Computational Electromagnetics
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Applications of Computational Electromagnetics
Lecture - 14.04
Inverse Problems – Non-Linearity

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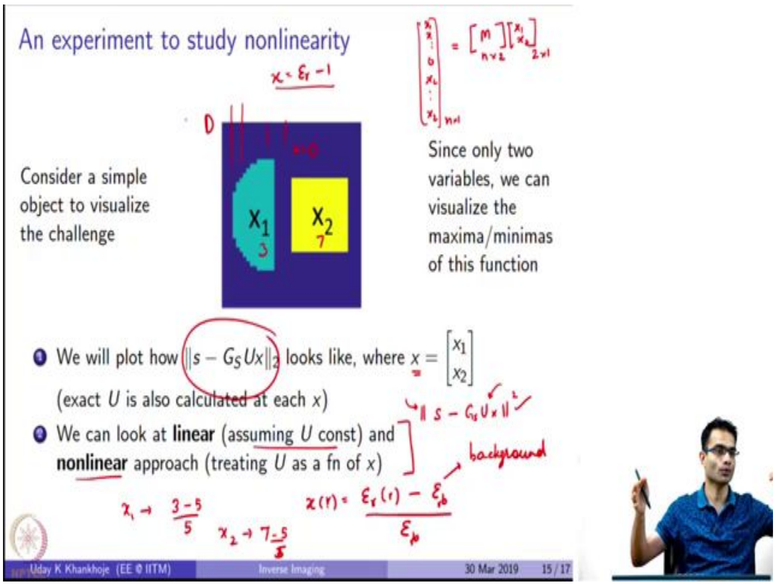

So, to study this nonlinearity we are going to do a simple numerical experiment ok, very simple numerical experiment. What is that experiment? This is my domain D; my domain D has only two objects ok, two objects x_1 and x_2 and further what I am assuming to make my life easier that I know the boundaries of these objects ok.

I am going to make my life really simple I want to restrict myself to two variables because why would you do that? Because I can visualize 2D I cannot visualize 10D right. So, I want to restrict myself to 2D so that I can visualize it ok. So, now, what should I do? This is my D this is my two variable problem so, my contrast x is simply x_1 , x_2 and what I can do is I can plot this for different values of x .

Student: X.

Just to see is there one parabola and there are several parabolas what is there right and its in 2D so, I will not have a line, but I will have a surface. Not just that, I can also look at the direction of the gradients right. What will the gradient direction tell me?

Student: Maximum change.

It will clearly me maximum change and it will tell me that if I did, for example, the gradient descent algorithm which direction will the search go right. So, there are two simple gradients I can think of from the previous page.

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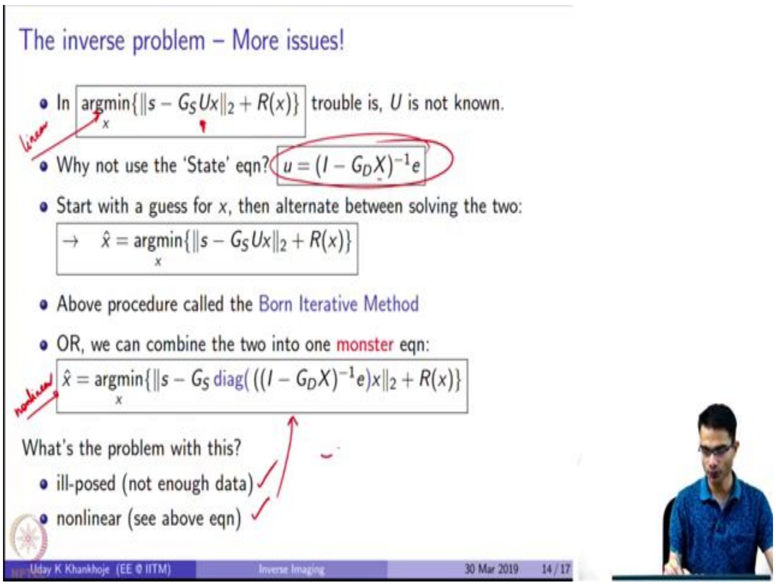

One is that I can take a gradient of this guy where I assume U is constant. So, it's a simple gradient. So, this is what I call the linear gradient or I can be brave and take a gradient of this expression and that will be called a non-linear gradient; obviously, non-linear is exact, but it's computationally very tedious.

So, that is what I am going to study in the next numerical experiment all right yeah ok. So, I am saying assuming U is constant; that means, my gradient I am taking the gradient of $\|s - G_S U x\|$. That means, U is constant.

So, when I take the gradient only the x dependent terms are this guy and non-linear means over here U is a function of x.

Student: If I continue (Refer Time: 02:50). So, this system is like 2 homogeneous for determinant.

2 homogeneous objects in some medium and their permittivity is x_1 and x_2.

Student: Ok.

No ok. So, what I am saying is that this entire object has permittivity x_1 not one pixel, but this entire blue object is x_1.

Student: Ok.

And this entire square object is x_2 those are the two variables.

Student: So, rest (Refer Time: 03:16).

So, you are saying that contrast vector should have been actually the size of the number of the pixels of this right this whole.

Student: (Refer Time: 03:22) like how this is it be similar.

So, actually so, this actual x would have been a very long vector with $[x_1, \cdots, 0, x_2, \cdots x_2]^T{}_{n\times 1}$. That is how this contrast vector would have been. But if I know how to make my life easier I am going to give you this additional info that this entire object is this x_1 this entire object is x_1 x_2. So, I can write this in terms of some matrix which is composed of 1s and 0s multiplied by x_1 x_2.

Right if this is size $n \times 1$ and this is size 2×1 I can get this to be an $n \times 2$ matrix which is this have 1s and 0s in the right places. So, that I get this. So, that I work with just two variables right. So, this m matrix will then get multiplied over here. So, these kinds of tricks we do to make the problems more visualizable other wise you know you will just throw up your hands and say n variables, I cannot visualize it you will not getting intuition.

Student: Then it will be three variable right.

What is the third variable?

Student: Medium permittivity of (Refer Time: 04:25).

0 remember.

Student: Right.

$x = \varepsilon_r - 1$. So, if this is vacuum its $x = 0$ which is given to me I am not going to try to find it.

Student: Yeah.

Do not. So, you see the question is if you want to study this in its full generality then you will have to make this to be blood, you will have to do everything all that. So, you will be so, make a very complicated problem and get no intuition problem. So, we are solving a simpler problem where we know everything except these two these itself is going to be difficult. So, let us visualize this right. So, that is why I am saying only two variables.

Student: So, sir initially we know that we do not know the value of x_1 x_2.

We do not know the value of x_1 x_2.

Student: We do not now.

We do not know the value of x_1 x_2. So, I am going to try to solve this problem to find out what is that x_1 x_2 which gives the best agreement with s the scattered fields that is the experiment ok. So, a lot of a priori information has been given to you the location of the objects the fact that they are homogeneous and the fact that there is nothing else that is the perfect example of a priori information.

Student: So, what we are assuming U constant.

Assuming U constant means that even I look at this objective function and I take its gradient I assume U to be constant and not a function of x. So, then $\|s - G_S U x\|^2$ this is a linear functional. So, to take gradient is very simple that is what I mean by assuming U constant.

Student: But which is U is not constant.

It is not constant. So, what I do is that I assume U to be constant at that iteration find out the new x right. So, I assume U to be I calculate U from x assume it to be constant and put it in over here find out the gradient which gives me a new value of x that I plug in and get a new U put it back in keep it constant. So, it's like a poor mans we have doing this thing. Ideally what you should do you know that U is a function of x. So, put it put the whole thing over here and that is what I am calling a non-linear approach ok.

So, these are the two different cases ok. So, now, let us let us see what happens. So, we have setup the problem very well now let us look at what will happen.

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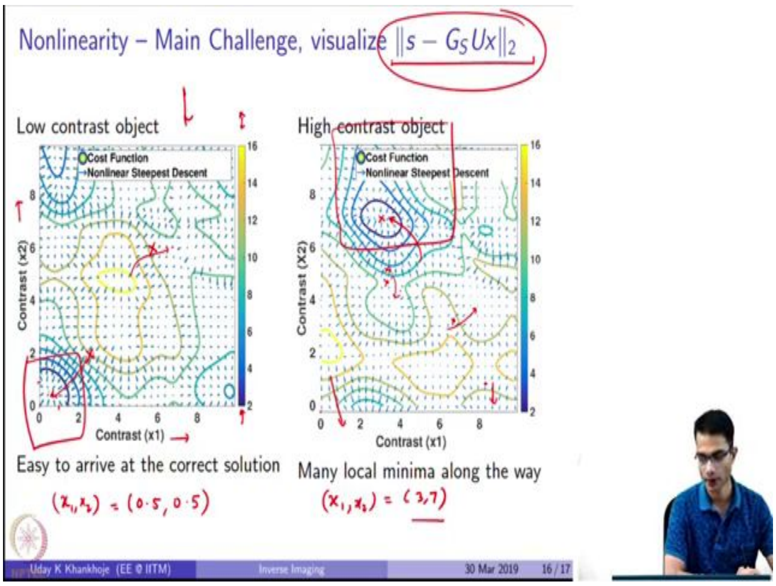

So, what I am plotting is the value of this objective function $\|s - G_S U x\|^2$. What are my axes? x_1 x_2. So, let us first look at the left hand side this is a low contrast problem whether true solution is $x_1 = x_2 = 0.5$ ok. So, dark blue color corresponds to lowest value and yellow colour corresponds to highest value of cost function ok.

So, what I am showing this contour plots right. So, it's like a topographic map of a hill right. So, yellow value is the peak, blue values are the valleys and what are these arrows showing you these arrows are showing you the direction of the negative gradient because I want to always go in the direction of negative gradient that is the steepest descent that will take me to the minima.

So, you notice over here if I start from the origin which was our guess for born approximation I fall into approximately the correct answer half half right. If I had started from let us say one one one, but I still go in to the correct valley, but I still get the correct answer? Yes because in this region over here all the gradients are guiding me to the correct valley. So, its like I have one valley over here right. So, even if I started from here, what will happen? These arrows will nicely guide me back in over here.

So, there is only one valley I start anywhere in the neighbourhood of that valley I reach the correct answer very good. On the other hand, supposing I said no I will start my search from let us say (6,6) supposing I have start from here where do was the gradients taking?

Student: (Refer Time: 08:30).

They take me in some weird direction over here and then here the gradients become very small in magnitude the length of the vector is the magnitude of the gradient. So, I reach here and what happens I cannot proceed any further the gradient is 0, but what is the value of the cost function? It is not minimum right. So, where you initialize matters because now this is a non-linear optimization problem, but if I started anywhere close to 0 not being very strict about it, I reach the correct answer that much is clear. Now let us take another problem ok.

So, before coming to this let us just summarize over here. So, for a low contrast object it does not matter where you start from and you arrive at the correct answer. Why? Because the true minima and the global minima are very close to the origin. So, if I started from $(0,0)$ or its neighborhood I reach the correct answer.

Student: If we started around 0 8.

If I started $(0,8)$ then I use the wrong answer right that is correct and many other points we have started $(8,1)$ also I reach the wrong answer. So, there are many regions in this which will take me to the wrong answer, but since the literature started by assuming started with the theory of born approximation they always started with $(0,0)$. So, they always reach the correct answer. So, they got a satisfactory answer.

And then at some point they realized oh you know realistic tissue need not have very small values of contrast, but are methods are failing you can because you made a simple 2 D problem you can you can see this until why it is failing; its failing because there are so many hills and valleys in between that you are not able to reach the correct answer. And we can see that in the next problem on the right whether true answer is $[x_1\ x_2] = [3\ 7]$ right. So, the correct answer is setting over here if you started from $(0, 0)$ what will happen?

Student: Disaster.

Disaster right. I am going down I am going to get negative values. So, there is no hope actually I I just do not know where to start from if I start from some random point if I start from here again I go down I if I start from let us say here and I reach over here gradients are 0 right ok. So, it's a lot of computational work to produce this diagram I am doing this diagram to aid my understanding

Student: This is.

This is a diagram as a 2 D plot of values of $(0, 0)$ $\|s - G_S U x\|^2$.

Student: About that x 1 x 2.

For where the true $[x_1\ x_2] = [3\ 7]$ and s is corresponding to that and now different different values of x and U what does the value of the cost function looks like. So, in real life I will never have access to this because first of all it will be a may be a 10000 variable problem. So, how do I visualize 10000 dimensions.

Student: (Refer Time: 11:26).

Yeah this is for that for its for this only.

Student: Suppose this case you something else (Refer Time: 11:31).

Yeah exactly for this case all I had to do was make $[x_1\ x_2]$ from half, I made it $[3\ 7]$ and boom I am in deep trouble, I have no good way of knowing where to go wait that is that is. So, that is that is one huge problem over here. Only if I start somewhere in you know roughly this domain over here then will I land up at this problem over here. So, that is why I need some a priori information.

Student: Exactly nice theorem that we do.

No. So, all your weight of machine learning and you know deep learning and all of that can be applied to this to try to learn where might be the correct valley from a lot of data to tell you where to land up in ok. This is why this is a very challenging research problem and realistic tissues realistic landmines and all they will have high contrast right. So, that is why you have to live with this problem.

As was one suggestion was suggested over here um. So, for example, here we have been made $[3\ 7]$. So, this was $[3\ 7]$ and this was a challenging problem right. Now this contrast was defined as $\varepsilon_r - 1$ right, now that is a simplistic definition the true definition of contrast is actually like this

$$\chi(r) = (\varepsilon_r(r) - \varepsilon_b)/\varepsilon_b$$

but ε_b is the permittivity of a background medium.

So, a background medium was vacuum it is $\varepsilon_r - 1$, but if I know that the true permittivity is roughly going to be in this high range then what I can do is, I can fill the medium with some kind of what is called matching liquid or matching material whose ε_b is let us say I can take for example, something like 5 let us say. So, then what will the contrast become?

So, $(3-5)/5$ right for x_1 will become this and x_2 will become $(7-5)/5$ right. So, these are all numbers which are small right minus $(2/5 =- 0.2$ and $2/5 = 0.4$. So, my problem has become lesser I have mapped it sort of this problem I made the permittivity smaller ok, but many times this is not going to be possible because I cannot produce at will a liquid which has permittivity 4 or 5 hardly it reveal right, moreover in the landmine detection problem, forget it. Not really.

So, the question is what we start with m equal to n and then use that. So, it may help you in sort of getting into the right quadrant, but the problem is when I start with m equal to n, I have an m dimensional space and I have got some solution over there now when I want to go for a higher resolution let us say I go to you know 100 m. Now I have 100 m dimensional space how do I relate that m dimensional space and this 100 m dimensional space?

Student: Compares with value you have some many how the.

Yeah I have some value of the thing.

Student: No change every time.

So, in some sense I have to down sample that solution I mean I have let us say x_1 x_2 here, now you are saying I want to retrieve this at a.

Student: Higher resolution.

Higher resolution 3×3 resolution now what do I do? I have got two resolution answer, now I want to map these two as an initial guess for a higher resolution its a 3×3, then what do I do I have to?

Student: To answer in the.

How do I map this to that?

Student: One grid to this one grid.

Yeah, one grid here to one grid there it can be done, but that is again in one retrieve match to do it and that is done and that is helps us sometimes to get into the right valley, but these valleys can be sensitive supposing I started in this valley I reach the correct answer, but if I started let us say instead of starting here I started over here and then I move my go here. So, it may or may not work.

Student: But it can be a good approximation.

It is a good approximation, in fact it is commonly done ok.