Digital Signal Processing

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Lecture 65:
Linear Phase (3)
-Relationship between the zeros of a linear phase FIR filter

There is another aspect to FIR filters in terms of relationship to zeros.

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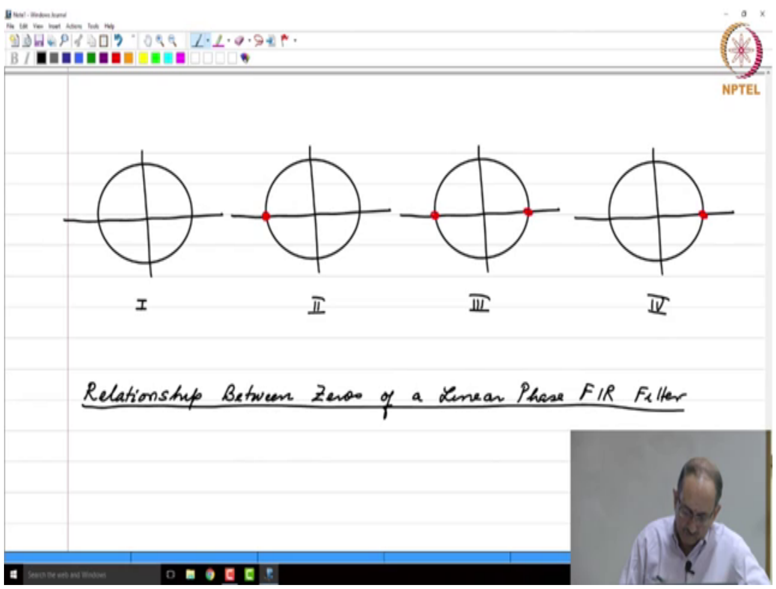

So, Relationship between zeros of a linear phase FIR filter.

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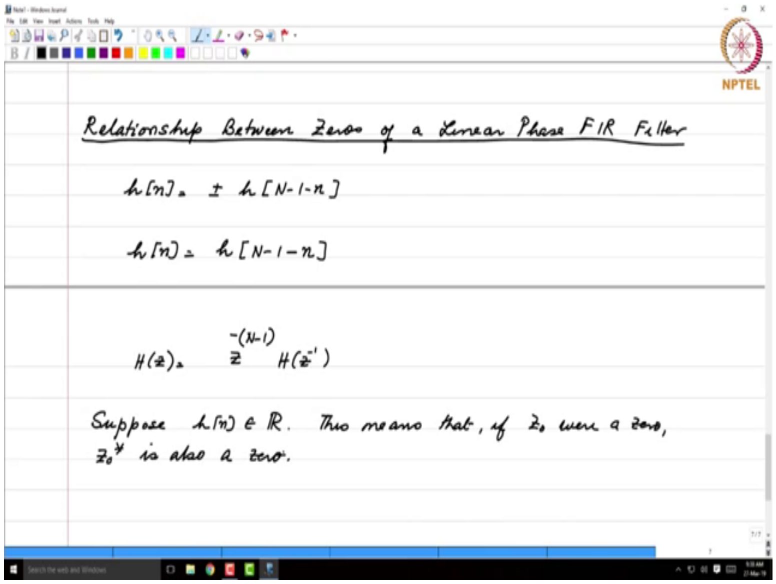

So, recall that $h[n]$ has to be either $\pm h[N-1-n]$, and this was actually one of the earlier tutorial problems where you were asked to relate the Z-transform of these two sequences. Just to quickly recall, so if $h[n] = h[N-1+n]$, then $H(z)$ is, in this particular case, all you are doing is here you have shifted it by $N-1$ samples to the left. If you consider $h[n]$, then $h[N-1+n]$ is nothing but $z^{-(N-1)}H(z)$.

Now instead of $+n$. if you now take this expression and then replace $+n$ by $-n$. wherever z is there you have to replace z by z^{-1}. Therefore, if this were now the case, all you need to do is replace z by z^{-1} therefore, this is now the relationship. Therefore, if this were the symmetry in the time domain, in the transform domain this is how the transforms are related. In addition, suppose $h[n]$ where real value, so this means that if z_0 were a zero, then z_0^* also is a zero, because zeros are to occur in complex conjugate pairs because the sequence is real.

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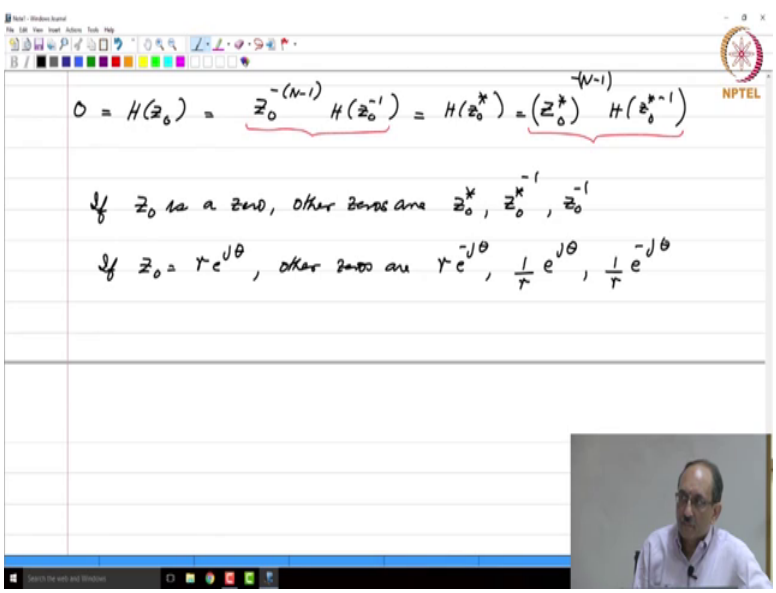

Therefore, $H(z_0)$ will be $z_0^{-(N-1)}H(z_0^{-1})$. And because z_0 being a zero means z_0^* also is a zero. So, if z_0 were a zero, $H(z_0) = 0$. And $H(z_0) = H(z_0^*)$ also because of symmetry, because the sequence is real. If z_0 were a zero, z_0^* also is a zero. If z_0^* is a zero, then this must also be $(z_0^*)^{-(N-1)}H((z_0^*)^{-1})$.

All I have done is, I am using this equation and I am also using the fact that if z_0 were a zero, z_0^* also is a zero. So, immediately you can conclude so, $H(z_0) = 0$, this is what we started off with. But then $H(z_0^*) = 0$ because of real valued.

From these two equations, remember, each of these terms is 0. Since, this cannot be 0, this has to be 0 therefore, $H((z_0^*)^{-1}) = 0$, $H((z_0^*)^{-1})$ also is 0. And the implication is, if z_0 is a zero, other zeros are z_0^8; this follows from real valuedness and then $(z_0^*)^{-1}$ and $(z_0)^{-1}$. These two follow from the symmetry equations. And hence, if z_0 had the form $re^{j\theta}$, then other zeros are the complex conjugate $re^{-j\theta}$ and then $(1/r)e^{j\theta}$ and $(1/r)e^{-j\theta}$.

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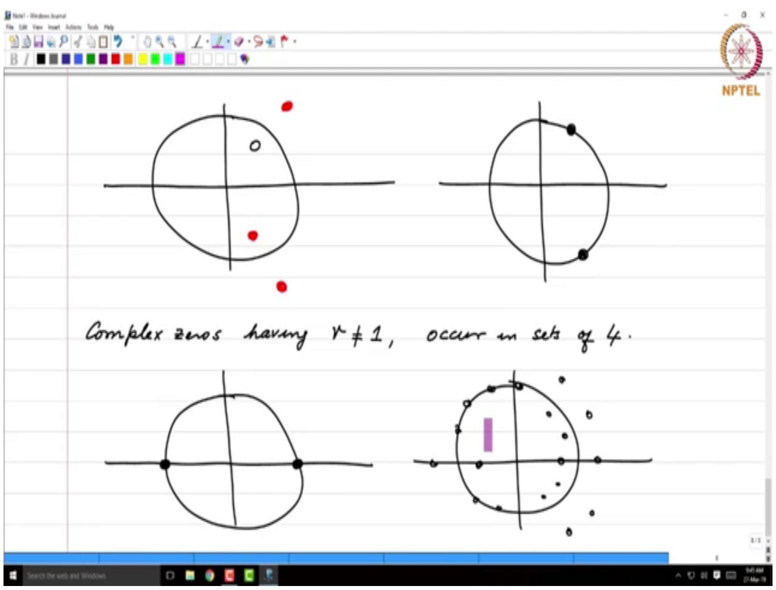

Hence, if you had a zero here, then you are guaranteed to have a zero here. You are guaranteed to have a 0 here because of real valuedness, this zero being here because of symmetry has to have a zero here. And these zeros are in their reflected positions $re^{j\theta}$, $(1/r)e^{j\theta}$, $re^{-j\theta}$, $(1/r)e^{-j\theta}$. Therefore, complex zeros having $r \neq 1$ occur in sets of 4.

On the other hand, suppose you have a zero on the unit circle, then its corresponding complex conjugate is this. Notice that in this case, the reflected position, zero is the equation that has to be satisfied the four equations that I have to be satisfied are satisfied by these 2 roots themselves. You do not need 4 independent or distinct roots, these 2 roots satisfy all 4 equations. Suppose you have a zero here, this is its own complex conjugate, this is it is own reflected root.

Therefore, you have a root at $z = 1$, $z = 1$ satisfies all 4 equations. The other location where that single root will satisfy all 4 equations will be $z = -1$. So, if a root is here, again it is its own complex conjugate, it is its own reflected root. Now, if you go back and look at my notes, I had given the is pole-zero plot for the linear phase FIR filter which I had designed using the $firpm$ command 32^{nd} order filter and there you would have seen roots like this and then you would have seen roots and there was even I think in this particular case, they were roots like this.

Therefore, of unit circle zeros that are complex occur in sets of 4, zeros on the unit circle occur in complex conjugate pair satisfying their own reflected roots condition. And real valued zeros, you just need to, for it is reflected position. In this particular case, there are no zeros at $z = 1$ or $z = -1$. So, go back and look at the exact pole zero plot for the 32^{nd} order FIR filter case, they will follow these conditions that we had just listed.

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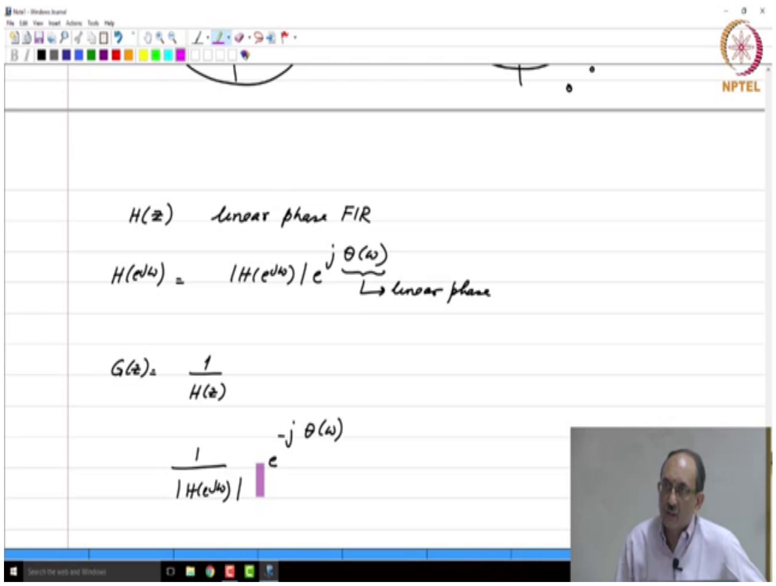

Now, it is good to take stock of things we have seen linear phase FIR filters. That is what the main topic of this has been in the last few lectures. Then, you also mentioned IIR filters. Here, we pointed out symmetries sufficient, but not necessary for the IIR filter case.

So, the question is, are there linear phase IIR filters? Suppose, you have $H(z)$ which is linear phase FIR and this frequency response can be written as $|H(e^{j\omega})|e^{j\theta(\omega)}$ and this of course is linear phase.

And suppose you consider $G(z) = 1/H(z)$, what can you say about $G(z)$? What are the things that you can say about $G(z)$? First, is $G(z)$ IIR or FIR? It is IIR because zeros become poles and poles become zeros. Therefore, if you had an FIR filters something like this and then, this will be. So, this is clearly it has uncancelled non-trivial poles, so this will be IIR. Is this linear phase or non-linear phase? Because this will be $(1/|H(e^{j\omega})|)e^{-j\theta(\omega)}$; if $\theta(\omega)$ were linear phase, this also has to be linear phase. So, this is indeed exactly linear phase.

And now we have seen examples and make the general remark that given the choice between IIR and FIR, you would prefer IIR because it meets the same magnitude response with a much lower order. And hence, given these two choices, you want to pick the IIR filter. You also want linear phase and here is an IIR filter with exact linear phase.

So, clearly this is something that is what you want. So, IIR filter had linear phase and IIR filters in general have lower order than FIR filters, then why did we even spend time learning about FIR filters linear phase, all those things? This IIR filter can be implemented without any trouble, this is rational transfer function, correct. Therefore, it can be implemented using a linear constant coefficient difference equation. Why do not you think about that.