Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering (EE)
Indian Institute of Technology, Madras

Applications of Computational Electromagnetics
Lecture – 14.12
Antennas Pocklington's Integral Equation – Part 2

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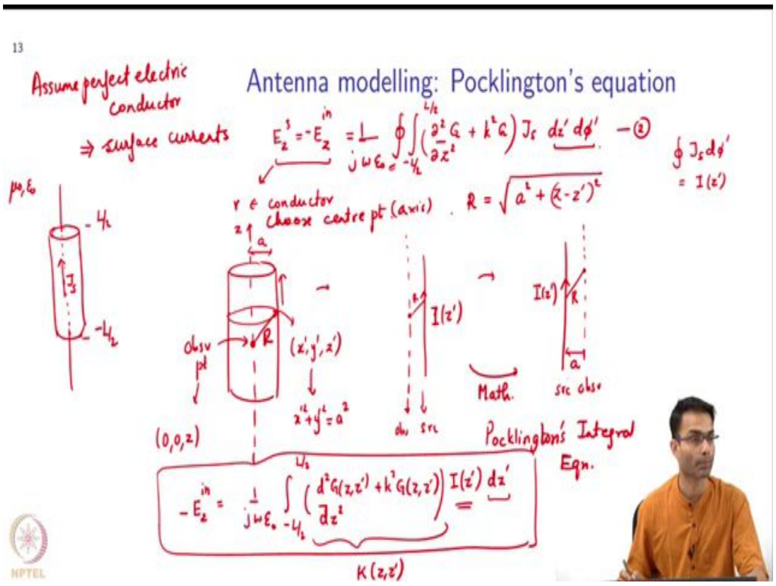

Right; so, again simplifying assumption we can make is that the conductor is a very very good conductor, let us so assume perfect electric conductor.

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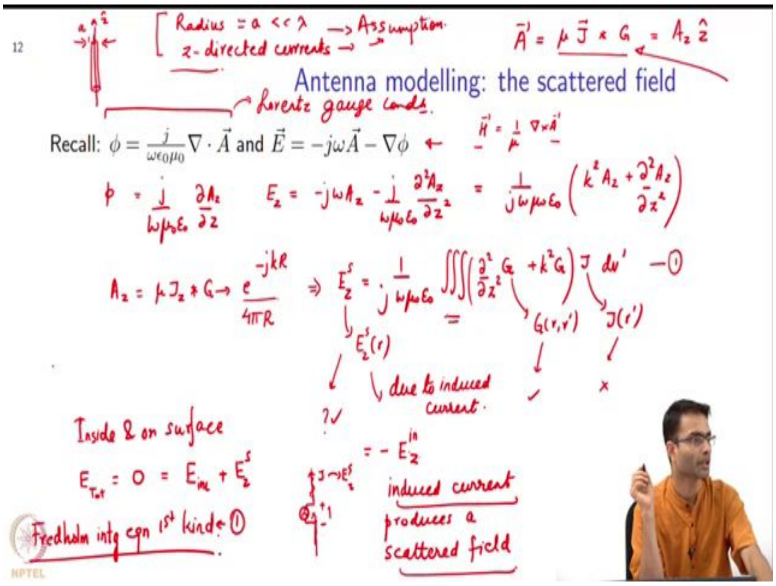

That assumption we will lead to what, if you look at this geometry over here in a conductor where do the currents lie? surface right; so, this implies surface currents right. So, if I were to this was my one more here right this is my exaggerated of course, mu naught epsilon naught over here, now I am going to have only currents on the surface right.

So, I can draw like this. So, I am going to I can call those as J_S pure surface current J_S. So, then that integral will get reduce to; this three-dimensional integral will get reduce to a this 3D integral will become 2D integral right, surface integral right. So, this will become a what form, so I can write down the electric field $E_z^s = -E_z^{in}$.

And now in this I can write as $-E_z^{in} = \frac{1}{j\omega\varepsilon_0} \oint \int_{-L/2}^{L/2} (\frac{\partial^2 G}{\partial z^2} + k^2 G) J_s dz' d\phi'$. There is no the r part of it has been absorbed as a delta function and it is become a pure surface that ok. So now, this is a 2D integral.

Any other assumption that I can make; by the way when we when we use this equation over here we are constrained that r must belong to the conductor right otherwise this boundary condition is not going to be valid right. So, where can I choose my observation point to test this I mean to write this question what can be a good choice?

Student: On the surface.

On the surface, what is the problem with the on the surface.

Student: 0.

So, I mean should I choose it like the, you have one choice over here, you can put on the dotted line (axis of wire) or you can put on the surface itself.

Student: Dotted line.

Dotted line is a better choice because singularity of green functions can be avoided right, and the boundary condition is valid inside the conductor also field total field is 0 right. So, we will choose the centre point axis so, what becomes like this again exaggerated drawing ok, so this is my observation point ok.

And the R is going to be ok, and what is this, so this is my z axis over here right. So, what is my value of R that I am going to write; so, R remember is the distance between r and r'. So, the primed coordinates are where the current term is right its $J_s(r')$. So, J_s is flowing all the way up and down over here.

So, that is primed coordinate. So, what is the location of the source point (x',y',z'), what is the location of the observation point (0,0,z) right. This observation point I am choosing to be along the dash line right. And this guys points are (x',y',z'), but I am choosing these source points to be only on the surface of the because a current is purely a surface current, so $x'^2 + y'^2 + z'^2 = a^2$.

In Green's function I have the distance between r and r', so that is what I am writing as $R = \sqrt{a^2 + (z - z')^2}$. So, if I fix my observation point where is this integral going; it is going over the entire surface and length 2D integral fine. So, totally cleared so far; can I make some other simplifying assumption to because if I can reduce a 2D integral to a 1D integral, it would be even nice a right hm.

Student: (Refer Time: 06:39).

Say again.

Student: (Refer Time: 06:42) cylindrically.

Cylindrically symmetric right, this problem is cylindrically symmetric; so, I need not consider all of these J_s separate I can consider and equivalent I right. So, I can further do this I can say instead of considering the pure surface current let me consider just a filamentary current over here right. So, I can call this to be some to use a different symbol, I will call this some $I(z')$ and my observation point to be here. So, even my value of R does not change is this the same right. So, that will collapse this contour integral over here in to I, there are these constants that will appear which you can take care right; J_s defined over the entire surface so, all that is there.

Student: Sir, now we calculated value by of these enforcing.

They calculating; we are enforcing the boundary condition along the centre point of the axis.

Student: Finding the values of (Refer Time: 07:42).

Finding the value of what?

Student: I.

No, the I continues to live on the surface of the conductor because that is where it is that is its support the surface of the conductor right, I am enforcing this equation at the observation points which is my choice. So, I could have chosen on the surface, but that would I given me a singularity in Green's function, I choose instead the axis of the cylinder because there I will always a have non-zero capital R; it is our choice and we choose it in a way that we get to avoid maximum pain fine ok.

Now so, what happens to this equation? So, this equation then becomes

$$-E_z^{in} = \frac{1}{j\omega\varepsilon_0}\int_{-L/2}^{L/2}\left(\frac{\partial^2 G}{\partial z^2} + k^2 G\right)I(z')dz'$$

$I(z') = \oint J_s d\phi'$

We are assuming all the currents are purely z directed there is no screw component vertically in it. Because this was the 2D integral and this is now a 1D integral.

Student: For d z prime.

d z prime is still there; so, what I am saying is this over here?

It is a pure surface correct, and it is the same for all ϕ. Now looking at this equation over here you may as well I mean if I did not tell you that the configuration was like this and I told you consider a symmetric situation like this where I have this; let me draw it again right this was my current over here, supposing I called it like this, it's the same right whether I put the once, I have got it down to one line of observation and one line of current right this is the observation and this is the source.

Once I boil it down to two lines it does not matter where the origin is because everywhere inside this over here I have capital R which is the distance between these two. So, may as well (Refer Time: 10:54) and put the source here and observation over here ok, it is a symmetric situation over here. No, we are not putting the source and observation on the same line, we have always separated by A; now.

Student: No.

Right; so, this going from here to here is just not physics, but math mathematically its equivalent ok, physically you can still think of it like this and that is your what should I say that that is your I mean the integral equation that, we have got as a result over here is the final form. It just helps in evaluating these integrals ok; I mean it's not a big deal. We are not going to go ahead and calculate it in closed form right now, but is just a simplification.

Student: (Refer Time: 11:50).

z' goes along the origin yeah, along the z axis itself that is all and the observation point is now a distance a away here, totally symmetric situations. Any way this equation that you get is what is called the Pocklington's integral equation alright, so it is named after the person who came up with this. So now, you can appreciate what would happen if this you know, I did not make the assumption of very thin conductor, then I would not get a 1D integral, I would have to solve it over the ϕ coordinate also, so it is a two-dimension integral.

Then if I made the further assumption that it is not a perfect conductor, then I can no longer make the approximation that the currents are on the surface, but there living in the volume right. So, then I have a full 3D volume integral which is fairly involved to solve ok. So, that is why we are, I am making taking you to the simplest integral equation that we can solve over here there are right ok.

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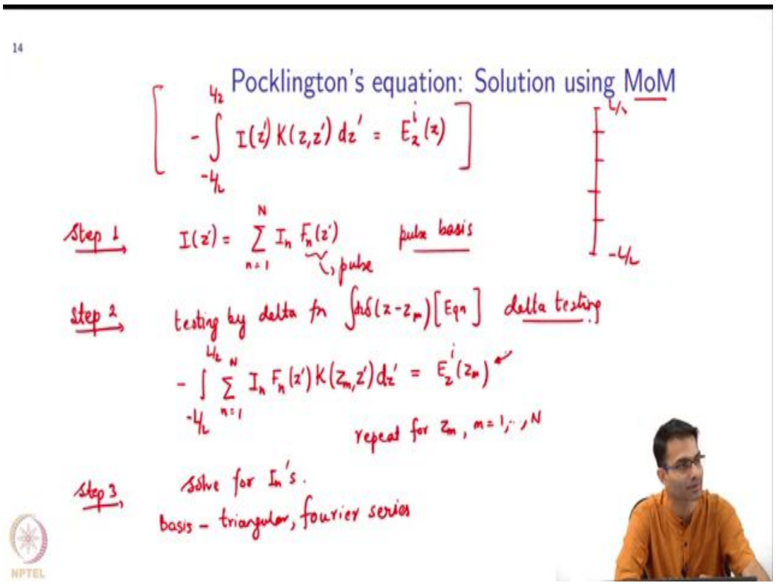

Now, this equation over here we will just write it once again so, this

$$-\int_{-L/2}^{L/2} I(z')K(z,z')dz' = E_z{}^i(z)$$

This whole expression over here $\frac{1}{j\omega\varepsilon_0}(\frac{\partial^2 G}{\partial z^2} + k^2 G)$, I am going to call it the kernel some capital K so, this has become this is in short form over here. So, this is a integral equation which in the module on integral equations if no we have seen how to solve this. So, what would be the first step over to solve it using MoM; the Method of Movements objective is to find J or I in this case. So, step 1 would be.

Student: Discretize.

Right discretize right, so this is my current, from -L/2 to L/2 chop it up like this right. So, I can say that $I(z') = \sum_{n=1}^{N} I_n F_n(z')$; I can say it has some constant value I_n and some kind of a pulse basis right. So, this is a pulse right, so, what I am doing I am doing pulse basis right and then what do we do? next step this just this supposed to be a revision for you step 2 would be testing.

Student: Testing.

Testing by delta function right testing by ; so, what is that mathematically how do I do in mathematically.

Student: Employed.

Right so, both left hand and right hand side of functions of z right so, I have to take a $\delta(z - z_m)$ and integrate over which co-ordinates, dz right. So, that is or that is your pulse basis and delta testing ok. Repeat for all m that gives me a n cross n system of equations right.

So, that is my step 3 solve or I_n's ok. So, you see this part was really easy because you have already studied these integral equations over here. Now you could have done better kind of basis functions right, you could have done for basis functions you could have done triangular basis functions, you could have done entire domain basis functions for example, Fourier series.

Student: Fourier series.

Right you could have done a Fourier series basis functions in fact for a antenna problems Fourier series is may give you better convergence faster because in general the waves look like sinusoidal functions because they coming from the back side from a transmission line. So, they are already sinusoidals the antenna distorts is a little bit, but the deviation may not be much. So, you make get faster convergence with Fourier series and so on, and for testing,

testing its you could do delta testing or you could do Galerkins method where you use the same basis function right.

So, the field is so, MoM in general does not tell you which one to do. So, if you see you know a paper or text book saying we solve it using MoM, we should ask what was the basis, what was the testing it is not necessary that is always pulse basis and delta test ok. So, so far we have not really spoken too much in detail about this how do we calculate this right hand side, the incident field right.