Digital Signal Processing
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Lecture 60:
Zero- Phase Filtering, Linear Phase (1)
-Zero-Phase filtering
-Waveshape distortion caused by allpass filtering

Let us continue with phase response. So, we saw what the definition of group delay was.

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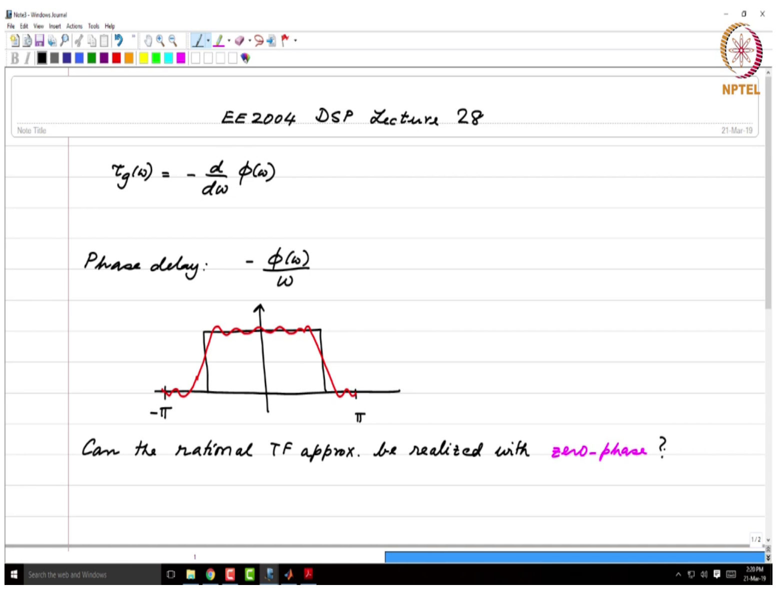

And we introduced $\tau_g(\omega)$ as being the negative derivative of the phase and we saw what the physical implication of this is. There is a related concept called phase delay. And, this is given by $-\phi(\omega)/\omega$ and one of the tutorial problems explores these two concepts in more detail. Suppose, you have a narrowband signal which means it has a certain envelope and inside the envelope, there is a high frequency oscillation.

And if you pass this through a general system, you will be asked to show that the envelope gets delayed by τ_g and the carrier inside will get delayed by the phase delay τ_{ph}. So, the envelope gets delayed by the group delay and the carrier or the phase that gets delayed by the phase delay. So, under certain assumptions, you can show that this is what falls out, if you assume a general system like this.

We will not talk about phase delay other than mentioning the definition whereas, group delay we will talk more about group delay as we go along. So, what we have seen so far is phase response in general is non-linear and the corresponding non-linear phase response gives rise to whatever group delay that such a phase response gives raise to.

And, we saw what the physical implication of group delay is, it delays components. Now, if you recall so, this was the ideal low pass filter and then we made the statement that the Paley Wiener theorem does not allow you to realize ideal filters that are causal. So, we saw that the next best thing is to approximate them using rational transfer function and such an approximation may look like this. This may be the rational approximation to the ideal filter, that is, the rational transfer function approximation to the ideal filter. And we have seen the implications of phase response.

So, the question that we would like to ask at this point is, can the rational transfer function approximation be realized with zero phase? The reason why we are asking this question is if the phase were zero, then your group delay will be zero. Therefore, whatever delay or distortion that arises out of group delay will not be present therefore, it will be nice to have a frequency transfer function with zero phase. So, that you do not have to worry about phase response affecting your output. Now, let us see whether zero phase transfer function can be realized.

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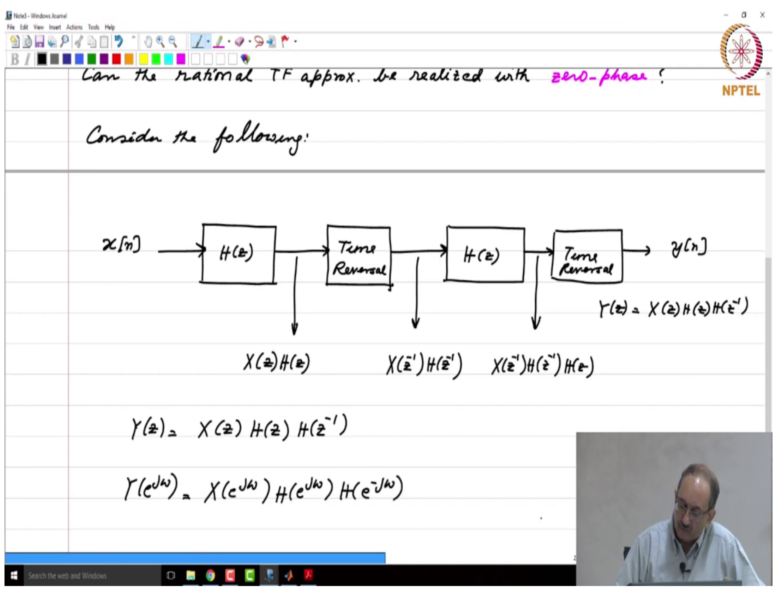

So, now let us consider the following. So, you have an input $x[n]$ and then you pass it through a system $H(z)$, then you do time reversal, then you pass it through $H(z)$ once more. And, then you do time reversal once again and then you get the output $y[n]$. Now, let us see what is the consequence of these series of operations. At this point, the output is $X(z)H(z)$. At this point, if you time reverse, you are going to replace z by z^{-1} therefore, you have $X(z^{-1})H(z^{-1})$.

Again you are filtering by $H(z)$ therefore, this becomes $X(z^{-1})H(z^{-1})H(z)$. Again, you are going through one more time reversal. Therefore, at this point $Y(z)$ is, now you have to take the input Z-transform and then replace z by z^{-1} therefore, this becomes $X(z)H(z)H(z^{-1})$. So, this is what the output's Z-transform is, that is, $Y(z) = X(z)H(z)H(z^{-1})$. So, this is what we get and now let us look at the outputs frequency response. So, this is nothing, but $X(e^{j\omega})H(e^{j\omega})H(e^{-j\omega})$.

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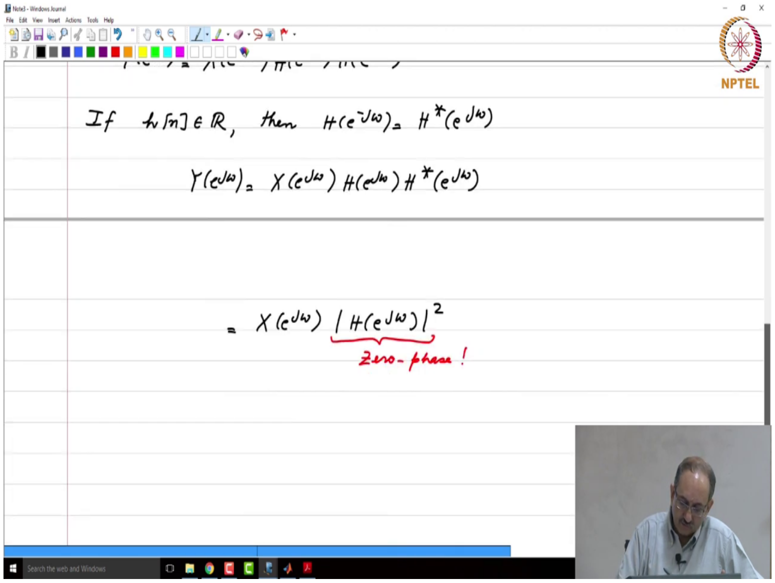

Now, if $h[n]$ were real valued, then $H(e^{-j\omega})$ is the same as $H^*(e^{j\omega})$. Therefore, $Y(e^{j\omega})$ is now $X(e^{j\omega})H(e^{j\omega})H^*(e^{j\omega})$. And, clearly this is $X(e^{j\omega})|H(e^{j\omega})|^2$ and immediately you are able to see that this is zero phase.

Therefore, if you design your filter such that this is the frequency response that you finally want, then you would have achieved zero phase filtering. Clearly, this is too good to be true. So, what is the catch here? So, if this were possible, then zero phase filtering is possible, then we need not worry about phase response at all. You can then take out the parts of the syllabus pertaining to phase response, right.

Student: (Refer Time: 09:02).

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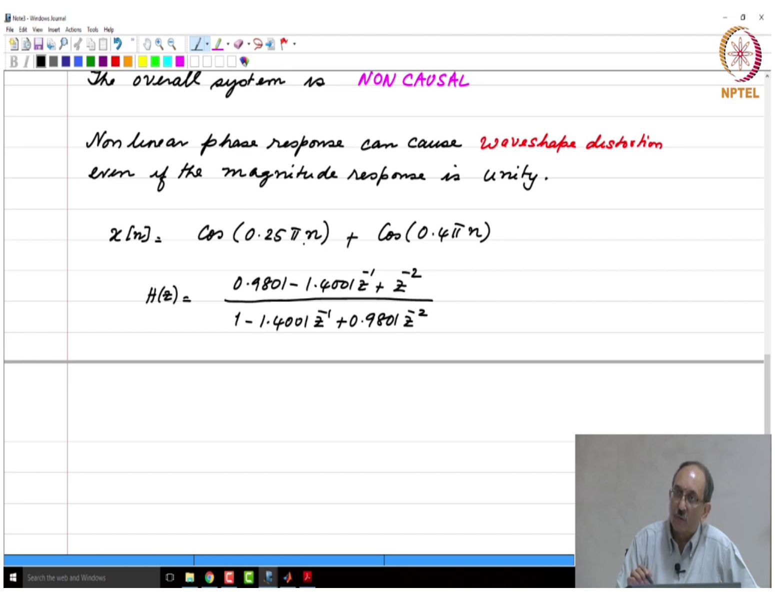

Very good, the overall system because of the time reversal operation is non-causal and therefore, not realizable in practice. Therefore, even though zero phase filtering seems possible, the operations involved require the system to be non causal and hence this is not practical. Therefore, you have to deal with phase response and phase response being non-linear gives rise to this group delay. And, the other important point is that, even though the magnitude response may be constant, if the phase response is non-linear, it can cause waveshape distortion.

So, the non-linear phase response can cause waveshape distortion, even if the magnitude response is unity. So, it is not just the magnitude response being unity is enough for you to not cause distortion. So, the individual components will go through without any amplitude change, because the magnitude response is unity. But, the phase response being non-linear can cause waveshape distortion which is something that you do not want. Now, to see this, suppose, you have $x[n] = \cos(0.25\pi n) + \cos(0.4\pi n)$.

And, let me pass this through a filter which is allpass, allpass clearly has unit magnitude response. Therefore, if I have this and therefore, the corresponding numerator because its allpass is this. Now, if I take this signal and then pass it through this allpass filter, If you look at the input wave shape and the output wave shape, even though the frequency response magnitude is unity, because the phase response is non-linear, this will cause distortion in the output and the output wave shape will not be the same as the input wave shape.

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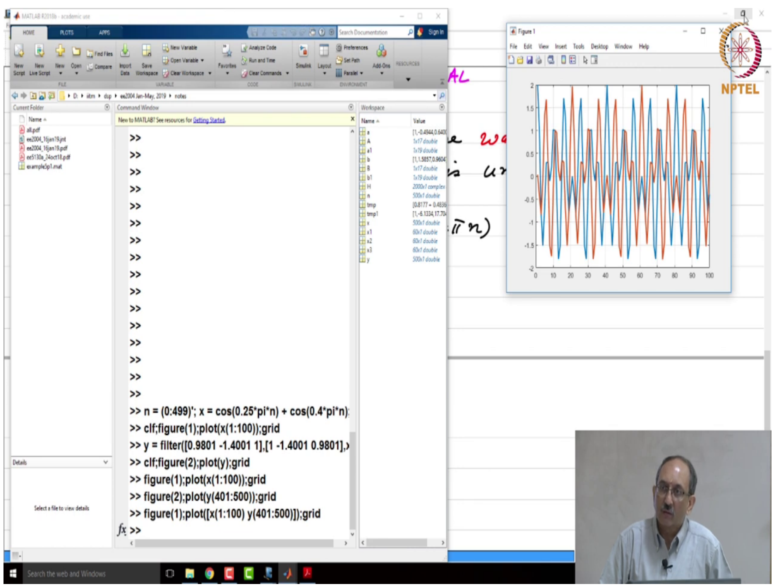

So, let us look at this in MATLAB. So, I have $\cos(0.25\pi n) + \cos(0.4\pi n)$. So, I have created 500 samples of this input. So, let me plot a first 100 samples. So, this is my input, so this is $\cos(0.25\pi n) + \cos(0.4\pi n)$ and now I am going to pass this through an allpass filter. Therefore, I am going to pass it through this filter which is exactly what I had written earlier, numerator is $[0.9801\ -1.4001\ 1]$ and the denominator is the time reversed version of this. So, I am going to filter it and now let me plot the filtered output.

So, this is, let me plot the entire filtered output to point out one thing that, if you are not aware you should know about. So, the point I want you to note is that initially the filter is at rest and you are applying this input suddenly, there will be initial transients and that is why you see here in the beginning; there are transients here. And, if you recall the response to suddenly applied inputs, you will have output mode will contain input mode plus natural modes. You can decompose this as the steady state response plus transient response, the transient response is given by the natural response of this all pass filter.

So, that is what this portion is and then in this portion, you will have the steady state response. Now, let me zoom in here or rather its probably better if I got it separately. So, now let me plot the last 100 samples. So, let me plot from 401 to 500. So, this is the output and if you look at these two, you see that the wave shape is very different and there is no way that you can take the output and you can delay it by integer or fractional amount, it does not matter.

No way can you make the output and the input aligned with each other. The gain is exactly 1 at both these frequencies and yet no amount of delay will cause these two to be, yes.

Student: (Refer Time: 17:00).

No, they do not look similar at all.

Student: (Refer Time: 17:06).

So, what is the, what is in it that you think is happening here?

Student: (Refer Time: 17:19) time delay. So, that is (Refer Time: 17:22).

Let me, since you think there is a time delay. For example, if you notice here, you have something that is here and then its coming down whereas, here it is from here it is going up, right. So, it is really not; so, you can try this, you can try various delay factors. Since, I have given you what the transfer function is and I have also given you what the input is, no way will you be able to take one waveform and shift it and get the other. The shift can be either integer or an integer plus fraction, does not matter. You will not be able to align these two together.

Student: (Refer Time: 18:46) the delay but.

Why do not you try it and then you let me know. Since you have the numbers, try it and then see whether you are able to match it. Remember, if you are able to match, it you should be able to match it precisely; not approximately, all right. So, if you pursue this example more, you will convince yourself that even though the magnitude response is unity, there is still waveshape distortion.

You cannot do away with the waveshape distortion that this non-linear phase response gives you. Therefore, in general, we have seen that zero phase filtering is not possible. So, you have to deal with phase response and magnitude response being unity is not enough, the non-linear phase response can cause waveshape distortion.

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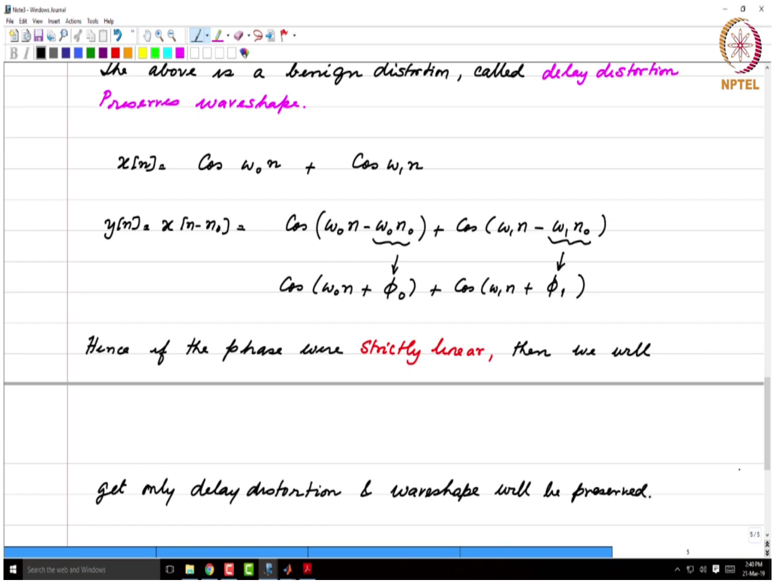

Now, suppose $y[n] = x[n - n_0]$, then this strictly mathematically speaking, the output is not identical to the input, but the output and the input are related by a mere delay. Therefore, the above is a benign distortion called delay distortion. So, if you filter the input by a system, since you cannot have zero phase; zero phase is ruled out because it leads to a non-causal operation. If you had non-zero phase, then you will definitely incur some distortion at the output.

But, if this distortion happens to be delayed distortion, then this distortion is benign. So, the output and the input are identical except for a delay. So, this delayed distortion preserves wave shape therefore, it is really not a terrible distortion. Therefore, we ask the question, what kind of phase response will give rise to this kind of delayed distortion? And, again let us take the simple two component cosine to give a feel for what is going on. Suppose you have $x[n]$ and you had $\cos(\omega_0 n) + \cos(\omega_1 n)$ and if $y[n] = x[n - n_0]$.

So, this is $\cos(\omega_0 n - \omega_0 n_0) + \cos(\omega_1 n - \omega_1 n_0)$. And this is nothing, but $\cos(\omega_0 n + \phi_0) + \cos(\omega_1 n + \phi_1)$. And, you are able to see that this phase is proportional to frequency, right, this phase is clearly proportional to frequency. Hence, from this simple example as an illustration of the general principle, we state that hence if the phase were strictly linear then we will get only delay distortion and waveshape will be preserved.

And, this can also be inferred from the definition of group delay. If the phase response is piecewise linear, then the group delay will be constant. And, hence if the input has components that falls in the pass band of the filter were then gain is nominally 1, if the phase were linear, then the output will be just a delayed version of the input, because the phase is linear; the past band being nominally 1 means the gain is 1. Therefore, the gains are not altered for the individual components and the output is merely a delayed version of the input, for those components falling in the past band.

Student: Sir.

Yes.

Student: Actually phases are delayed by (Refer Time: 24:51).

No, because all we are doing is, we are replacing n by $n - n_0$. So, you replace $n - n_0$ in both of these terms. So, you get $\omega_0 n$ and $\omega_1 n$. But the delay when you replace n by $n - n_0$, the phase component is $\omega_0 n_0$ and $\omega_1 n_0$ in for these two cases. So, this means that this phase is proportional to frequency therefore, you can think of the phase response as being linear.

So, you were saying right; so, this is indeed correct. So, all this tells you is if the phase response were linear, then all the components get shifted by the same amount. Therefore, the higher frequency signal when it gets shifted by a by the same number of samples, the corresponding phase change will be proportionally be more. Therefore, if the phase response were linear, then you can expect the input to be merely delayed which also is consistent with the fact that a linear phase response gives rise to a constant group delay.

So, this is the motivation for you to think about systems having linear phase because, they have the advantage that they will only introduce delay distortion. Zero phase is not possible, not realizable. If have are non-linear phase, you will have waveshape distortion. If the phase were linear, you can have only delayed distortion which is benign. So, this is the motivation for looking at linear phase.