if you remember what I done was that my current source was along the z axis and I has taken the observation point to be this radius apart; a apart right, that is why this a squared was there. And we are done this to avoid, what? The Green's function singularity, I could have chosen the points to be on the same on the surface of the antenna right it's possible and we have dealt with singular integrals right. So, this, but to make math simpler we had done this to avoid singularity.

Here, however, there is a certain symmetry is missing over here, because I mean the second antenna is on one side, so I cannot in good I mean being honest, I cannot take some you know distance d apart and do testing on that point. So, what is my sort of alternate, what is the solution that I will do for source points and testing points? Remember, here, this was my source and this was my testing point. So, here I have no choice, I have to live with the singularities.

So, I will where the current is flowing that is going to be the place where I will do testing, I will get the singularity in Green's functions I will deal with it not a problem. But now let us start with the boundary condition that is what is going to guide us. So, let me rewrite it boundary condition is for example, equation 1 right. So, equation 1 will give me

$$E_z^{s,A}(z) + E_z^{s,B}(z) = -E_z^{i,A}(z)$$

Now let us just do a one to one mapping of this, what will be the first term become? So, this is our shorthand notation for the left hand side. So, integrals form where to where? This is the field due to the induced current on the same antenna on A. So, what should be the limits of integration? $-L_A/2$ to $L_A/2$. Which current will this be? I_A or I_B?

Student: I A.

$I_A(z')$; so, let us say z' is the coordinate that goes up and down over here ok, $K(z,z')\,dz'$ this is the scattered field produced by A. Next is the field produced by the current induced on antenna B; so, limits of integration? $-L_B/2$ to $L_B/2$.

To make life more clear, I will say some other coordinate because this is a dummy variable, but let us make it a different symbol and $K(z,z'')$ ok, this $z'\,z''$ is the relative coordinate on I mean say relative to z, B is equal to $-E_i^{(z,A)}(z)$ right. So, this we are assuming, for example, a delta gap or whatever you want does not matter what it is, that is my first equation. Second equation becomes second equation would be coming from which boundary condition? This guy, the only difference is; so over here 1 sec, so one thing I should make clear here z belongs to which place; this boundary condition is valued for which z? $z \in A$.

$$\int_{-L_A/2}^{L_A/2} I_A(z')K(z,z')\,dz' + \int_{-L_B/2}^{L_B/2} I_B(z'')K(z,z'')\,dz'' = -E_i^{(z,A)}(z) \qquad z \in A$$

Now I have to take $z \in B$ right. So, $z \in B$ the first term is the field produced by the current in A falling on B right. So, this is going to be limits of integration $-L_A/2$ to $L_A/2$.

Falling on B that is all right; so, my z is going to be on B, but z' has to go over whichever current is radiating right. So, this is going to be

$$\int_{-L_A/2}^{L_A/2} I_A(z')K(z,z')\,dz' + \int_{-L_B/2}^{L_B/2} I_B(z'')K(z,z'')\,dz'' = 0 \qquad z \in B$$

So, this is the expression that you should try to interpret what is just saying that, the left hand side just look at the left hand side, what is the left hand side saying? I will tell you what is the field at any point z. If I carry out this integral; right, z' runs over the current element and z is where it is and I am matching it on the boundary by equating it to $-E_{inc}$ that is the meaning of this right. And remember, how did this thing come? It came by convolving G and the current which gave me A, from A I got E. So, this is the value of the electric field that any point z and to solve it I am matching it on the boundary. So, similarly this is the field produced by the current element A at some location z, this plus this is what is being made equal to either 0 or the incident field ok.

Student: Sir, they both I_A, I_B unknown.

Both I_A, I_B exactly right; so, I_A that is what I want to find out I_A, I_B are unknown.

K is Green's function with this operator right this so, $\partial^2/\partial z^2 + k^2$ acting on G that is my K. It looks complicated, but it can be evaluated ok. So, we are almost there; so, let me put a few subscripts here this K over here, this is a on a right; that means, the source and the testing K point were A. In this K , what is the source point? Source point is the second element falling on K . Here what is the source point? A falling on B this is B falling on B right. So, the notation was K source to observation point that is the interpretation right just otherwise everything looks like K , but in fact, the z in the z' correspond to the source in the destination. So, what is the source and what is the destination that is to make it very explicit right.

So, in $K_{A\rightarrow A}$ or $K_{B\rightarrow B}$ I have a Green's function which has a R. So, what will that value of R be?

$$R = \sqrt{(z-z')^2} \text{ or } \sqrt{(z-z'')^2}$$

Because is the self term, and in $K_{A\rightarrow B}$ or $K_{B\rightarrow A}$ what is my R over here?

$$R = \sqrt{d^2 + (z-(z_B + z''))^2}$$

I just have to find out the relative distance between right, z double prime goes from $-L_{B/2}$ to $L_{B/2}$. It is offset by some value z_B.

So, this is the absolute z coordinate on current element B and current element A was anyway on the z axis was this was z right, so this is the distance. So, I mean it is not complicated, it is have to keep geometry very carefully in mind and this whole thing over here will boil down to a coupled matrix equation this first term over; so, what is the unknown over here? I_A and I_B. So, I_A for example, this whole thing can be written as say some $F_{AA}\, F_{BA}$ or let us call it AB; BA make sense because source to observation then $F_{AB}\, F_{BB}$. Once you calculate all the matrix I mean the integral $[I_A\; I_B]$ is equal to $[h\; 0]$, this is after the MoM procedure has been carried out.

So, what will be the size of these sub matrices? So F are all sub matrices, what will be the size of for example this or this?

Student: N cross (Refer Time: 11:48).

Yeah so, this will be $N_A \times N_A$ right similarly this will be $N_B \times N_B$ right. So, this overall matrix is what size?

Student: N A.

$(N_A + N_B) \times (N_A + N_B)$. Because I have discretized A into N_A elements B into N_B elements, so I get the bigger matrix and I solved out. Yeah, after having done all of this what was my objective? I have done all these great math I have got some solution I have solved it got $I_A\; I_B$ now what? What was my original objective?

Student: Driving point impedance.

Driving point impedance, how I find that? So, I want to find out for example, Z_{Ad} equal to the grand question.

Student: Sir, what is the right hand side in the?

H is whatever is coming from the vector form of this guy is what is coming over here at all the testing point see, Z_{Ad} then becomes at what point am I asking this question? At the connection terminals of antenna.

Student: A.

A; that means, at this point right. So, voltage A divided by what?

Student: I A of z equal to 0.

That is all, and why is it all? Because I_A is not the current in the absence of the other object, I_A is come other solution of everything all the mutual coupling has been taken into account. So, I do not it slightly different from this circuit model where I need to first find out Z_{11} then Z_{12} and the ratio of currents here, I am directly solving this problem finding out the current at the input terminal that is my driving point impedance right.

So, you could find out the mutual coupling by solving the problem without element B you get some impedance right. So, this sort of completes how to deal with two antenna elements and this opens up a vast area of research I will just very briefly mention it over here.

Instead of element B being a current element, you could have, for example, a human body and you wanted to study what is the role of a human body on the input impedance of your antenna you are holding a mobile phone over here close to your head. So, inducing some currents on your head and changing the input impedance right. So, everything inside the antenna will change depending on what objective place it close to. So, people have used this as a model for not just like a one wire element, but make it a complicated object find out what is the mutual coupling over there ok.

So, in general and electromagnetic problems are very challenging because of these induced currents floating around everywhere and this is one rigorous way of dealing with it when the geometry is very general need not be some very specific thing ok. And this procedure can be generalized to N as many elements as you want in a and like an antenna array. So, the only thing to be careful about here is that, we will have to work out the Green's function with the singularity because I am choosing the source and testing points to be up along the same axis.

Student: Sir, is it 2D Green's function?

3D Green's function like this is like our original problem over here what was this was my 3D Green's function know; so, the current element is one dimensional, but at least two yes, one dimensional only a long of finite length along the z axis its very thin and radius, but its radiating in 3D space that is why I am using the 3D Green's function and making the simplifying assumptions of z directed and very small radius that volume integral boil down to a line one dimensional integral. Just because its one dimensional integral you should not think that it is a 1D problem; it is a one dimensional element radiating in three dimensional space. So, as long as your wire is very thin, this is exact.

Student: Same way that x y themselves.

Yeah, that is why at this A square comes in.

Student: (Refer Time: 16:28).

Yeah, what I wrote was for this what, the $-z$ should have.

Student: It is a z d.

Yeah.

Student: z is the.

Yeah, so you are right; so, this instead of z this should be $z_B + z''$ that is what you are saying yeah. Actually yeah, we have to move to the relative frame of the antenna element, so, even the original thing is fine as long as you are sure that z and z' are in the same. If z', z is also on the current on the axis and z'' is also in the same axis, the singularity should be there.