**Computational Electromagnetics**
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**Applications of Computational Electromagnetics**
**Lecture - 14.16**
**Antennas - Mutual Coupling - Part 1**

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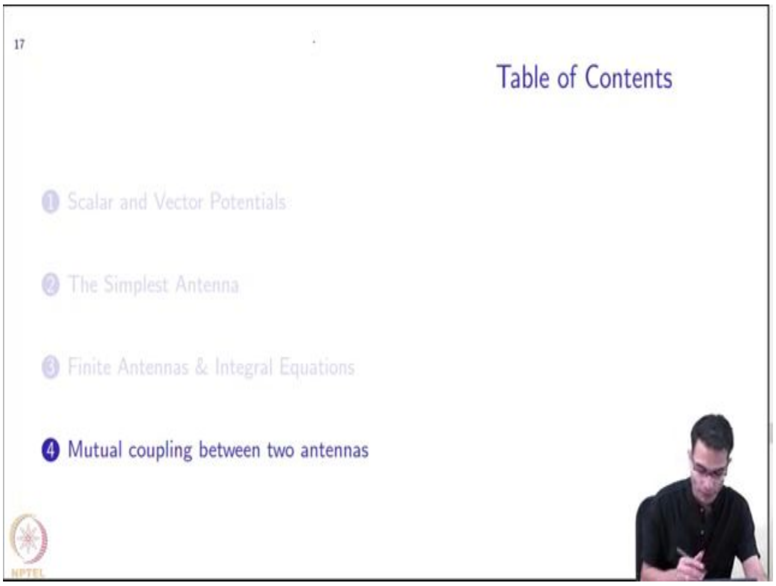


So coming to the Mutual Coupling between two antennas ok; so, this is another major application of CEM right.

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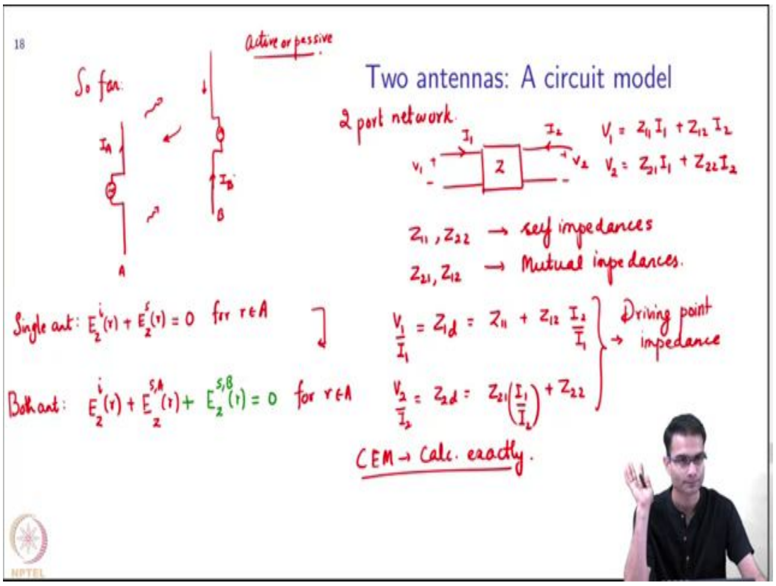


So, what have we this sort of let us look at what we have done so far. So, we had one antenna like this let us call it antenna A ok, and what was this antenna A doing, it was radiating a field in free space. So, and the input impedance that I calculated was assuming, I mean what were the boundary conditions that we applied over here, to solve this problem what was the boundary condition, we said that $E_z^i(r) + E_z^s(r) = 0$ for $r \in A$ in the conductor, it is a perfect conductor the total field is $0$ right.

Now, what happens if there is some other object in the vicinity of this antenna, that object will also have some current induced, I mean the field will fall on it, induce some current that induced current in turn will radiate another field, that field will come and fall on this antenna. So, this boundary condition will not be strictly true because there are more fields over here.

If this boundary condition is not true, my solution changes right my; that means, my current distribution changes if the my current distribution changes what happens to my impedance? It will change right. So, the input impedance that you have calculated for a single antenna in free space you were very happy about it, you put it into an electromagnetic environment where there are various objects and suddenly you find that your you know your performance is not so good right.

So, one major reason for that would be there is the since the environment around has changed the input impedance of the antenna has changed therefore, the matching condition is no longer valid. So, how do you calculate this? So, again there is a vast theory of approximate methods, but again in CEM what we will try to do is give you an exact way of calculating this, there is more it is more work, but it is a more rigorous than in the other case. So, what we will do is now we will add another antenna element over here ok.

So let us say there is one antenna radiating and I have introduced one more antenna in its vicinity ok. So, another antenna comes in over here we will call it antenna B ok. This antenna can be active or passive. So, what does that, what does active antenna mean? That is it is also radiating; that means, it also has a generator, passive antenna could be is just sort of in receiving mode or like a one of these Yagi-Uda antennas, there is one driving element and there are various rods behind it which are there right. So, that is an example of a passive element. So, this situation is general over here.

So, how does this what will happen physically is, this guy radiates a field, current is induced on it right and then that in turn will radiate another field ok. So, there are two ways of thinking about it, one way is that a like a I mean like a series A sends something to B, B induces B radiates sends to A, this changes the current in A and so on, back and forth right. It will not continue forever, there will be some steady state solution to this ok. So, in the steady state there is going to be some current in A and some current in B right. So, how will this boundary condition get modified? Then, if I ask you what are the fields at any point on A?

So, you will have $E_z^i$ ok, that is always there, that is the voltage I mean the field produced by the voltage source, then you have the self field right. So, so far this is what I had in the single antenna case now right. So now, there is going to be a plus $E_z^s$ for scattered right. So, the field scattered by antenna B right. So, this was a single antenna and this is both antennas. So, in terms of a circuit model, how can I think of this? So, actually a very simple two port network also drives home this picture very well right. Remember in our earlier case, I had the generator and one load connected across it.

Right now the situation has changed a little bit, I have something like this. So, $V_1$ and $I_1$ goes in over here, $V_2$ and $I_2$ appear over here right. So, this is I mean for Electrical Engineers this is very standard. So, I can write

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad\qquad V_2 = Z_{21}I_1 + Z_{22}I_2$$

So, what are these $Z_{11}$ and $Z_{22}$? These will be called the self impedance right. So, these are the impedances in the absence of the other antennas. This is what we had so far when there was a single antenna nothing else to disturb it and then $Z_{12}$ and $Z_{21}$ these are called the mutual impedances. From a circuit from an antenna or a circuit designers point of view what is the impedance that he or she needs to match, is it $Z_{11}$, $Z_{22}$ or what?

It is neither Z 1 1 nor Z 1 2.

Student: $Z_{in}$.

$Z_{in}$ right so, that is $V_1/I_1$ at port 1 that is the impedance right. So, that is going to be $Z_{1d}$, I will tell you what $d$ stands for and that is going to be equal to

$$V_1/I_1 = Z_{1d} = Z_{11} + Z_{12}I_2/I_1$$

$$V_2/I_2 = Z_{2d} = Z_{21}(I_1/I_2) + Z_{22}$$

This $Z_{1d}$ is called the driving point impedance. So, you can see that the driving point impedance depends on knowing $I_1$ and $I_2$, I need to know what these currents are only then I can find out what is the matching network to design right. So, again, how do we calculate this?

So, again this was method which is called the Induced EMF method and various other methods, they are used to approximate, if I can get an approximation of the current distribution then I can calculate this driving point impedance and all that right. And, that that works for very special cases of you know parallel conductors only Z directed and so on, but if you have some general configuration it is not accurate. So, CEM again allows us to calculate these exactly ok. So, is the set up clear? Half when I have two antennas in the present of each other there is going to be you know some current, $I_A$ over here and some current $I_B$ over here and these are steady state currents after all reflect multiple reflections they have.

And our objective is that in the presence of $I_A$, I mean in the presence of the other antenna tell me what is $Z_{1d}$ tell me what is $Z_{2d}$? So, let us proceed.

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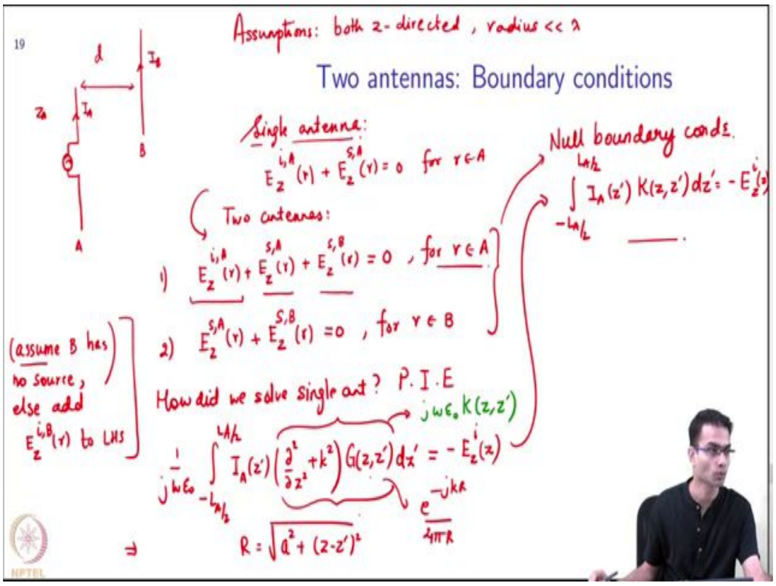


So, the boundary conditions that we have let us redraw this antenna over here. So, this is antenna A ok. For simplicity, what I am going to do is the antenna B, I am just going to make it passive ok, but I can easily add a generator to it ok. So, this is my antenna B and there is $I_A$ here, $I_B$ over here ok, some distance d apart. Now, since this is the very first introduction of CEM to mutual coupling, I need to make some simplifying assumptions which is not required, but it is just to keep the derivation simple.

So, the assumptions I will make are both are z-directed. So, both currents are z-directed and radius is much smaller than wavelength ok. So, this just allows the math to become a little bit easier, we are ok. So, with a single antenna, we will keep comparing with the single antenna case and the build the theory of two antennas to each other with the single antennas our boundary condition was $E_z^{(i,A)}(r)$. So, $(i,A)$ is incident field for element A.

$$E_z^{(i,A)}(r) + E_z^{(s,A)}(r) = 0 \text{ for } r \in A$$

Now we are going to for two antennas, I need to find out the boundary conditions. So, very simply there will be. So, let us take two cases $r \in A$ and $r \in B$ ok. So, the first case, what will I write? ${E_z}^{(i,A)}(r) + {E_z}^{(s,A)}(r)$ this is the self field and some field scattered field due to B being measured on A, ${E_z}^{(s,B)}(r)$, this must be equal to 0 right ok.

$$ {E_z}^{(i,A)}(r) + {E_z}^{(s,A)}(r) + {E_z}^{(s,B)}(r) = 0 \text{ for } r \in A $$

Second boundary condition, what will be the first term? So, there is going to be ${E_z}^{(s,A)}$ from A right. So, let's just write it in this way so, this is the field that is produced due to current in A falling producing a field and that field falling on B and ${E_z}^{(s,B)}(r)$ is equal to zero for r belonging to B.

$$ {E_z}^{(s,A)}(r) + {E_z}^{(s,B)}(r) = 0 \text{ for } r \in B $$

So, we assumed B has no source. If it has a source I there will be an ${E_z}^{(i,B)}$ that is all very simple to deal with ok. These two conditions there is a name for them they are called Null boundary conditions. So, let us remind ourselves, how we solved the single antenna problem.

Student: Sir.

Yeah.

Student: Why will get into incident field for?

No, I am just assuming, if there was, there will be one more term we will add it right. There is absolutely no difficulty, we can add like this term over here right a similar term ${E_z}^{(i,B)}(r)$ will be added to the left hand side of this.

Student: But it is in the.

So, I have some voltage source in A which is generating a field and that is the first term, the second term is the field due to the current flowing in the antenna itself and the second term is the field due to the current in antenna B right. Now, and this is true for any point on the antenna A, when I move to antenna B the same logic holds. So, if there were source in antenna B then I have to add the source term that is all, is it clear? So, let me write it here else add $E_z^{(i,B)}(r)$ to LHS.

So, that there is no confusion ok. So, let us remind ourselves, how did we solve single antenna; it was the Pocklington's integral equation right; Pocklington's integral equation and let me just write it down over here

$$1/(j\omega\varepsilon_0)\int_{-L_A/2}^{L_A/2} I_A(z')(\partial^2/\partial z^2 + k^2)G(z,z')\,dz' = -E_z^{i}(z)$$

This was our very simple thing. Now, this character over here is going to appear many times. So, I am going to make a shorthand notation for it. So, this term over here, I am going to call it $j\omega\varepsilon_0 K(z,z')$ just a shorthand notation for this term. So, what happens now or let me maybe I should write it over here. So, this will become

$$\int_{-L_A/2}^{L_A/2} I_A(z')K(z,z')\,dz' = -E_z^{i}(z)$$

Just our more compact way of writing the same equations. And, here my $G(z,z')$ had what kind of terms? It was

$$G(z,z') = e^{-jkR}/4\pi R \quad \text{where} \quad R = \sqrt{a^2 + (z-z')^2}$$

So, it is a function of $(z,z')$. Now, we go to the case of the integral equations for two elements right.