

Computational Electromagnetics
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Applications of Computational Electromagnetics
Lecture – 14.09
Antennas - Radiation Patterns

(Refer Slide Time: 00:14)

The Hertz Dipole: Visualizing fields

Cr: Stutzman [1]

Field pattern: $F(\theta, \phi) = \frac{E_\theta}{E_\theta(\max)} = \sin\theta$

Power: $P = |F(\theta, \phi)|^2$

at a given r
i.e. on a given Sphere.

$\vec{H} \propto \hat{\phi}$
 $\vec{E} \propto \hat{\theta}$

(r, θ)
 $(0, 0)$
 $(1, \pi/2)$

$\theta = 0$
 $\theta = \pi/2$

$\vec{r} = P(\theta)$

\vec{r} plane

\vec{r} plane

The next thing we would like to do is to visualize what these fields look like ok. So, there should be a bracket over here yeah. So, what is so, the little bit of terminology our hertz dipole is here sitting at the origin, it is pointed along the \hat{z} direction.

(Refer Slide Time: 00:37).

The Hertz Dipole: Far fields

$\vec{H} = \frac{I \Delta z}{4\pi} jk \left(1 + \frac{1}{jkr}\right) \frac{e^{-jkr}}{r} \sin \theta \hat{\phi}$

$\vec{E} = \frac{I \Delta z}{4\pi} j\omega\mu \left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right] \frac{e^{-jkr}}{r} \sin \theta \hat{\theta}$
 $+ \frac{I \Delta z}{2\pi} j\omega\mu \left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right] \frac{e^{-jkr}}{r} \cos \theta \hat{r}$

$kr \gg 1$

$\vec{H} = \frac{I \Delta z}{4\pi} jk \frac{e^{-jkr}}{r} \sin \theta \hat{\phi}$

$\vec{E} = \frac{I \Delta z}{4\pi} j\omega\mu \frac{e^{-jkr}}{r} \sin \theta \hat{\theta}$

TEM wave
radiation fields

Purely real.
radiated power.

Far field, fields $\propto \frac{1}{r}$

$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H} = \frac{1}{2} \left(\frac{I \Delta z}{4\pi}\right)^2 \frac{k\omega\mu \sin^2 \theta}{r^2} \hat{r}$

$P_r = \iint \vec{S} \cdot d\vec{s} = \int_0^{2\pi} \int_0^\pi \frac{1}{2} \left(\frac{I \Delta z}{4\pi}\right)^2 \frac{k\omega\mu \sin^2 \theta}{r^2} r^2 \sin \theta d\theta d\phi$

$\frac{|E|}{|H|} = \eta$ characteristic impedance.

Now, if you look at the expression for the magnetic field in the far zone, over here what do you see? Sin theta what direction $\hat{\phi}$ right. So, in this it is the magnetic field this is my $\hat{\phi}$ direction right. So, in the equatorial plane, the magnetic field is in the x y plane.

So, that is why this plane is called the H -plane for the hertz dipole because we had H is proportional to something along $\hat{\phi}$. Now, on the other hand you look at the electric field what way is it? It is pointing along which direction in the far field $\hat{\theta}$ right. So, electric field is proportional to something along $\hat{\theta}$ right. So, $\hat{\theta}$ is downwards in the equatorial plane. So, if I look at this plane over here which is shown right. So, it is like what if we think of earth this is like longitude right. So, any longitude is considered is called the E-plane because the electric field is sort of in that plane so right.

So, this is sort of standard terminology which is used ok. So, what happens is that you know many times you want to talk about what is the radiation pattern we have heard the word likes what is the radiation pattern of this antenna. So, towards that what is useful is to find out what is the normalized field because we do not want to be dealing with physical constants and currents and all over that thing. So, to that end you define something called a field pattern which is your E_θ/E_θ^{max} . So, let us just write down the expression for E_θ . So, I had what varies E as a function of θ just which is $\sin \theta$ right everything else is a constant with respect to θ . So, what is this over here? $\sin \theta$ right; so, if I plot if I take let us say the

E-plane ok. So, I am going to plot the E-plane over here and I want to plot in polar coordinates what does this you know field patterns squared is normalized field pattern squared. This is called the power pattern there is the field pattern and this is the power pattern. If I want to plot this in the, in this plane what would it look like? So, well my $\theta = 0$. So, how do we do a polar plot, you take some theta right and plot. So, at $\theta = 0$ what is the value? 0 right. $\sin^2\theta$ is going to be 0.

So, I am going to start from here and when I go to $\theta = 90$ right on the x axis or the y axis does not matter, what will it be 1 right. So, it is going to look like and then come back here right. So, this is at $\theta = \pi/2$, this is at $\theta = 0$ right and this, is it going to be symmetric about the z axis? It will be symmetric about the z axis.

So, this is going to look like a figure of 8 right. So, in any E-plane this is what the field power pattern looks like. So, how do you interpret this? So, someone says tell me what is the power at some given theta ok so they give you the theta. So, they give this theta. So, what you do you go and find out what is this length and that length is proportional to your power normalized power pattern right. So, that is just how to interpret a polar plot.

Student: Sir.

Yeah.

Student: What is that (Refer Time: 04:41).

This locus of points is the power as a function of theta plotted in polar coordinates, not in Cartesian coordinates.

Student: (Refer Time: 04:53) p equal to 1 (Refer Time: 04:55).

Yeah. So, this will be 1.

Student: Yeah. So, this x e.

is x.

Student: (Refer Time: 05:02).

x axis this axis the spatial x axis.

Student: No in the.

This could be x or y axis or any axis in the equatorial plane because so, this is the E-plane right E-plane contains the z axis it is like a longitude it can be anywhere. So, how do you are your question is how do I.

Student: (Refer Time: 05:28) theta 2 z.

Theta, no do not interpret this as a Cartesian plot, a polar plot where this length over here is what is equal to $P(\theta)$. Like the like you do when you plotted RCS right. So, you start from $\theta = 0$, $P(0) = 0$. So, where was that point at the origin right then $\theta = \pi/2$. So, $\theta = 0$ in the r we have plotting as a function of (r, θ) . So, $\theta = 0$, this gave me 0, because power was 0 and 0 right that is the origin $\theta = \pi/2$ right, what is the power? $P = 1$ at $\theta = \pi/2$ right. So, that brought me over here in (r, θ) coordinates.

Student: So, that you measure the (Refer Time: 06:27).

Yeah. So, it is telling you that power most of the power is going along which direction?

Student: (Refer Time: 06:34).

Equatorial plane right; now, you might ask I just took the ratio of E to $E(max)$ what about H , I mean to find power I should not just be squaring the electric field right I should do? $\vec{E} \times \vec{H}$. So, did I make a mistake?

Student: No (Refer Time: 06:52).

Yeah, because, if you look over here in your far fields over here if you take the ratio of E and H you get η right. So, here I did not write this over here, but if I take $|E|/|H|$, I will get the characteristic impedance right. So, if I wanted the actual power then to multiply by $1/2\eta$ all of that stuff.

Student: So this is for far field here.

This is far field yeah. So, usually we are considering we when the talk about visualizing fields you know we are talking about far field unless you have a specific application where you want near fields, then that is a different thing. Then, I mean the other sort of nice thing about visualizing far field patterns is that this field pattern here does not depend on r .

So, it is like a universal plot whatever r you take as long as I mean the far field it will always look like this ok. This was in the E -plane what does it look like in the H -plane. So, what is the H -plane now the xy right. So, it is like a top view of this guy. So, xy how does the power pattern look in the H -plane. So, what is first of all what is the xy plane characterized by in terms of θ ?

Student: (Refer Time: 08:29).

What does the xy plane look like? I mean for H -plane is the equatorial plane right that is the xy plane, what is the value of theta for the equatorial plane? $\pi/2$. So, what happens to the power pattern at $\pi/2$? $P = 1$ constant right. So, what should it, what should the polar plot be?

Student: (Refer Time: 08:51) polar circle.

Circle right. So, no matter what direction, I no matter what ϕ I take right, this is my ϕ no matter what ϕ I take, I always have 1 right. So, this length is 1 so 1 right, what is it telling you that if I had an observer over here who went around the H-plane, what would he or she find the power to be constant right not changing. On the other hand if he or she went to the poles, what would they find that when they reach the pole 0 power and maximum power on the equator right. So, you are for example, your Wi-Fi router has the two sticks right they are like dipole antennas right. So, if you thought that you would get maximum reception by pointing the antenna at your device. In fact, you are pointing a null you will get no power.

So, you are better off that is why you will find that the devices are kept with the poles with the dipole antennas standing vertical. So, that it feeds maximum people in the plane and there is no, I mean monkeys are not yet using Wi-Fi one the roof. So, we do not care about the z

direction for that right. So, I mean I derived this for hertz dipole, but it will also be roughly true as you go to longer antennas.

Student: (Refer Time: 10:06).

Yes, $1/r$ electric field.

Student: We have $1/r^2$.

No, this F is normalized you know. So, the r cancels off this is at a given r , $E_{\theta}(max)$ will also have a $1/r^2$ right, $1/r$ this is maximum as a function of θ .

Student: That is good.

Right so, then you can say at a given r ; so, on at a given r i.e. on a given sphere.

Student: So, F will remain constant as a function of r .

Yeah, F remains constant as a function of r which is why it is very useful. If I wanted to find out absolute power what I would have to do is find out $E_{\theta}(max)$ which will have my r^2 dependence in it and multiplied by this $|F|^2$ guy.

How will this look like in 3-D? I have plotted two sections yeah. So, what food group does it resemble?

Student: Apple group.

Not donut is more like it, yeah apple also apple, donut whatever depending on your preference right so.

Student: Apple does not go to 0 at the centre.

Yeah apple does not go to 0 at the centre.

Student: (Refer Time: 11:16) the antenna also will have a bump.

It has a bump, but apple does not go all the way to 0 right very tight donut can go to 0.

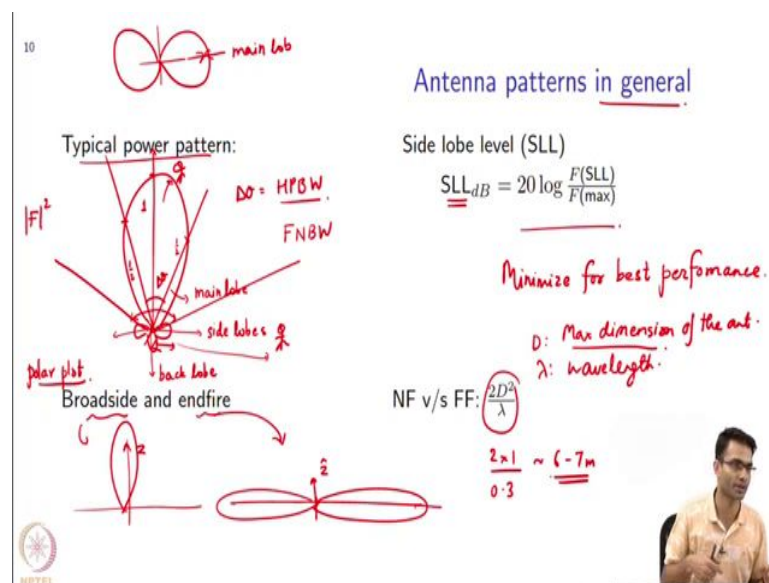
Student: (Refer Time: 11:24) there is no hole.

There is well there is a hole of infinitely small radius.

Student: Yeah.

Yeah. So, it is a weird dipole weird donut ok. So, that is how we visualize the fields fine. This, I mean this framework these terminologies they are all used in the field of antennas ok, and we can spend a long time discussing various definitions you know, but we shall not do that here ok.

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So, now let us what I have done was we looked at the hertz dipole in general. So, let us look at some typical antennas ok. So, in general, so, we are moving away from hertz dipole right. So, typical power pattern how does it look like, this is what we will find in the literature. So, you will find something like this many papers you will see diagrams like this right.

So, what does this mean, first of all what coordinate system is this plotted in? It is a polar plot ok. So, when you see this do not try to interpret this as a x y plot right. So, the origin is over here. So, what is this saying that if I draw this right in this way. So, this is the direction of beam maxima and then I have something else over here right this is another thing and this is yet another thing.

So, this guy is called the not surprisingly called the main lobe of the antenna ok, all realistic antennas will have a main lobe and these things on the side they are called not surprisingly side lobes and opposite direction of the main lobe is what is called a back lobe ok. Typically this is normalized such that this number is 1 right because this is where $E_{\theta} = E_{\theta}(max)$ right, that is the maxima of your, what I am plotting over here is $|F|^2$ the power pattern.

Student: We had the 0.

We had what as 0?

Student: (Refer Time: 13:37) talking about this.

We are not we are no longer talking about the hertz dipole we are talking about some general antenna and this is a hypothetical power pattern for this antenna, it might look like this right so, this.

Student: See what does the (Refer Time: 13:51).

So, I what did we have in the hertz dipole case? So, we had something like this right.

Student: Right.

So, in this case this was the main lobe and the maximum direction in which power was going. So, the hertz dipole has no side lobe and it has no back lobe, it only has a main lobe or you can say I mean or another way of calling it is that, it has a side lobe equal, I mean a back lobe equal to the main lobe whichever way we want to think about it ok.

But, realistic antennas are more like this what I have drawn over here, there is one main lobe that you are designing, which is going to be your intended beam that you will use. And there is something which you cannot help, there will be some energy that is getting leaked out into the side lobes ok, that is part of the game you have to live with it and even worse is the back lobe, you cannot do much about it this is what happens in realistic antennas. How do people characterize these beams you want to find out; so, for example, if you wanted to do point to point communication which for example, in 5G will happen; in the case of 5G you will have multiple beams each one talking to one user and all.

So, you want to be able to produce thin beams which are focused at one point and maybe you can electronically steer it will not. So, the way to characterize this is to find out that theta at which this becomes half the power is a power right. So, when power becomes half. So, I can find out, so this is also half. So, this over here this becomes the half power delta theta. So, this delta theta becomes it is called the Half Power Beam Width. This is one way of characterizing a beam. Another way I mean there are several ways the other popular way is to find out that theta at which you get a null.

So, in this case it will be this direction right here I have a null. So, then this over here is it becomes it is called the First Null Beam Width. Various definitions all of them are basically attempts to capture how focused the beam is. So, that is your typical power pattern which you even you open papers you read that is what you will find and two popular configurations of radiation patterns are given names broadside and endfire ok. So, the broadside radiation pattern is something which is focused in which direction?

Student: (Refer Time: 16:45).

Right so, something that is along the axis of the antenna itself most of the energy is going over there that is your board side and end fire is in the direction over here like this right. So, this is your end fire over here so, this is your z axis. So, hertz dipole looks more like a which of the two does it look like.

Student: End fire.

End fire ok. So, this is just nomenclature. So, in communication problems for example, 5G I will give an example of 5G is the best example or even your regular 4G that you are working with right now; when you are sending a beam to a particular direction you do not want that signal to go to someone else.

But if you have a side lobe that same signal is also going to another user that is standing over here right, some users here your intended signal is supposed to go here you have the maximum beam very good, but your leaking your one part of the beam over here. If I had a very sensitive receiver, I will be able to record what is happening over here. So, in antenna

design typically people want to reduce this side lobe level. So, how do you characterize this is using this expression over here. What is the F for the side lobe level?

So, what is this extent over here divided by $F(max)$. So, you want to minimize for best performance right. SLL is a very commonly used word in antenna and design that's why I mention it over here. And finally, this we had glossed over this distinction between near field and far field when does it actually begin right.

(Refer Slide Time: 18:45).

The Hertz Dipole: Near fields

$$\vec{H} = \frac{I \Delta z}{4\pi} jk \left(1 + \frac{1}{jkr}\right) \frac{e^{-jkr}}{r} \sin \theta \hat{\phi}$$


$$\vec{E} = \frac{I \Delta z}{4\pi} j\omega\mu \left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right] \frac{e^{-jkr}}{r} \sin \theta \hat{\theta}$$

$$+ \frac{I \Delta z}{2\pi} j\omega\mu \left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right] \frac{e^{-jkr}}{r} \cos \theta \hat{r}$$

$\left. \begin{array}{l} \vec{H} = \frac{I \Delta z}{4\pi} \frac{e^{-jkr}}{r^2} \sin \theta \hat{\phi} \rightarrow \frac{1}{r^2} \\ \vec{E} = \frac{I \Delta z}{4\pi} j\omega\mu \left[\frac{1}{(jkr)^3} \sin \theta \hat{\theta} \right] \frac{e^{-jkr}}{r} \\ + \frac{I \Delta z}{2\pi} j\omega\mu \left[\frac{1}{(jkr)^2} \cos \theta \hat{r} \right] \frac{e^{-jkr}}{r} \rightarrow \frac{1}{r^3} \end{array} \right\}$

$\int_{\text{Vol}} \vec{E} \times \vec{H} = \frac{1}{r^5} (\sin \theta \hat{r} - \sin 2\theta \hat{\theta}) (jk) \rightarrow \frac{1}{r^5}, \text{ purely imag.}$

$\leftarrow \text{Energy transferred between E \& H fields} \rightarrow \text{reactive fields.}$



There is no good answer for it because, it depends if you look back and these expressions. This approximation here the hertz dipole itself had no dimension because Δz was very small, but when I make a realistic antenna a rule of thumb is this factor $2D^2/\lambda$ where D is the max dimension of the antenna.

So, for distances greater than $2D^2/\lambda$ and λ is the wavelength at distances that are far away, I mean at distances that of greater much greater than $2D^2/\lambda$, your far field begins. So, just a typical example, we have all seen those triangular vertical base stations right on your tops of buildings and all. If you wanted to find out what is the far field distance, where does the far field begin? So, what would you think D is roughly for that, what is the maximum dimension

of that? The base stations that you see on top of bridges buildings that serves your mobile network, what is the maximum dimension that you think, how much.

Student: Dimension of antenna.

Dimension of antenna is what I am asking here.

Student: Height.

Yeah, height max see D is the maximum dimension do not ask me the width radius now. What is the maximum dimension of that structure?

Student: 200 meters.

200 meters.

Student: Sir I will be (Refer Time: 20:18).

So, you guys have not seen it very close right it is not bigger than 1 meter right, we I am not talking about some FM station or something.

Student: (Refer Time: 20:28) yes that is it.

No, I am talking about these white colored things that are kept vertical.

Student: Yeah that is are not vertical.

1 meter right no I mean order of meters right that is what I am to because that is, so that is 1 meter 2 into 1 and wave length is what frequency does it work at your mobile phone.

Student: 2.4.

2.4 is Wi-Fi.

Student: Wi-Fi.

Yeah. So, this is a 4 is.

Student: (Refer Time: 20:55).

About say 1 gigahertz. So, that is 30 centimeters. So, 0.3 meters right. So, how much is that 20 by 3 roughly?

Student: Some.

Some 6-7 meters; so, if you want to be safe from radiation damage to yourselves should be at least this distance away from a base station. While safety apart, you can say that when I am, further than this distance away from that base station, the far field approximation is correct ok. So, this is a typical way in which we can use this formula right.

Student: So, users (Refer Time: 21:35) 25 height. So, that (Refer Time: 21:38).

Yeah, it is kept at a height the base station is kept at a height. So, that it is not blocked by buildings and so on, it can easily reach users ok. Line of sight is not a deal breaker because the waves still diffract through the windows and doors. Any questions about this?