Computational Electromagnetics
Prof. Uday Khankhoje
Department of Electrical Engineering
Indian Institute of Technology, Madras

Applications of Computational Electromagnetics
Lecture – 14.06
Antennas – Potential formulation

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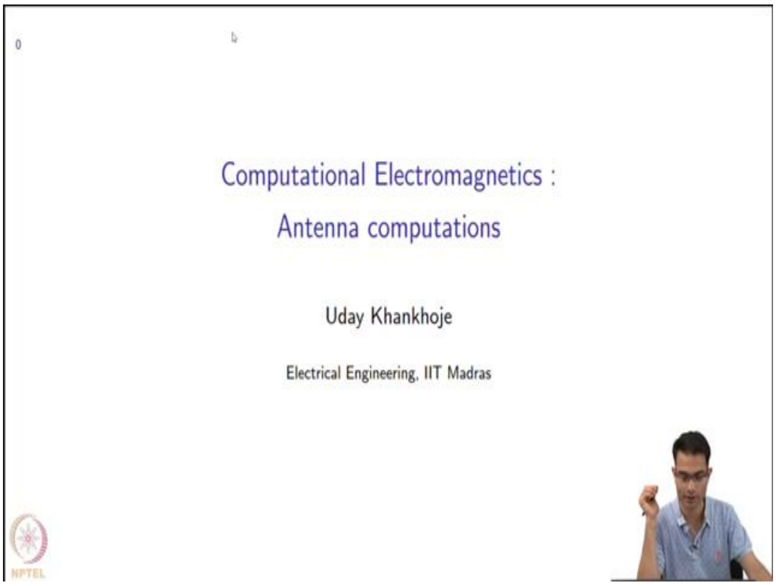

Right. So, in this module we are going to look at some Applications of Computational Electromagnetics to modeling Antennas what is their impedance, how do they radiate?

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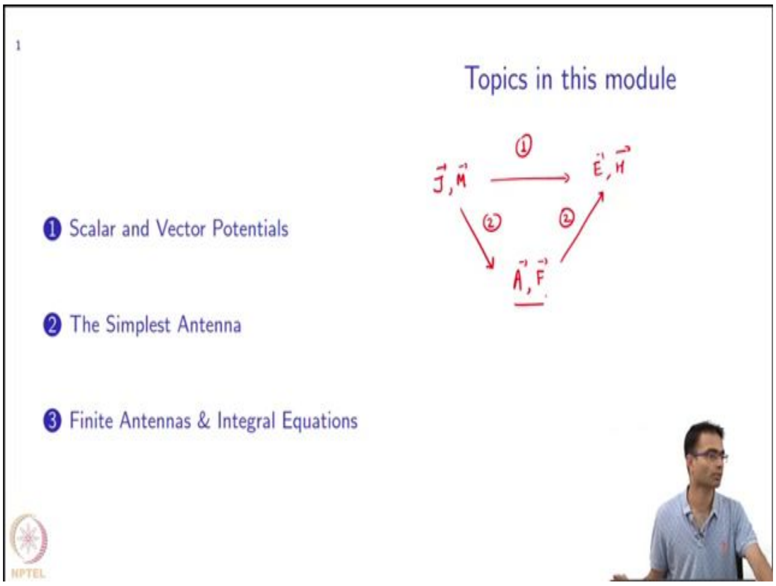

So, to start with we will actually take a slight bit of a detour. So, far what we have been seen in from Maxwell's equations is that, if you give me current I can calculate E and H that is our straightforward route right. So, you give me J and maybe even M if you have a magnetic current and out comes E and H this is a standard route. But there is another route where you go from these currents you would calculate something else in between. These are magnetic and electric vector potentials and from here you go to E and H.

So, these are two routes; this is route 1 and this is route 2. The source and destination of these routes remains the same, but the intermediate steps are different and for antenna problems this is generally true that this second route is easier ok. In our scattering problem so, far we have not encountered this A vector right you can also formulate this scattering problem in terms of the A vector ok.

So, all basically those four Maxwell's equations, how you treat them? So, since we have not yet looked at these vector potentials we will introduce them ok. And then we will deal with the simplest antenna which is your hertz dipole and then go to more complicated cases right.

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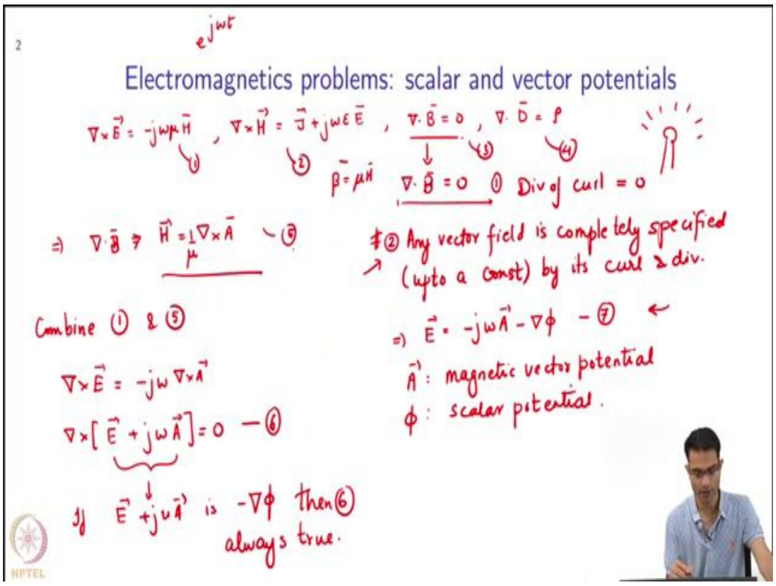

So, let us look at these. So, scalar and vector potentials. So, let us say write down our Maxwell's equations.

Now what we will deal with is we are dealing with non magnetic materials and moreover. So, when we are talking about antenna problems here, the antenna sitting over here it is radiating out in free space and this is usually free space that we are talking about.

So, of course, μ is that of free space and in general we are not talking about magnetic material over here. So, this condition over here I can rewrite as simply $\nabla.\vec{H} = 0$. So, what sort of the starting points of this vector and scalar potentials is this equation over here ok. Looking at this you can easily I mean by using your standard identities in vector calculus, this is satisfied by any vector of the form of what form? Can I replace H by something and make a statement that will always be true?

Curl right. So, the idea is that divergence of curl is always 0 regardless of what the field is. So, I can write $\nabla.\vec{B} = 0 \Rightarrow \vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$

And I do not I mean A could be anything right this relation will always be satisfied. So, what I am doing is from Maxwell's equation I am finding out something which is consistent right this is not inconsistent with Maxwell's equations. So, this is the first idea right the second idea that we are yet to use, but we will use is that we are done this right in the beginning of the module studying in vector calculus, that any vector field is completely specified up to a constant if you give its?

Student: Curl.

Curl and divergence ok. So, its completely specified by its curl and divergence ok. So, for this vector field A what have I specified?

Student: (Refer Time: 04:55).

Right. So, far I have specified the curl. So, 1 degree of freedom is still there with me the divergence which we will sort of exploit later on ok. So, now, that I have this what can I do with it? So, let us number our equations was 1 2 3 4 ok. So, now I can combine 1 and 5 because from 5 I get a relation for H correct. So, what become what happens now?

$$\nabla \times \vec{E} = -j\omega \nabla \times \vec{A} \Rightarrow \nabla \times (\vec{E} + j\omega\vec{A}) = 0$$

So, this E this relation over here is always going to be true.

Student: Gradient of.

If it is like the gradient of something right. So, if $\vec{E} + j\omega\vec{A} = -\nabla\phi$ is of the form. So, this is equation 6 right if E plus j omega A is of the form let us say minus grad phi then 6 is always true ok. So, what does that give me it gives me? $\vec{E} = -j\omega\vec{A} - \nabla\phi$ let us call that equation 7. So, what did we start with? We started with A, from A I have supposing I give you A I have equation 5 which gives me H and once I get H I can for example, take curl of H and get E right and I also have a relation for E in terms of A and ϕ right.

So, A is called the magnetic vector potential and ϕ is called the scalar potential right. So, far we have not said anything about this point number 2 ok, so, we will come to it. So, what could we do next?

Student: Curl of first equation.

Take curl of first equation and then that will allow us to substitute right.

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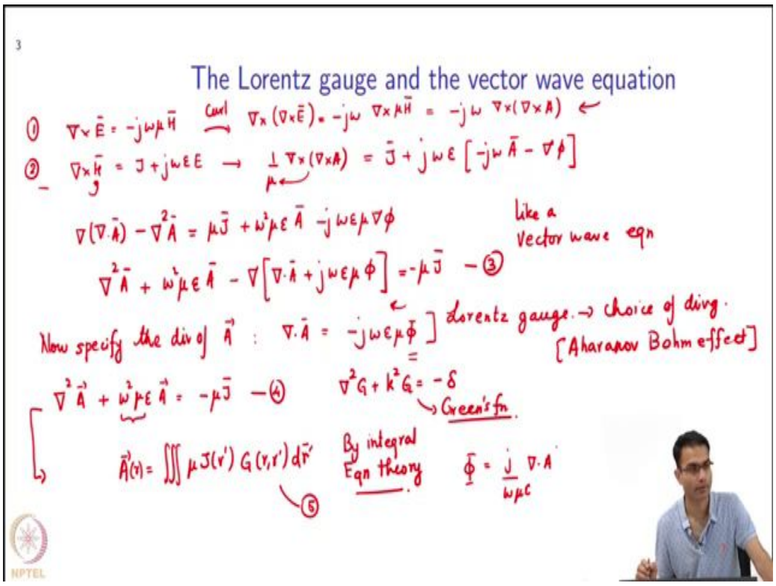

So, let us write it down over here. So, my first equation was $\nabla \times \vec{E} = -j\omega\mu\vec{H}$. Now if I take curl on both sides I get

$$\nabla \times \nabla \times \vec{E} = -j\omega(\nabla \times \mu\vec{H}) = -j\omega \nabla \times \nabla \times \vec{A}$$

Student: Identity (Refer Time: 11:10).

We should apply the identity can I not do something else, so, right.

So, then what? So, this is taking us to one route where possibly we will have to apply the vector calculus identity to E itself. This was equation the route for equation 1. If I take equation 2 itself and do not apply any curls what happens here? So, let us take So, $\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ ok.

$$\frac{1}{\mu}\nabla \times (\nabla \times \vec{A}) = \vec{J} + j\omega\varepsilon[-j\omega\vec{A} - \nabla\phi]$$

$$\nabla(\nabla.\vec{A}) - \nabla^2\vec{A} = \mu\vec{J} + \omega^2\mu\varepsilon\vec{A} - j\omega\varepsilon\mu\nabla\phi$$

$$\nabla^2\vec{A} + \omega^2\mu\varepsilon\vec{A} - \nabla[\nabla.\vec{A} + j\omega\varepsilon\mu\phi] = -\mu\vec{J}$$

So, that route would also bring us to this same point this is the shorter route.

Is this true only if epsilon and mu are functions of are not function of space let us look at me epsilon for example.

Yes over here if I when I did this over here, I assume that you are right its over here implicitly when I took this the del operator I made the assumption that ε is a not a function of space and why is this sort of ok? Because I am trying to solve Maxwell's equation for an antenna usually radiating in free space. So, free space I can make that assumption. I have also made the assumption that μ is not a function of space, because when I substituted for H over here there was a one by μ over here which I took outside the curl right. So, it came from here and went here ok.

So, let us this just this equation let us call it equation 3 right. So, does this equation 3 look like does it look like something that we have seen before.

Student: (Refer Time: 16:27) not such.

It looks a little bit like a Helmholtz equation it is not Helmholtz equation why?

Student: That is a.

That this big term over here right. So, its somewhat. So, more generally we will instead of calling it Helmholtz equation we just call it a vector wave equation right. So, its like a vector wave equation do I know how to solve this we do?

Student: From of this.

This is a horrendous equation I do not know how to solve it right, but here is where I use fact number 2; fact number 2 is divergence and curl both have to be specified. So, if I now specify the divergence of A in and what is the most natural choice for divergence of a negative of this term, $\nabla.\vec{A} = -j\omega\varepsilon\mu\phi$ ok. So, supposing I specify this degree of freedom was there in my hand right. So, then what happens to equation 3 I get $\nabla^2\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}$. I know how to solve this.

So, this choice remember this was the choice, I need not have made this choice and I could have chosen this although even more difficult problem its not unsolvable numerically I can solve it, but this choice is what is called the Lorentz gauge. So, remember this is a choice of the divergence there is another gauge called coulomb gauge where some other choices made for divergence of A ok.

So, it depends on the problem we want to basic philosophy in most EM problems is make those choices which makes life easy. So, here making this choice makes life easy because you know I get this nice vector wave equation and do we know how to solve this? Yes what is the straightforward way of solving this kind of an equation?

Student: Green's function.

Green's function in more generally integral equation methods right particularly so, because this guy is constant its not a function of space. So, I know how to solve this equation it will give me the solution will be what supposing I want to solve this.

Student: Sir.

Yeah.

Student: Suppose is sorry.

Yeah.

Student: And we can if we get (Refer Time: 19:15).

Exactly. So, if we solve for a well we will come to that we will come once let us settle how to calculate A ok.

Student: Sir why should be unique for programs also it will be same.

No coulombs gauge does not make this choice for divergence of A it makes some other choices. So, remember so, what finally, what should it be consistent with I should get this same E and H regardless of whatever choices I have made and you will get the same E and H, your form for A and ϕ will be different, but that does not matter ok. So, actually let me mention there is a slide bit of I should say I do not know if I should use the word controversy, but ambiguity in what I said. As far as electrical engineers are concerned this A and ϕ are purely mathematical constructs which are helping us to solve the problems that is how electrical engineers look at it.

Physicists; however, attribute some physical importance or significance to this A and ϕ, electrical engineers say all we can measure of our fields physicists say that no there is some significance to A and there is something call the Aharonov Bohm effect which tries to measure or give a physical significance to A and they say that in some highly specific quantum regime there is some physical meaning for A. So, A also should be unique right.

And so, this gauge condition and all has to be seen little bit more carefully. In usual antenna problems all we never run into that issue. So, I mean we will not go further, but I will write the name over here you can look it up Aharonov Bohm so, but that is an interesting detour along the direction. So, any other questions ok. So, we know how to solve this equation and the solution is by Green's function can you tell me in words what is the solution?

Student: In any D.

In any D.

Student: (Refer Time: 21:18).

It is the convolution its the impulse convolution of impulse response with the forcing function right. So, what is the. So, if I have defined $\nabla^2 G + k^2 G = -\delta$. So, if I have a minus sign here in the definition, then $\vec{A}(r) = \iiint \mu J(r')G(r,r')d\vec{r}'$

3D I mean you can if G if the problem is 1D 2D 3D that is those many integrals is what we will have right. So, this is by integral so, if you assume 2D 3D Green's function, but the problem is 2D.

Student: So, the (Refer Time: 22:42).

You have to do a little carefully because if one direction there is no physics happening in one direction, in some sense there is an infinity in that direction when you try to integrate in that direction. So, something is happening in xy, but nothing changes in the z direction. So, if I integrate along the z direction for a function that does not change what will I have get? I will get infinity. So, if you can deal with that and cancel it off correctly then you will be fine. But its a little bit more work you will do that way. So, let us just sort of summarize what all we have .

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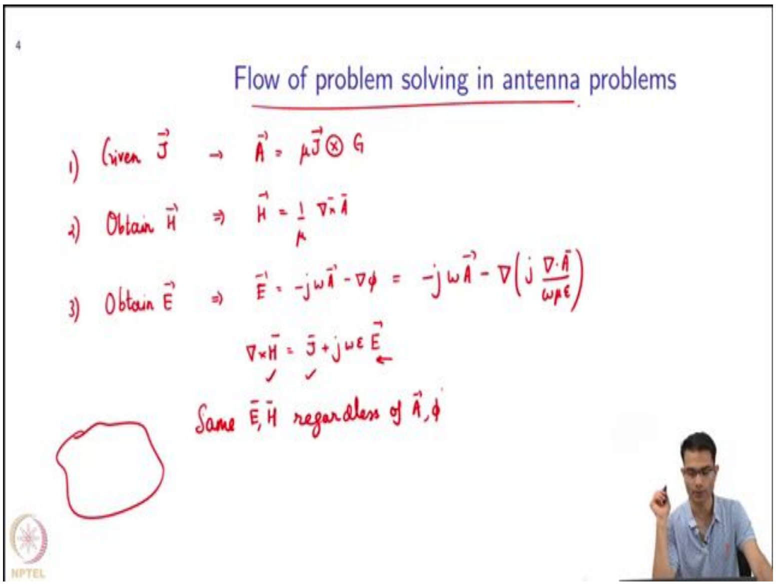

So, sort of the flow of problem right. So, what is given to you is if any problem is given let us say the current J, I give you the current that is flowing in some antenna what can you do? I can find out A as the convolution of μJ and G from this equation over here equation 5 right. So; that means, I get A. The second thing once I get A obtain what? H from it why because what is the definition of right. So, I can use this relation over here H is.

$\frac{1}{\mu}\nabla \times \vec{A}$. Third is what is the other third thing that you are interested in electric field in general that is our objective find the electric and magnetic fields. So, E is going to be what choices do I have?

Student: (Refer Time: 24:22).

So, in terms of this definitions over here where it go. So, $\vec{E} = -j\omega\vec{A} - \nabla\phi$. So, from here I can write down that $\phi = j\frac{\nabla.\vec{A}}{\omega\mu\varepsilon}$. So, this will become $\vec{E} = -j\omega\vec{A} - \nabla(j\frac{\nabla.\vec{A}}{\omega\mu\varepsilon})$. So, this is one way of doing it, is that a simpler way of doing it of finding E?

I know there is a electric vector potential, but you are recalling from memory recalls from logic.

Student: Maxwell's equation.

Maxwell's equations right. So, what is if I know H can you give me E yes what is E? So, I can use curl of H because I have already calculated H in step 2 is $\nabla \times \vec{H} = J + j\omega\varepsilon\vec{E}$. I know H I know J; I get E right. So, all I have to do is take curl of H.

Student: (Refer Time: 26:03).

Student: Difference of the gauge here right.

No ok. So, first of all statement of uniqueness theorem was not that if I know epsilon mu the fields are unique, statement of uniqueness theorem was if I know the tangential E and H fields or some combination thereof over a close contour then I know the fields everywhere and they are unique.

So, its not enough for me to just know ε, μ I mean there are infinite fields given free space for example, right every time I change the current pattern I get a different radiation theory. So, the gauge condition is not violating the is not violating the uniqueness theorem, because whatever different gauge condition I will choose I get the same E and H. If I get the same E and H the tangential fields on this contour are the same. So, uniqueness theorem does not get violated.

Student: (Refer Time: 26:59).

You get the same fields, its like a intermediate variable.

You put the same thing right. So, let me write this down same we go ahead. So, in any antenna problem this is going to be what we will do.