Computational Electromagnetics
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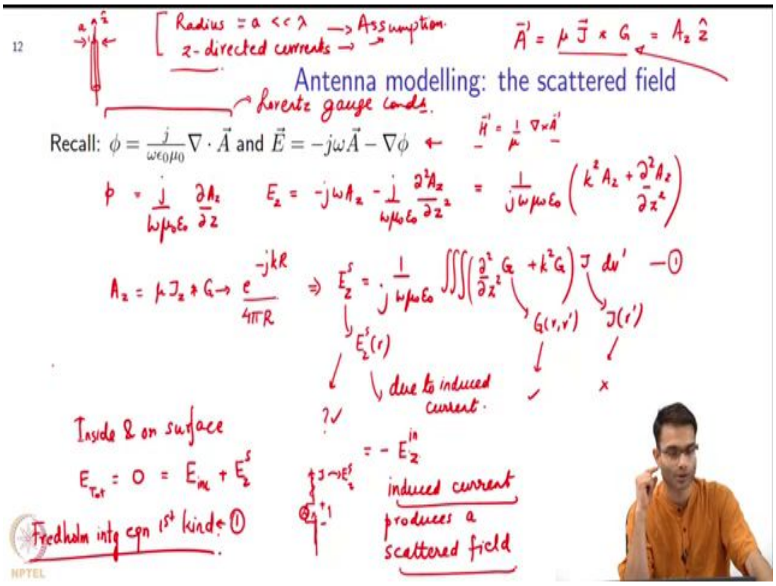
Applications of Computational Electromagnetics
Lecture – 14.11
Antennas Pocklington's Integral Equation – Part 1

(Refer Slide Time: 00:14)



Right so, we will continue with our discussion of the modeling of an antenna and we finally come to the part where we get to use Computational Electromagnetics right. So, what we said earlier was that in a realistic scenario imagine you know an antenna I connect some voltage source to it ok, some RF source, I wanted to broadcast some information. The trouble we said is that we do not typically know how is the current distributed along the length of the wire; I mean, I know what is the voltage at the current at the point of connected, but across the length I do not know what is J as a function of space.

So, how can I calculate A, how can I calculate H, how can I calculate E. So, what we will do is we will use the CEM theory to find out what the current is and what will of course, help us is boundary conditions ok, that is the one thing that always fixes things in nature right boundary conditions will force. For example, tangential fields to be conserved, fields to be zero on a conductor and so on ok.

Now, let me on you that modeling realistic antenna is quite complicated. So, in order to give you a first demonstration of how to do this computationally again we will make simplifying assumptions. So, one of the simplifying assumptions will be we'll take our current source to be let us say along the z axis, but very thin will make the wire to be very thin ok. So, thin wire over here with radius to be a, alright. Later on we can sort of appreciate how to make it bigger, but you will find that even making the; I mean keeping this radius to be thin is itself a challenging problem.

So, only thing is we are not making it infinitely thing its small compared to lambda right. So, this radius of wire is equal to a which is much much less than lambda that is the assumption ok. Under this assumption, we can again a sort of make a claim that the current is going to be z directed right; the current was going to go be going vertically up because the radius is very small. So, we are not going to have things like a current that is spiraling and moving up right, that may happen as the radius becomes bigger then as happening. But, if this dimension is very small mostly the current this is going up, so z directed currents. This is also an assumption.

So, these are the two assumptions that we will make and based on this we will see how can we solve this right. Now, we when we had derived the integral equations in terms of this magnetic vector potential A and scalar vector scalar potential ϕ. $\phi = \frac{j}{\omega\, \varepsilon_0\, \mu_0} \nabla . \vec{A}$ as a choice which we had made for the divergence of A and what it we call this? Lorentz gauge conditions right. So, this was our; how will this get simplified under our assumptions, what components? Remember A is a what form, A is the form $\vec{A} = \mu \vec{J} * G$ (convolution) right. So, what form will what how can I simplify this a little bit more.

Student: (Refer Time: 03:59).

Louder.

Student: Divergence will be.

Divergence will be.

Student: Will be.

A is only in the z direction right; so, $\phi = \frac{j}{\omega\mu_0\varepsilon_0}\frac{\partial A_z}{\partial z}$, only the z components right because A is coming from J over here and if J has only a z component, A will also have just a z component right.

The next thing we had was a relation for the electric field in terms of the potential right that was this relation over here $\vec{E} =-j\omega\vec{A} - \nabla\phi$. So, we are not going to go to the magnetic field we are going to go the second route to the electric field ok. I could have also gone from H and then from H to E. Any guesses why they are going directly to E from A and ϕ, why I am a going to E?

Student: (Refer Time: 05:19).

Say again, the magnetic vector potential is.

Student: (Refer Time: 05:24).

No, this A is the magnetic vector potential.

Student: (Refer Time: 05:28) we have to find A.

Yeah.

Student: We have a relation between h and u.

Yes.

Student: (Refer Time: 05:35).

No.

Student: (Refer Time: 05:36).

We can use no, that Ohms law is not ohms law sort of valid in the resistor, here there is a little bit more complicated right J is not going to be proportional to E.

Student: (Refer Time: 05:52).

My question is that normally what did we do to get the fields once I got A, how would I write $\vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$ right.

Student: We can find the (Refer Time: 06:04).

And I can find.

Student: (Refer Time: 06:08).

No; my question is this, I have from A I want to get to fields right. So, I can go from once get A I can go to H and from H I can go to E by taking curl of H, but instead I am writing E directly in terms of A and ϕ. Here I have to do more work right I have to also find ϕ, earlier if I used only A, I could get H straight away ok. So, this is the part of sort of looking ahead when I write down my Maxwell's equations, how will I enforce anything what will help me to solve the problem? Boundary conditions, boundary conditions are what are going to help me.

So, if your what is a reasonable assumption for a metallic wire that is a perfect conductor; on a perfect conductor what kind of boundary condition can I imposed can I imposed on H or an E?

Student: E.

E right because inside a conductor electric field is 0; so I that is the goal, that is the sort of property I want to utilize fields inside a conductor as 0. So, for that I need electric field that is why I am going from A to ϕ that is why I have written ϕ over here, and ϕ added together is going to give me E. So, that is just sort of looking planning ahead why I am writing the electric field.

Student: Components (Refer Time: 07:25).

Magnetic there are.

Student: Tangential (Refer Time: 07:28).

No, how do you apply tangential conditions on H in a conductor; in a electric conductor.

Student: For (Refer Time: 07:40) to be normal.

Yeah; so, we will it is simpler to do deal with field which is itself going to 0 that is the simplest boundary condition right. So, how does this E get modified now, $E_z = -j\omega A_z - \frac{j}{\omega\varepsilon_0\mu_0}\frac{\partial^2 A_z}{\partial z^2}$

So, let us take all of these terms outside over here. So, I am going to get

$$E_z = -j\omega A_z - \frac{j}{\omega\varepsilon_0\mu_0}\frac{\partial^2 A_z}{\partial z^2} = \frac{1}{j\omega\varepsilon_0\mu_0}(k^2 A_z + \frac{\partial^2 A_z}{\partial z^2})$$

Now, what? Now I have to use this property over here right $\vec{A} = \mu\vec{J} * G$, I will remember what my objective is to somehow solve for the current right.

$$E_z(r) = \frac{1}{j\omega\varepsilon_0\mu_0}\iiint(k^2 G(r,r') + \frac{\partial^2 G(r,r')}{\partial z^2})J(r')dV'$$

Student: (Refer Time: 09:25).

G is of what form $G = \frac{e^{-jkR}}{4\pi R}$.

Student: (Refer Time: 10:05).

Yeah.

Student: (Refer Time: 10:10) A does not (Refer Time: 10:12).

Yes, that is why we use this assumption that is not it is not a function.

Student: Could be z directed, but it is not a (Refer Time: 10:20) was.

Yeah, because they are all going this way there is no swirling around over here.

Student: So, they could know that there, but still be constants there you know if a is not small.

If the radius is not small then they could swirl around, then they it will become a function of x and y and z right, imagine a helix right where I am on the helix also depends on x and y you know not just purely z. So, then that as this assumption will become I mean it will not be correct over here right.

So, in general 3D convolution right, I have this is the most general expression right G convolved with J ok. I am integrating over the primed coordinates ok.

So, we have got an expression where I have the electric field on the left hand side is it known; so far is the electric field known? So, far it seems like it is not known I mean what is electric field. So, I mean we have come to that; so, I am going to put a question mark over here G is known J is not known ok. So, what can we I mean there are too many unknowns in this problem if I do not know the if I do not know E also and J this know I can solve it, but do I know E? Yes, that because anywhere inside and on the surface what is the electric field.

Right, E total is equal to 0 right; now this electric field that I have written over here is it the total electric field in this equation 1, is it? This is the field that is happening so, imagine that imagine that there is a plane wave that is falling on the antenna alright, that plane wave is inducing a current on the antenna that current is this J.

So, how many total fields are there, there is the field that is produced by this current by the induced current and then there is a field that was already present due to a source in the case of a receiving antenna it is a wave that is falling on the source. In case of a transmitting antenna there is going to be a field that is produced by the generator right, it may be localized for example, I connect a voltage source across some point, so the electric field here, but not somewhere else.

So, in general this is going to be E incident plus E scattered right, so this is actually this scattered field over here; scattered means, scattered by the I mean due to the induced current.

Student: (Refer Time: 14:31).

Yeah, but that it does not cancel of the E incident in some sense I mean you cannot forget about the E incident ok. So, that is why the total field is composed of both of these and both of these together as 0. So, this equation 1 then get simplified in to this is equal to minus E incident right I am going to write E incident on top and put a z over here because I am only talking about the z common .

Student: (Refer Time: 15:01).

Yeah.

Student: And if I (Refer Time: 15:03).

Yes.

Student: That is sort of (Refer Time: 15:10).

No.

Student: Is begin call it because they are we are not exiting the force because I does not the.

Yeah ok; so, we will come to source modeling later, but let me just give you an example. Supposing this is my antenna ok, I have connected some voltage source across it, so this voltage source is going to produce some electric field inside over here right. So, that incident field is valid only over there right, outside the voltage source you are not going to find any electric field, but this has now produced a current over here right. So, you have both of them over here, this current in turn is going to produce a scattered field.

Student: (Refer Time: 15:52).

The incident field is going to be there; so, if I consider some point over here there will be a the incident field due to the voltage source and due to the field like this J is beginning to produce; both are going to be there. It is not that because I connected this now I can ignore these the induce the field produced due to induced current; remember right, induced current produces a field and they call it as scattered field, not induced field induced current scattered field these are the two terms you should keep ok.

So, now I know this question mark becomes a tick mark, I know this the left hand side I know the induced current and the scattered field therefore, and I know everything in this except J. Now, J is appearing inside the integral sign unknown so, what kind of an integral equation is this Fredholm integral equation of first kind right; alright. Now, if you were to go and solve this as it is, its going to be little problematic because this is a 3D convolution over here right. So, let us see how we can modify this.