Digital Signal Processing
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Lecture 72:
Sampling (4)
- F_s information needed to know true frequency

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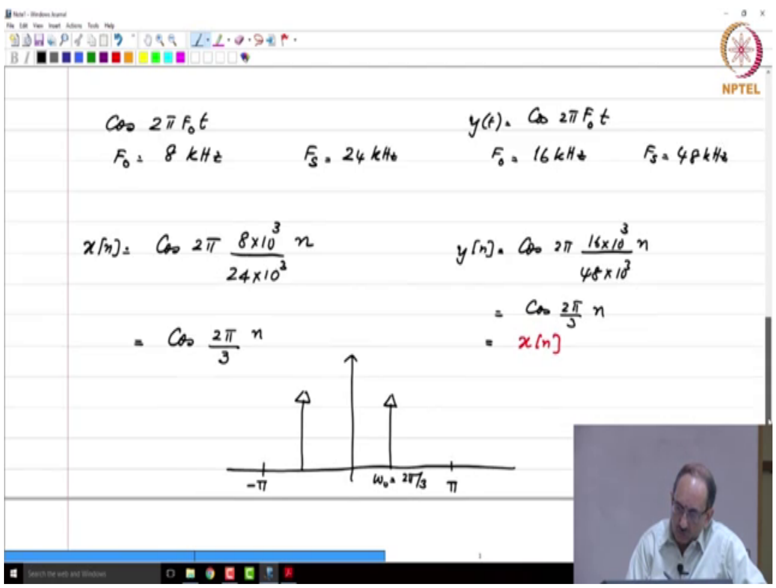

Now, let us consider another aspect. Let us consider $\cos(2\pi F_0 t)$ and we have F_0 to be 8 kilo Hertz. It is enough if you sample this at 16. So, just for this to illustrate, this point, let us assume the sampling frequency is 24 kilo Hertz. And hence, $x[n] = \cos(2\pi F_0(\ \ ))$, you are going to replace t by nT. So $\cos(2\pi F_0 nT)$, $T = 1/F_s$ and hence this becomes $1/(24 \times 10^3)$. And you are going to get $\cos\left(\frac{2\pi}{3}n\right)$. So, this the discrete time sequence that is going to result when you sample an 8 kilo Hertz cosine at 24 kilo Hertz.

Then somebody has a different cosine that F_0 happens to be 16 kilo Hertz and that person chooses to sample the signal at 48 kilo Hertz. So, you have $y(t) = \cos(2\pi F_0 t)$, where F_0 in this case happens to be 16 and sampled at 24. And if you look at the corresponding $y[n]$, so this after all is $\cos(2\pi\ 16 \times 10^3\ t)$, you are going to replace t by nT, $T = 1/F_s$ and F_s is 48×10^3. This of course is, $\cos\left(\frac{2\pi}{3}n\right)$ which is nothing but $x[n]$,.

And hence, if you look at the DTFT, be it either $X(e^{j\omega})$ or $Y(e^{j\omega})$. So, what is going to happen is, remember, we are talking about the infinite duration sinusoid, you have infinite set of samples and ω_0

will be $2\pi/3$. So, the point about this is that, given this spectrum, if I ask you what the original signal was, whether this was an 8 kilo Hertz sampled at 24 or 16 sampled at 48, you cannot tell.

Therefore, given the fact that ω_0 is $2\pi/3$, to what true analog frequency does this correspond to? That is, is this 8 kilo Hertz or 16? You cannot tell unless you have these sampling frequency information. So, going back from this to the true frequency, you need F_s information.

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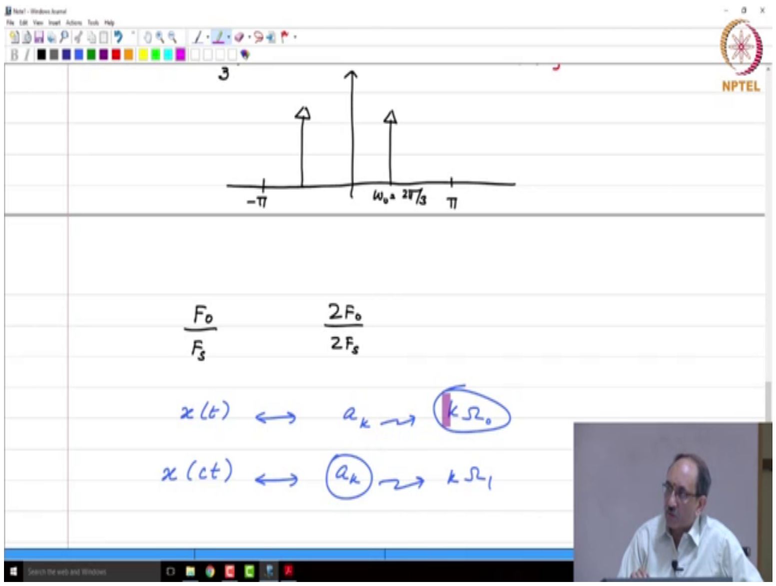

So, this is again, really if you think about this, this is no surprise because any frequency F_0 which is in Hertz which is the true frequency is going to get mapped to F_0/F_s in the DTFT domain, right. Because of this division by F_s, you are normalizing things and hence if you take $2F_0$ and sample it to $2F_s$, you will get the same value which is exactly what is happening here.

Therefore, to go from this DTFT spectrum to the underlying true analog frequency in Hertz, you need F_s information. And actually, you have seen this exactly happen in your signals and systems course. So, this is not the first time you are encountering such a thing, and where is it that you were encountering the same thing in your Signals and Systems course? Ok.

Hint: CTFS. More hint, properties of Fourier series. Why do not you just tell me the property? it is been so long ever since that we learnt about Fourier series, 1 year is a long time, right, I know we are coming to the end of the semester, its one and half years more, ok. So, what was happening there, you had $x(t)$ had Fourier series coefficients a_k. What were the Fourier series coefficients of $x(ct)$? a_k. So, if you look at the Fourier series coefficients as a sequence of numbers, you will not be able to tell whether these coefficients correspond to the Fourier series representation of $x(t)$ or $x(ct)$, just as a sequence of numbers.

However, if you plot this in the frequency domain with k, remember, these are occurring at multiples of Ω_0. And hence, a_k will then, in this case, correspond to the location $k\Omega_0$. However, in this case, this is the k^{th} Fourier series coefficient, but it will occur at $k\Omega_1$, because the periodicity of $x(t)$ and $x(ct)$ are different. And the fundamental frequency is $2\pi/T$, T is different in both cases. And hence, this k^{th} Fourier series component will occur at k times the fundamental frequency, but the fundamental frequency here is different.

And hence, as the sequence of numbers, you will not be able to tell just from a_k information whether it is the Fourier series representation of $x(t)$ or $x(ct)$. You need the periodicity information to map k to the actual location in frequency. So, this is what is happening here is no different. You need the sampling frequency information for you to know what this normalized frequency corresponded to in terms of actual Hertz, question.

Student: Yesterday (Refer Time: 08:28).

Student: So, in the Fourier (Refer Time: 08:34) certain amplitude (Refer Time: 08:37).

That, suppose I chose, it is a very good point you raise. Suppose, my signal is this. I sampled it at twice the frequency, sampling frequency as compared to earlier. Sure, my amplitude goes up by a factor of 2, but it so happens my original signal has been amplitude scaled by half and I get exactly the same spectrum. So, you cannot bank on that. By the way, to give some insight as to why when you sample it at a higher frequency, the amplitude goes up.

Remember, when you sample two things happen. One the periodic repetition is there, the other is amplitude scaling by $1/T$. Now, you need to have an intuitive sense as to why that happens, not merely relying on, ok, the formula say so therefore I will take it, because it is there in the formula. You also need to have an intuitive feel for why this is so.

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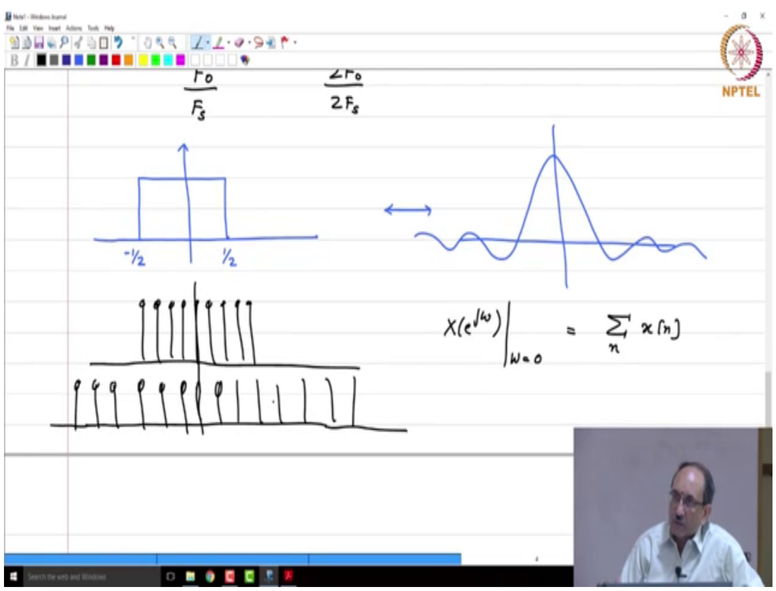

Let us take this example. So, we have a signal like this, say between $-1/2$ to $1/2$, and then this has a spectrum. Now, we are going to sample this. And remember, $X(e^{j\omega})$ at $\omega = 0$ is nothing but this. And you will get a certain number assuming you have sample like this.

Now, on the other hand, if you now sample this at a much denser rate, if you sample this at a much denser rate or rather a higher sampling frequency between $-1/2$ to $1/2$, if you sample it at twice the frequency compared to earlier, you will get twice the number of samples. And hence, remember, but if you plot this as a sequence of numbers, you will get something like this.

And now, if you look at the spectrum at $\omega = 0$ which is again the sum of all the sample values, now you will be summing up over twice the number of samples as earlier and you will get a larger number.

So, this is the intuition behind why that $1/T$ scale factor makes sense. It is not just mere formula.