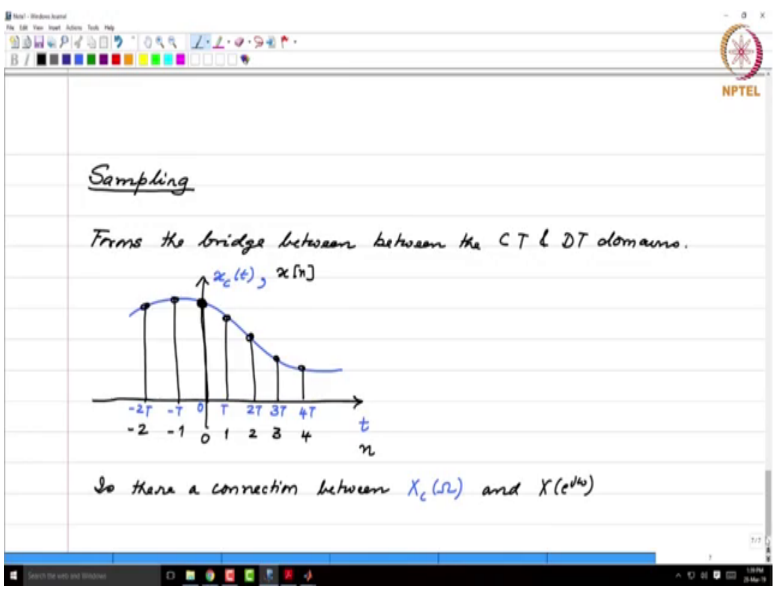
Digital Signal Processing
Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 68:
Linear Phase (4), Sampling (1)
-Introduction to sampling

(Refer Slide Time: 00:23)



Let us move on to the penultimate topic as far as the course is concerned, so this is Sampling. So, you already have an introduction to sampling from the earlier course. So, the initial part will be a recap of what you have already seen, but the goal that we are trying to achieve here is, we want to try and bridge the analog and digital domains. So, we have seen the transform of a sequence, namely the DTFT.

And if you think of this sequence as coming from an underlying continuous time signal, you also know that the underlying continuous time signal has the continuous time Fourier transform. And the question is, what is the relationship? If at all there is any relationship, between the CTFT of the underlying continuous time signal and the DTFT of the sequence.

So, this connection between the DTFT and the CTFT, you would not have seen in the earlier course. You would have just stopped with the expression for the sampled signal. So, we will close the loop in a manner of speaking showing the connection between these two transforms. So, this forms the bridge between the continuous time and the discrete time domains.

So, now, let us look at this continuous time signal. So, you have t here and you have $x_c(t)$. So, this is the underlined continuous time function or signal. And now you want to relate this to samples of this.

So, if you want to relate this to the samples, you will do something like this; you will sample it at even intervals. So, this is what is called uniform sampling.

Just to mention this point in this context, you can also sample the signal in a non uniform manner so, that is more advanced. So, we will only deal with uniform sampling. And the other implicit assumption we are making here is, we are assuming that the signal is, what was the assumption that we had made about the underlying continuous time signal?

Student: Band limited.

It was band limited. Not only was it band limited, what was the other assumption? Band limited of course, yes then?

Student: (Refer Time: 03:28).

Say it again.

Student: Finite (Refer Time: 03:31).

Finite, no that is all fine, right. So, you take uniform samples T seconds apart. So, what was the other assumption then? Or perhaps you did not even realize there was another assumption. You assume that the signal is low pass and what you have really learnt is, low pass sampling theorem. There is band pass sampling theorem also which normally you do not get to see in the first course.

So, those are the assumptions. So, this is the underlying continuous time signal and the stem plot shows the samples taken T apart. And typically, you will show this as the sequence $x[n]$ verses n and then you will plot them like this. Yes, question.

Student: Now what is the (Refer Time: 04:51) signal we go this?

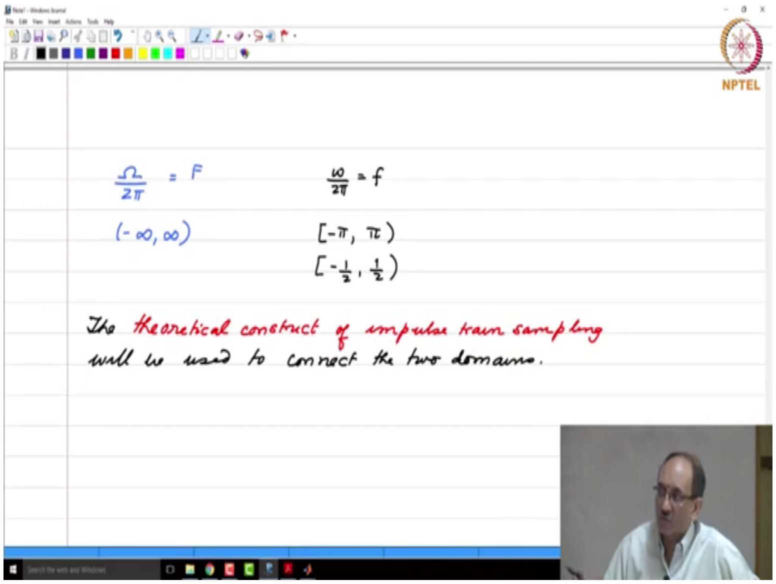
So, basically, if you consider its Fourier transform, the energy is, there is no component of the signal beyond a certain highest frequency. If your highest frequency is say Ω_c, all of the signal spectrum is between $-\Omega_c$ and $+\Omega_c$ so, this is low pass. On the other hand, if you had a signal spectrum between ω_1 and ω_2, then that is band pass.

So, now, what we are trying to see is we have the sequence $x[n]$ and that $x[n]$wn by this temp plot, the independent variable is n. And n can take on only integer values that we have seen. So, the sample at T maps to the index 1, sample at $2T$ maps to the index 2 and so on. So, the question is, is there a connection between $X_c(\Omega)$, which is the CTFT of the continuous time signal and $X(e^{j\omega})$, which is the DTFT of the sequence. Remember, $x[n]$ is the sequence of numbers whose values happen to be the values of the function sampled at T apart.

Another point to note here is that, when you map from the continuous time T to the index n, the value of T can vary. T can be small or large, it can be microsecond millisecond or it can even be in seconds which means, this point will move to the left or to the right depending upon this width because this is the underlying continuous time function.

Whereas, this sequence of numbers 1 will stay at 1, 2 will stay at 2 and so on. So, this sequence of numbers when you plotted versus n does not capture the sampling period. Whether T is in milliseconds or microseconds, $x[0]$ will be at the index 0, the sample at time instants T will map to the index 1, no matter what T is. And because we are trying to connect both discrete time as well as continuous time, we need notation for frequency in both the domains.

(Refer Slide Time: 07:40)



Therefore, we will have Ω and F as the frequencies in the continuous time case and ω and f as the frequencies in the discrete time case. Ω of course, lies in the interval $-\infty$ to $+\infty$. Therefore, F also lies in the interval between $-\infty$ to $+\infty$.

Ω is in radians per second, F is in Hertz. Whereas for the discrete time case, ω lies in the interval $-\pi$ to π. In turn, f lies in the interval $-1/2$ to $1/2$. Now to connect these two, as you have seen in signals and systems, we will use the theoretical construct of impulse train sampling. So, the theoretical construct of impulse train sampling will be used.

So, this will be used to connect these two domains so, connect the two domains. And this of course, is the theoretical construct because in practice, you do not do impulse train sampling. What do you do in practice then? Surely signals are sampled. So, even in your cell phone when you speak, there is an analog to digital converter that converts your continuous time pressure signal to samples. So, ADC is there. So, what does it do?

Student: It holds the (Refer Time: 10:03).

Yeah. So, it is sample and hold and then it samples the signal and then holds it and then circuitry converts this to sequence of numbers depending upon the number of bits that you use. And in this context, it is good to remind ourselves that, higher the sampling frequency needed, the more expensive does the circuitry become because if the signal has a very large bandwidth, if you want to satisfy the sampling theorem constraints, you have to sample it an extremely?

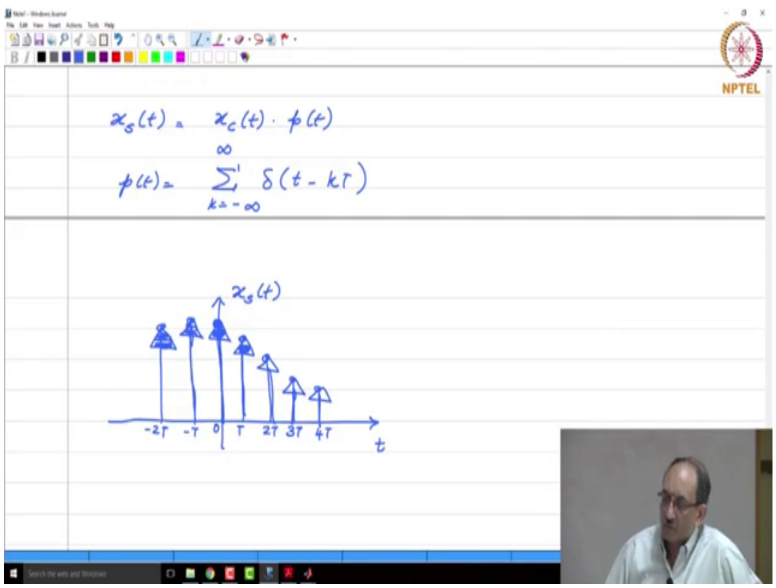
Student: High frequency.

High frequency; so, which means hardware required will be very more complex. The other thing is if you have very high frequency, it also means that, the signal will vary more rapidly, right. That is what means, when you have very high frequency content in the signal. And the assumption that you are making when you are sampling and holding, it means that while you are sampling, the signal is reasonably constant.

And if the signal varies too rapidly, even this assumption will not be true. So, this all physical systems have to latch on to a certain value and hope that value stays as it is before you can assume it has reached a stable value, lock onto it and then convert it to number of bits. The second thing is, when you convert this to number of bits, remember the next sample is going to come at you in no time because you are at a very high sampling rate.

So, the hardware has to respond has to convert this into bits and then be ready for the next sample to be converted. So, all of this puts lot of complexity and cost on the ADC. So, when ADC can be done reasonably well and not without too much cost, you would always prefer the sampled signal. Otherwise, there are still instances where you have no choice but to deal with the analog signal. That is because the implied adc requirements are too costly. Now let us get back to this. So, we are going to do impulse train sampling and what we have in mind is a picture like this.

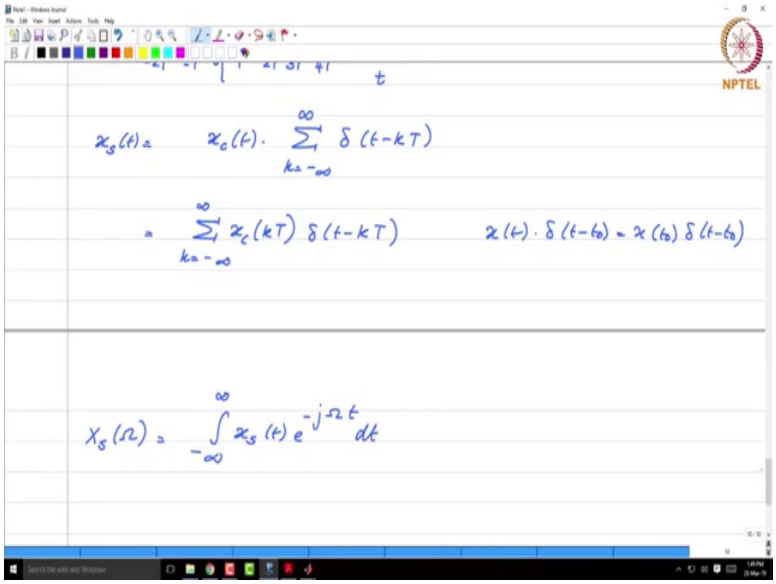
(Refer Slide Time: 12:42)



So, here we have the underlying continuous time signal. And now we are going to sample it using impulse train, which means, I now have this picture with me each of them is not just a number, but each of them is now an impulse. And the strength of the impulse is proportional to the value of the sample.

So, as we have seen earlier, the height of the impulse corresponds to the strength and this is now this and this is the impulse train sample signal and this is denoted as $x_s(t)$. Therefore, we have $x_s(t)$ which is obtained from $x_c(t)$ which is the underlying continuous time signal multiplied by $p(t)$; $p(t)$ happens to be an impulse train. And hence, $p(t)$ is nothing but $\delta[n - \ ]$, rather $n - kT$, where k goes from $-\infty$ to $+\infty$. So, we have a train of, sorry, this is $\delta(t - kT)$, this after all is a continuous time impulse train. So, $\sum_{k=-\infty}^{\infty} \delta(t - kT)$ is the impulse train.

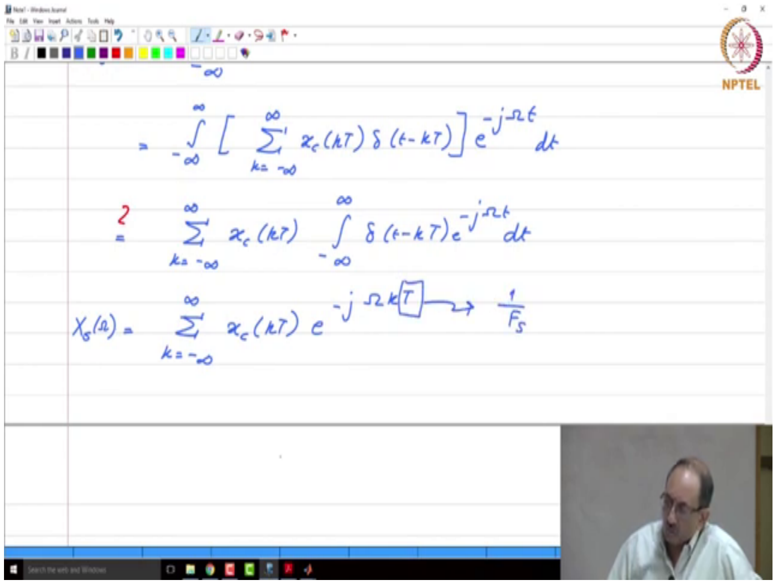
(Refer Slide Time: 14:36)



Therefore, $x_s(t) = x_c(t)\sum_{k=-\infty}^{\infty}\delta(t-kT)$. So, T is now your sampling interval. This is nothing but $\sum_{k=-\infty}^{\infty} x_c(t)\delta(t-kT)$, $x_c(t)$ can be taken inside. So, this is $x_s(t) = \sum_{k=-\infty}^{\infty} x_c(t)\delta(t-kT)$ and we will use the sifting property.

So, this now becomes $x_c(kT)$, because $x(t)\delta(t-t_0)$ is nothing but $x(t_0)\delta(t-t_0)$. So, from the sifting property, you get this. Now you have the impulse train sampled signal $x_s(t) = \sum_{k=-\infty}^{\infty} x_c(kt)\delta(t-kT)$. Now, it makes sense to talk about the continuous time Fourier transform of this continuous time signal; $x_s(t)$ still continuous time. Therefore, you can talk of its Fourier transform which is nothing, but $X_s(\Omega) = \int_{-\infty}^{\infty} x_s(t)e^{-j\Omega t}dt$.

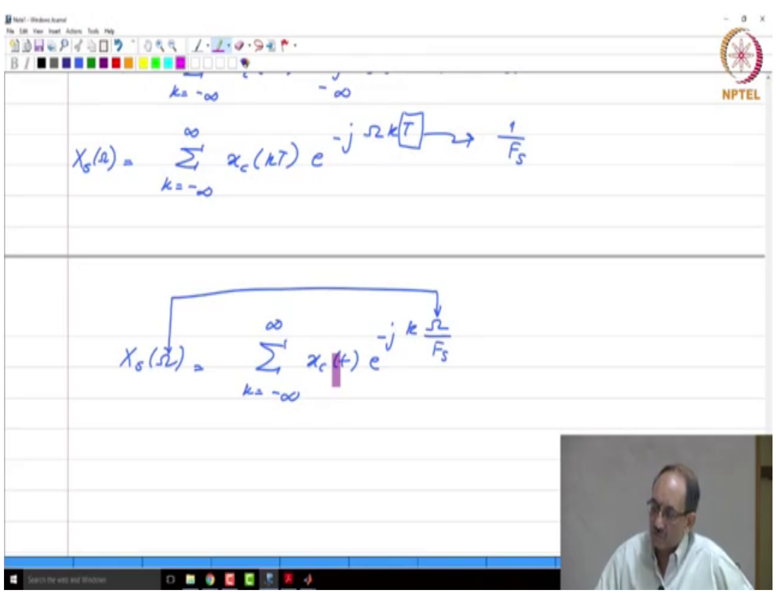
(Refer Slide Time: 16:01)



And we know what $x_s(t)$ is. So, this is nothing but $\int_{-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty} x_c(kT)\delta(t-kT)\right]e^{-j\Omega t}dt$ and then we will do what we always do, interchange two limiting operations. So, this now becomes $\sum_{k=-\infty}^{\infty} x_c(kT)\int_{-\infty}^{\infty}\delta(t-kT)e^{-j\Omega t}dt$.

So, again this is all review from what you have seen earlier. So, this just a recap, nothing new is here. So, this becomes this $\sum_{k=-\infty}^{\infty} x_c(kT)e^{-j\Omega kT}$ and $T = 1/F_s$.

(Refer Slide Time: 17:32)



And, $X_s(\Omega) = \sum_{k=-\infty}^{\infty} x_c(kT)e^{-jk\frac{\Omega}{F_s}}$ and this Ω is exactly this Ω. So, what we will further do is that, remember, our goal is to relate $X_c(\Omega)$ which is the continuous time Fourier transform of the underlying continuous time signal. Right now, we do not have $X_c(\Omega)$ in the picture at all, we have $X_s(\Omega)$ here.

But, we do not have quite $X_c(\Omega)$ for us to relate the original's unsampled signal spectrum with whatever you are trying to relate to. So, we have to get another expression of $X_s(\Omega)$ that relates that with the underlying continuous time signal. Then, we also need to relate this to the DTFT, that is the final step. We will look at another expression for $X_s(\Omega)$, which involves $X_c(\Omega)$ and then make some points. After making some observations, we will further tie the DTFT and the CTFT.