Computational Electromagnetics
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Review of Vector Calculus
Lecture – 1.1
Chain Rule of Differentiation

And welcome to the first lecture on Computational Electromagnetics which is the review of Vector Calculus. So, as we will find out in this course the whole language in which this course is thought or spoken is the language of Vector Calculus. So, it is important that we become comfortable with some basic ideas that are useful for electromagnetics. Vector calculus is a very vast area, we will cover only those parts which are relevant to electromagnetics.

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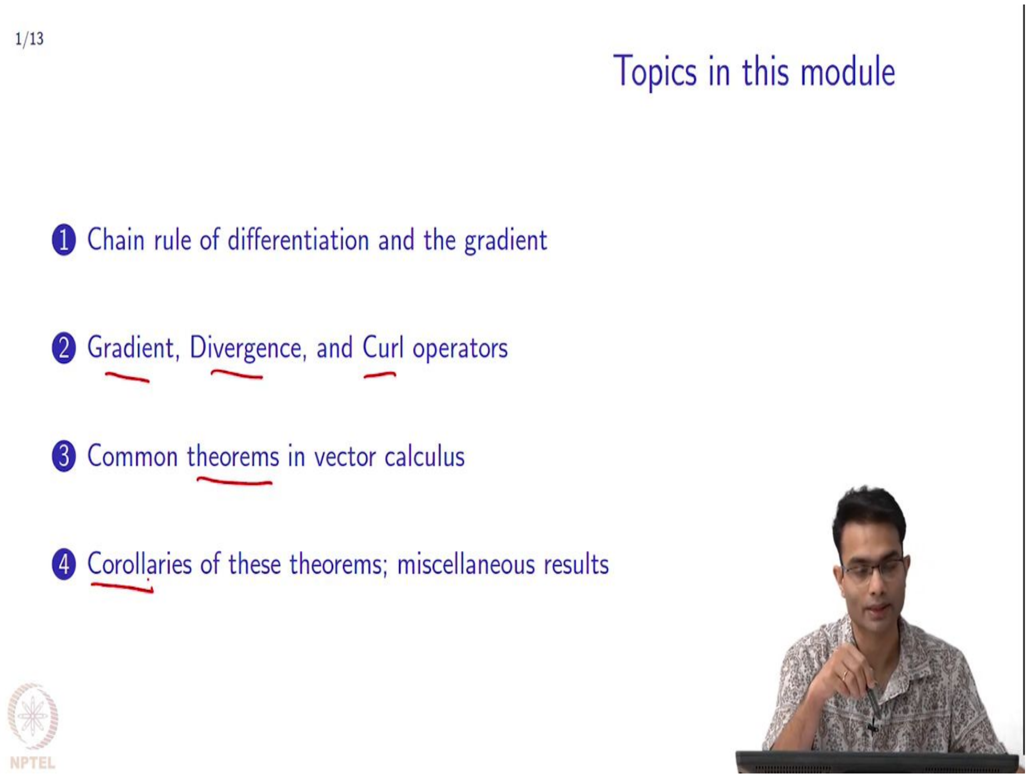

So, these are the topics that we will cover in this module. We will start with the Chain Rule of Differentiation, introduce the idea of the gradient, move to the three most popular operators that we will find the gradient, divergence and the curl. There are some very important theorems in vector calculus that we will talk about and then some of the corollaries which are like the tools and techniques used in this area.

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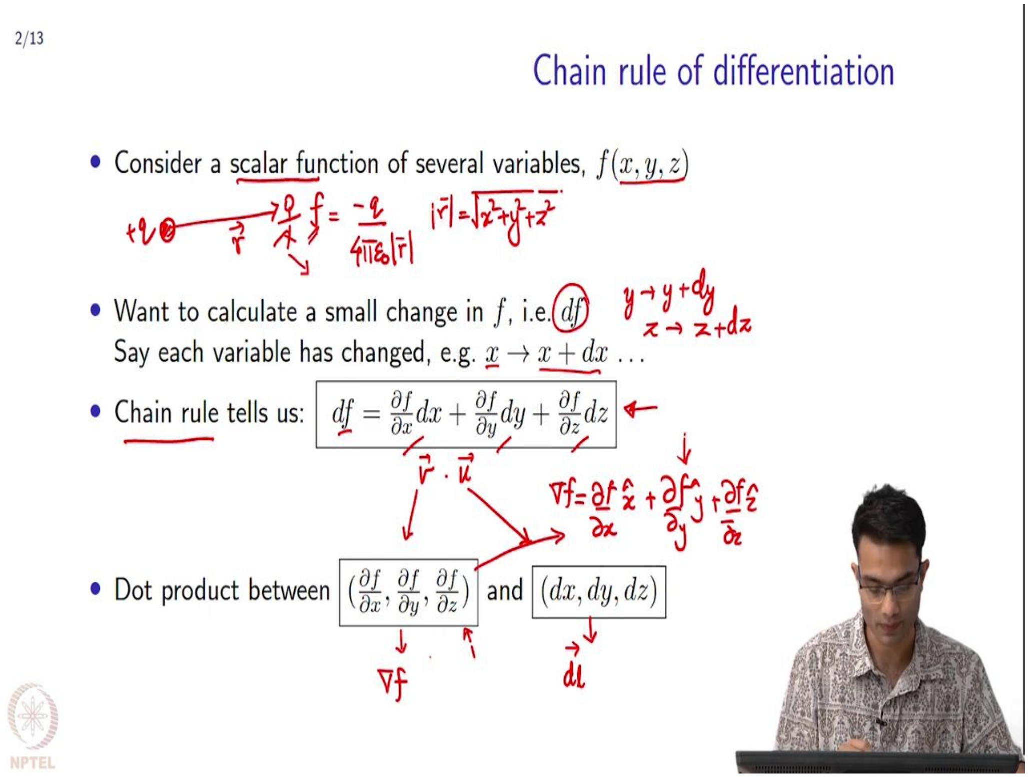

So, starting with the Chain rule; so, let us start from the beginning all right. So, let us consider scalar function. Scalar function and let us say it is of three variables. It could be of any number of variables, but we will stick with three because where even a three dimensional space it is easy to visualize. So, something simple could be for example, here is a charge plus q and I am standing over here and there is some distance over here, there is a vector r that connects the source to where I am, and this observer here could be recording; let say the electrostatic potential right.

Now, we all know that you from you know high school that potential is simply given by a simple expression minus $q/4\pi\varepsilon|r|$ where |r| is square root of $x^2 + y^2 + z^2$. So, now, you have this function f over here which is function of three variables x, y and z.

Now, let say that this observer wants to change in some wants to move in some direction and this direction could be arbitrary and I want to know how does the potential change. So, in other words what is this delta f; if there is some arbitrary change in the direction of the observer ok. So, in other words to make it precise so, let say x change from x to $x + dx$. Similarly y will go to $y + dy$ and z goes from $z + dz$. We want to keep it as general as possible.

So, again this is something that we studied in calculus high school so, called chain rule. So, chain rule tells us how what is the total change in a function given changes in the composite sort of variables. So, this df will be given by a set a summation of the changes due to a change in x due to change in y and a change in z right. So, this hopefully is familiar to all of you chain rule over here.

Note that these are partial derivatives, because the function depends on all three variables this only text the derivative with respect to one of the variables. Now, when I look at this expression while this is correct there is a more convenient way of looking at the same thing. So, if I look at this expression over here, I can think of it as the dot product between two vectors right. So, I can say that this is let say the dot product between two vectors v dot u and that for example, v could be this vector and u could be this vector right.

So, what I have got is this change over here df is the dot product of two vectors and this vector over here is something which appears time in again. So, this is defined as the gradient of f. So, it is a vector and so, it has three components over here and this is the shorthand notation for it this over here is the displacement right. So, this is the observer's displacement I can as well write this as something like dl.

Usually, in this course I will not be putting a arrow on top of the gradient of f, but it is understood that it is a vector, another thing I want to point out is that this is a shorthand notation. So, I have written it in brackets with three component; that means, that it is a vector with three components. I can as well write it as like this gradient of f is equal to x hat, y hat and z hat. You can see that this takes more time to write then this; so, I am going to use this more often or not it is understood when you see something with in brackets with commas it is a vector where I am talking about and its components.

So, this was about the chain rule which gave us the a very convenient way to arrive at the idea of a gradient so, this is the gradient.

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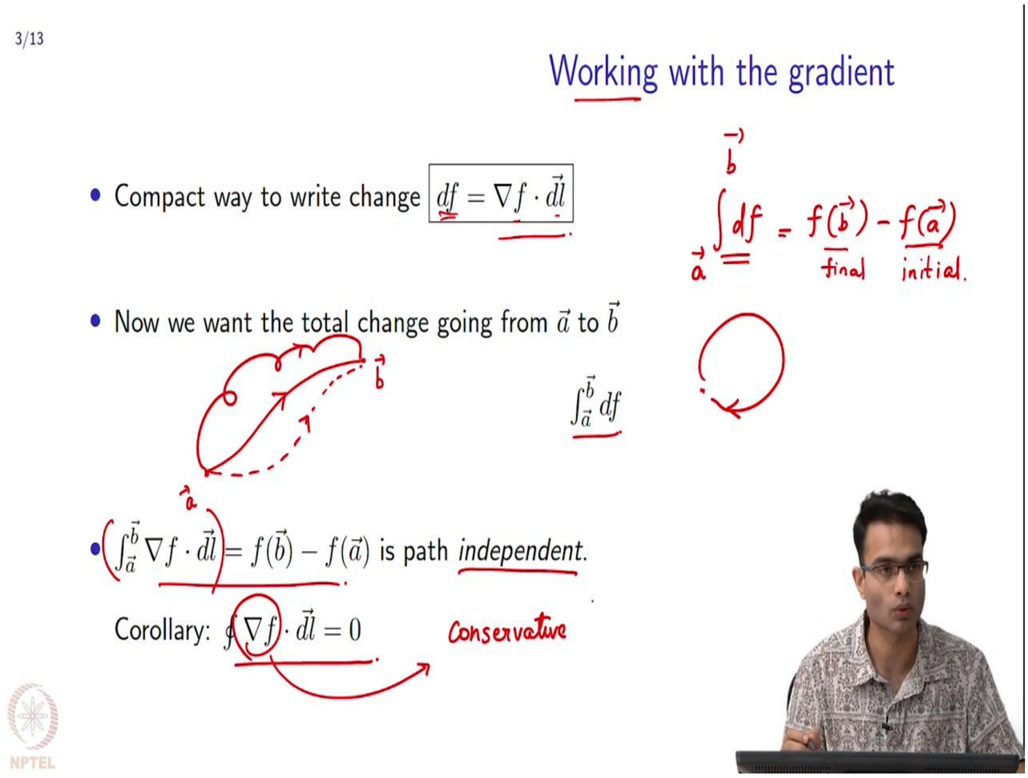

Moving on, now let us try to work with this gradient that we have defined. So, from the previous slide we have calculated this total derivative d f and wrote it as a dot product between the gradient over here and the displacement vector over here. Now, once you have written a differential element as it is called the next objective would be to find out what is the total change; let us say when I go from some point "a" vector to some point over here "b" vector. So, for example, I could go like this. So, this is the integral that I want to calculate. So, for example, this "df" could represent a differential or a small amount of work done and I want to find out the total work done so, I have to integrate along the path.

Notice that the limits of the integration are simply a and b, the root is not being specified ok. So, it is not this is not the only root possible I could for example, have gone like this or any complicated root let I could have gone like this, does not matter it is not specified over here. So, what you can see is that when I start integrating this d f right. So, I know that if I just write right; if you did not know anything else, you would simply write this as the value of the function at in this way fb minus fa right.

But now we also have a nice identity for d f which is in terms of the gradient right. So, this is the relation that we arrive at and the remarkable thing of course is that this expression over here is path independent. It did not matter which path I took because this is a complete differential over here, the result depends only on the final point and the initial point.

So, you can see immediately a very straightforward corollary of this is that if I am going from one point back to the same point right. So, in other words, the final and the starting point are the same, then this is true right the integral over close paths. So, when I put on the integral sign a loop over here like this it means that it is a integral over a closed path starting point initial point is the same and this is going to be zero right. Some of you may have encountered such expressions earlier. So, these kinds of forces if a forces of the type grad phi we call this a conservative force right

So, for example, we know that the Electrostatic force is a conservative field. So, it does no work over a close path not all forces are conservative; so, this is a special case. So, we have got some idea of how to work with the gradient now.