Digital Signal Processing

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Lecture 55:
Magnitude Response (4), Phase Response (1) - Comb filter

Okay let us get started, we were looking at Magnitude Response and then we saw first and second order systems; for the second order system we saw both the resonator and the improved resonator and the inverse of the resonator turns out to be the notch filter with the zero being on the unit circle and we also saw the improved notch filter where by introducing poles we were able to make the gain come back to 1, as soon as you cross the zero.

And we also saw the moving average filter and that response was $\frac{1}{N}e^{-j\omega(N-1)/2}\,\frac{\sin(N\omega/2)}{\sin(\omega/2)}$ and then based on the geometric interpretation of the frequency response; we were able to see that whatever magnitude response that the moving average filter had, the same response we could get via the geometric interpretation.

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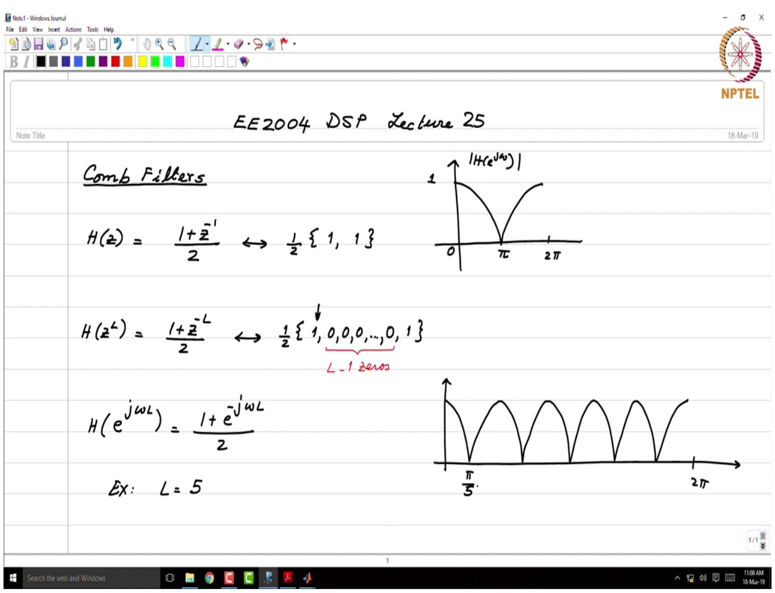

Now, let us look at the next class of systems namely, Comb Filters. This will be the last set of filters we will be considering as far as magnitude response goes; we will then move on to phase response. Let us consider $1 + z^{-1}$. So, this is a crude low pass filter because there is a zero at $z = -1$. What we will do is; we will divide by 2, so that the peak gain is 1. The peak gain occurs at $\omega = 0$, which is the same as $z = 1$; if you put $z = 1$, if you multiply by $1/2$, the gain at $\omega = 0$ becomes 1.

So, there is only reason for introducing this factor of half; otherwise this is a crude low pass filter, this is zero at $z = -1$ and if you look at the frequency response, it looks like this where I have plotted the response from 0 to 2π. This zero at $z = -1$ causes the frequency response to go to 0 at $\omega = \pi$ and this peak gain is 1 and this is the magnitude response; note that the corresponding impulse response is $\left\{\frac{1}{2}, \frac{1}{2}\right\}$.

It is pretty straightforward, just from the Z-transform expression you are able to see the inverse Z-transform has this time domain expression. Let us consider this $H(z^L)$ where I have replaced z by z^L, so this is $\dfrac{1 + z^{-L}}{2}$ and what happens in the time domain is; if you replace z by z^L, in the time domain just by looking at this, you are able to see that for $n = 0$ this is indeed 1 but then the next power of z occurs at this particular value; therefore, all these samples are 0 upto that point, until you hit the index cap L.

So, this is corresponding to $n = 0$ and this corresponds to $n = L$ and it is easy to see that there are $L - 1$ zeros. Therefore, if you replace z by z^L in the transform domain, you will introduce $L - 1$ zeros in the time domain. So, this is easy to see. The other implication you can draw from this that is when you replace z by z^L; if you consider the response in the frequency domain, this actually becomes $e^{j\omega L}$, because all you need to do is, to get the frequency response you need to replace z by $e^{j\omega}$; therefore, this becomes $e^{j\omega L}$, and this therefore, is $\dfrac{1 + e^{-j\omega L}}{2}$.

To get a feel for the picture associated with this, if you replace ω by $L\omega$, then all you are doing in the frequency domain is compression by a factor of L, because if you replace in continuous-time case, if you replace t by $2t$, the behavior of $x(2t)$ compared to $x(t)$ is compression by a factor of 2. Same thing is happening in the frequency domain here, you replaced ω by $L\omega$; which means you have compressed by a factor of L.

And hence, you can expect when compared to the previous case, when this was the frequency response between 0 to 2π. Now that you have compressed by a factor of L, remember this is still the DTFT of a certain sequence; which means, this is has to be 2π periodic, therefore you can expect this to repeat L times between 0 to 2π, that is all. and as an example, if you take $L = 5$, then we can expect 5 copies of this.

So, this is 2π and this has to be $\pi/5$, because you have compressed by a factor of 5, the previous point which was at π now will occur at $\pi/5$.

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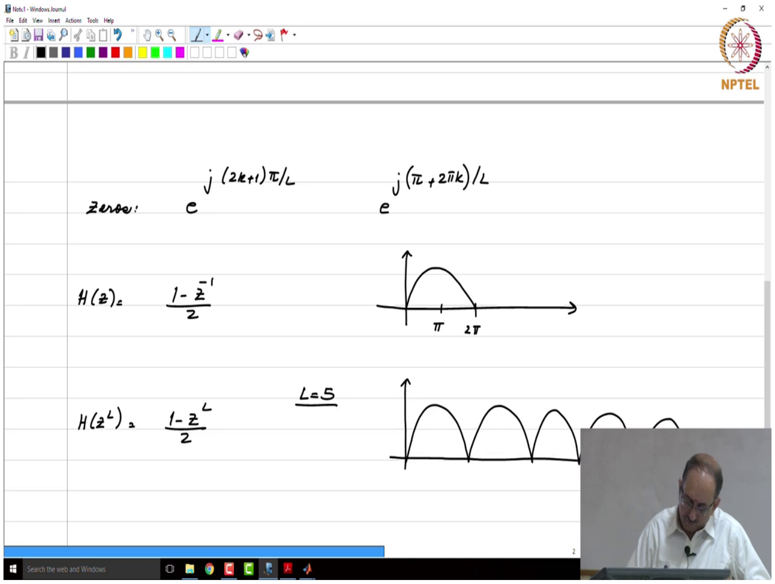

And if you look at the zeros of this, these are after all the L^{th} roots of -1, because you need to compute the zeros of the new transfer function where you have replaced z by z^L. So, this is the L^{th} roots of -1, and hence we can easily see that the roots are $(2k+1)\pi/L$.

And the way you get this is, $-1 = e^{j\pi}$. So, you have $e^{j\pi}$, and then if you add $2\pi k$, nothing changes and then to compute the L^{th} root, you have to divide by L, and so this is exactly the expression that I just wrote. So, these are the L^{th} roots of -1 and this is called as a comb filter, because these look like the teeth of a comb and these comb filters are used in music synthesis.

Another variant of this is instead of a crude high pass filter; if you start off with $H(z) = \dfrac{1-z^{-1}}{2}$; again the factor of $1/2$ is just to normalize the gain, then you can see that the original filter. So, this is a crude high pass filter; therefore, there is zero at $\omega = 0$ and the peak occurs at $\omega = \pi$. Therefore, the frequency response from 0 to 2π looks like this and in this case, if you replace z by z^L, you have $\dfrac{1-z^{-L}}{2}$. Again for $L = 5$, what was happening between 0 to 2π now L copies of that will have to happen between 0 to 2π. And if again if you take $L = 5$, you will have something like this, so this would be 2π.

And one use of this is to eliminate harmonic frequencies and hence if you had a signal that had tones at multiples of this, you can get rid of those tones by passing through this comb filter. Another variant of the comb filter is, you start off with this.

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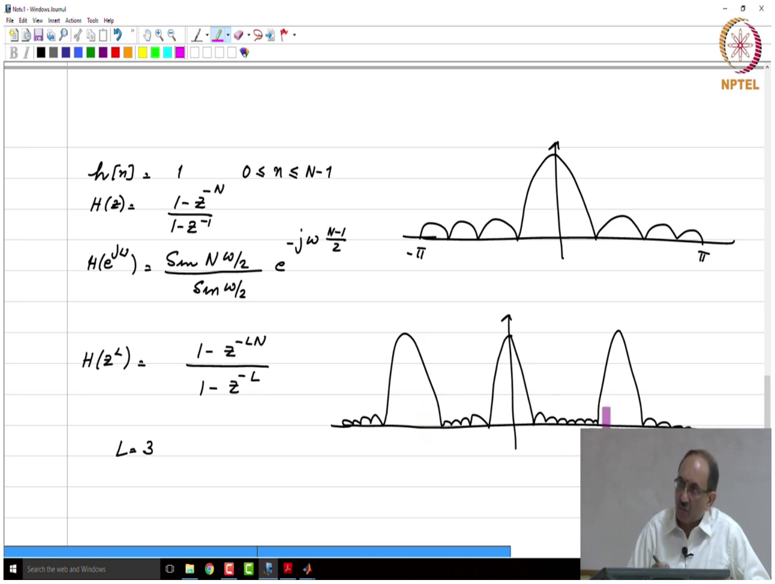

You start off with $\dfrac{\sin(N\omega/2)}{\sin(\omega/2)}$. So, this if you had $h[n] = 1$, $0 \le n \le N-1$; then $H(e^{j\omega}) = \dfrac{\sin(N\omega/2)}{\sin(\omega/2)} e^{-j\omega(N-1)/2}$ is this, because you will have $e^{-j\omega(N-1)/2}$. So, this is the frequency response. This is nothing but a scaled version of the moving average filter that we are just considered before we started looking at the comb filter and if you plot at the magnitude response, it will be something like this; where I have shown a rough plot assuming a certain value of N.

So, this is this, and $H(z)$ of course is $\dfrac{1-z^{-N}}{1-z^{-1}}$. And $H(z^L)$ is $\dfrac{1-z^{-LN}}{1-z^{-L}}$, all right. And now what will happen is, what is happening for the original filter between $-\pi$ and π; if you say take $L = 3$, then three copies of this will have to occur between $-\pi$ and π and here is a rough sketch.

So, this is z replaced by z^L, a rough sketch for $L = 3$. So, if you start off with this kind of frequency response, if you replace z by z^L, you will get something like this. Again this also comb filter and one use of this comb filter is, because it has peaks at regularly spaced intervals and comparatively much less gain in between, this can be used to pass harmonics. So, the signal had components plus harmonics at these locations, when you pass it through this filter, only the harmonics will roughly go through.