Computational Electromagnetics
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Applications of Computational Electromagnetics
Lecture – 14.07
Antennas – Hertz Dipole – Part 1

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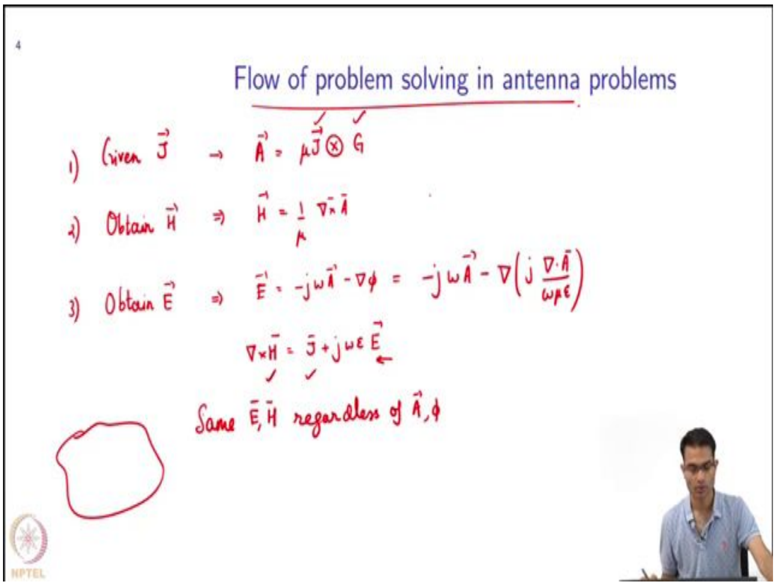

So, what we will do next is we look at a simple; very very simple antenna which is called a Hertz Dipole where this J term is extremely simple ok. So, it will just give us practice in this flow of derivations ok. Well, later on we will come to a little more interesting problem that if I give you; so, what is this Hertz dipole that I am talking about, it is an antenna of very very small dimension; very very tiny dimension.

So, the current over it is not a function of space is just some value. So, that is why I can do step 1, 2, 3 easily, when I go to real life problems I actually do not know what is the current on the antenna itself. So, I have to solve and find out the current first then repeat these; so, that is the step 0 added over here, and that is where computational EM helps us ok.

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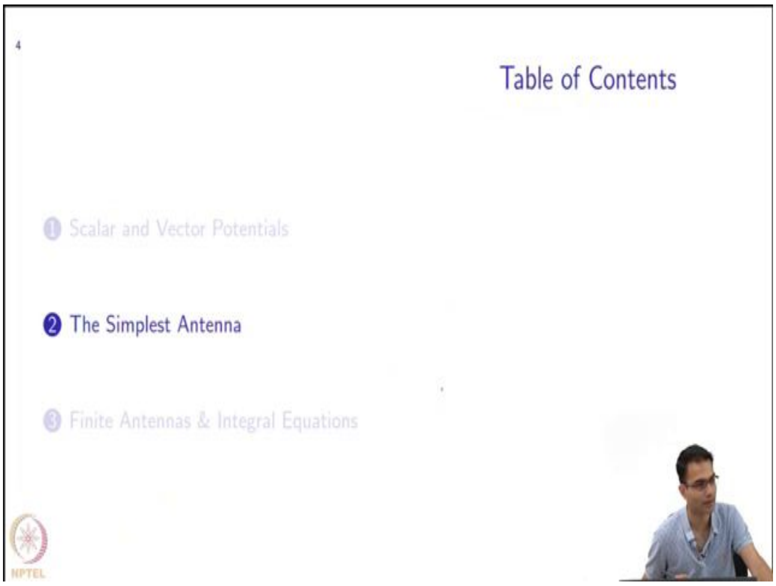

So, we will have a look at this simplest antenna which is your Hertz dipole ok.

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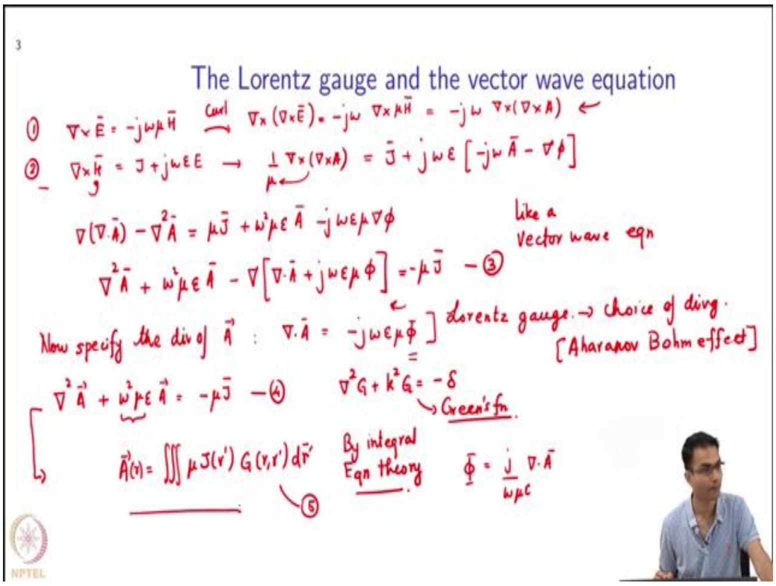

So, let us look back at our definition of A over here right, this is a definition of A.

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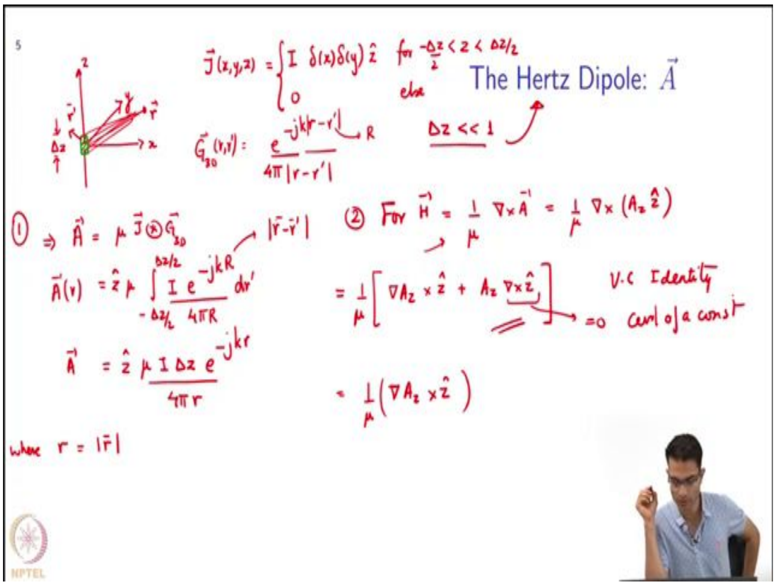

So, now what is Hertz dipole, as I've already indicated let us say this is my coordinate system and I put one tiny is a wire over here very tiny and of dimension Δz . And because it so tiny, I can say that the current in there is constant need not be a function space correct. So, the $\vec{J}(x,y,z)$ I can ask what is this current as a function of x,y,z. So, what can we call it, how will we write it?

Student: (Refer Time: 02:08).

I have to write it as a function of x,y,z; so, be little bit more precise and how I write the current has to be there have to be delta functions because it is very very small right. So, what about the x dependence of this, it is going be a delta in x then delta in y.

Student: (Refer Time: 02:35).

So, it will not be a delta in z because there will be a point so, this is not be a line current right. So, this will be $\vec{J}(x,y,z) = I\delta(x)\delta(y)\hat{z},\ for\ -\Delta z/2 < z < \Delta z/2$ it is like a tiny little step function ok. So, it is to drive home the point let us say it is very thin; infinitely thin and having some finite dimension only in one direction Now, the since this is a 3D problem, I need the 3D Green's function; so, what was my Green's function in 3D, you remember.

Student: (Refer Time: 03:27).

$$G_{3D}(r, r') = \frac{1}{4\pi}\frac{e^{-jk|r-r'|}}{|r-r'|}$$

So, from here I can write down A for this Hertz dipole, it is going to be we have written it as the convolution of J and Green's function.

Now, of these when I this convolution will have 3 integrals xyz; x, y will pop out right I will only be left with z right. So, this will become; $\vec{A}(r) = \hat{z}\mu \int_{-\Delta z/2}^{\Delta z/2} \frac{Ie^{-jkR}}{4\pi R} dr'$.

So, what would be a reasonable way to simplify this term? Remember, this Δz is very very small that is what makes its a Hertz dipole, what approximation can I make over here, remember R over here is $|r - r'|$.

Student: Taylor approximation

Taylor approximation something even simpler than that; so, what is r'? So, you have looking at this distance, this distance this all of these distances in this in convolution.

Student: Constant r constant.

Make it approximate $r' << r$. So, I can assume that.

Student: The constant.

Let us just constant;

$$\vec{A} = \hat{z}\frac{\mu I \Delta z e^{-jkr}}{4\pi r}$$

So, we got all are constants over here; yeah, I thinks all correct now I will just write it write one think over here where $r = |\vec{r}|$. So, the next think that I have to do is find out so, look back over here at the steps I had do I have my J now; I mean I had my J, I had my G, I calculated A. Next step would be to calculate?

Student: H.

H right; so for H, what is the relation I use, $\frac{1}{\mu}\nabla \times \vec{A}$ right.

Student: So, (Refer Time: 07:58).

So, we can call this as step 2 ok. Now you should you should use your coordinate system to your advantage, there are sort of simple ways of doing this and there are some very complicated ways of doing this and even in the literature we will find complicated and simple ways. So, we will take one elegant wave of doing this. So, curl of a now, what is a coordinate system in which you would want to do this, spherical is what you would want to do this in right you could as also write this as r as $\sqrt{x^2+y^2+z^2}$ and do it in Cartesian that would be a lot more work.

So, let us before we actually take the curl is do some simple sort of things over here. So,

$$\vec{H} = \frac{1}{\mu}\nabla \times \vec{A} = \frac{1}{\mu}\nabla \times (A_z\hat{z}) = \frac{1}{\mu}[\nabla A_z \times \hat{z} + A_z \nabla \times \hat{z}] = \frac{1}{\mu}(\nabla A_z \times \hat{z})$$

(This is a vector calculus identity,just like a product rule)

So, notice just by this simple modification instead of having to take a curl in spherical coordinates what do I have to actually take, just the gradient. It is much easier to calculate then curl right. So, let us see how that works out.

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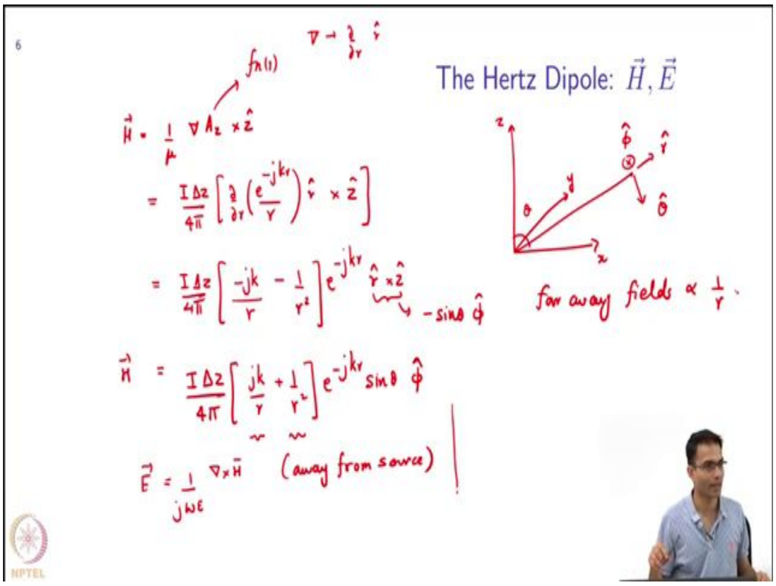

Now, also this is the gradient in spherical coordinates right so, let us write that down.

$$\vec{H} = \frac{1}{\mu}\nabla A_z \times \hat{z} = \frac{I\Delta z}{4\pi}[\frac{\partial}{\partial z}(\frac{e^{-jkr}}{r})\hat{r} \times \hat{z}\ ] = \frac{I\Delta z}{4\pi}[-\frac{jk}{r} - \frac{1}{r^2}]e^{-jkr}\hat{r} \times \hat{z}$$

Now, what is being asked over here $\hat{r} \times \hat{z}$ what will I get?

Student: (Refer Time: 13:06) first.

So, it will be $r\sin\theta$ right cross product and direction more importantly.

Student: (Refer Time: 13:19).

You have what are this spherical co-ordinates $(\hat{r}, \hat{\phi}, \hat{\theta})$ right. So, which way is your $\hat{\theta}$? Downwards, which way is the third direction is $\hat{\phi}$, is into the board or out to the board? Into the board right. So, $\hat{r} \times \hat{z}$ where will it always point; $-\hat{\phi}$ right? And you know it has magnitude $\sin\theta$ right. So, $\hat{r} \times \hat{z} = -\sin\theta\hat{\phi}$.

So, this magnetic field becomes $\vec{H} = \frac{I\Delta z}{4\pi}[\frac{jk}{r} + \frac{1}{r^2}]e^{-jkr}\sin\theta\hat{\phi}$ So, the reason that we got; did me have to do much effort to do get this? No, right and the reason is because you made these intelligent choices, here I did not go about you know trying to put in the curl in the spherical co-ordinates right I would have I have to do lot more algebra this is 0 curl of a constant.

Student: (Refer Time: 15:08).

So for example, if you look at the way Balanis does it, he takes a few more steps because it is doing it in a not in the smartest coordinate system, but the other author for this is (Refer Time: 15:23), the other book he does it in this nice way. So, that just by geometry you can see that this H is always pointing in the $\hat{\phi}$ ok. So, once I have got H, I am done I can get my electric field how; by taking.

Student: Curl.

Curl right. So, I can get E from here by simply the curl relation right. So, this we will complete this derivation in the subsequent module ok. So, just; so, one last thing I'd like to draw your attention to is that there are two different parts of this magnetic field, one has a 1/r dependence and the other has a $1/r^2$ dependence.

If I go very far away from the object, what term will dominate?

Student: (Refer Time: 16:28) r.

Only.

Student: r.

1/r will dominate right. So, the and the same without yet deriving it I can tell you that the same will happen for the electric field also, the field far away from the dipole will be dominated by a 1/r term. So, does that contrast with something that you have learnt from like class 9?

Student: (Refer Time: 16:52).

If I take if I take point charge what is the electric field far at any distance away from it Coulomb's law tells me.

$1/r^2$.

Student: (Refer Time: 17:04).

$1/r^3$ for a dipole right; yeah But here what do we see electromagnetic fields 1/r far away from the sort of this smallest building block of an antenna which is a Hertz dipole, gives me a 1/r ok. So, far away fields are of the form 1/r ok. This will be important as we go here and this is true for any antenna any weird shaped antenna go far away from it the fields are going for fall of has 1/r.