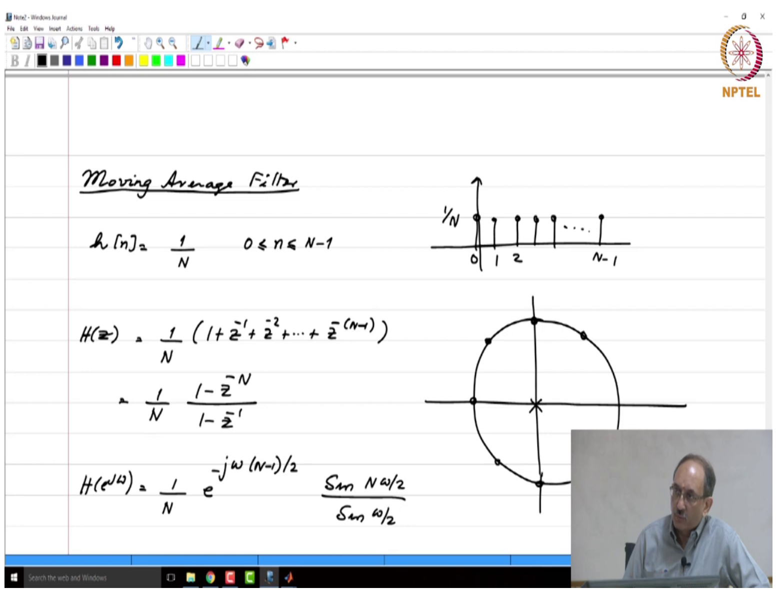
Digital Signal Processing

Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 54:
Magnitude Response (3)
- Moving average filter

(Refer Slide Time: 00:17)



We already seen this moving average filter and this impulse response is $h[n] = \frac{1}{N}$, $0 \leq n \leq N-1$ and have already seen the frequency response, let me this you have seen. So, the transfer function is $H(z) = \frac{1}{N}\left(1 + z^{-1} + z^{-2} + \ldots + z^{-(N-1)}\right)$ which is nothing but $\frac{1}{N}\frac{1-z^{-N}}{1-z^{-1}}$; we have seen this before, just recalling this result.

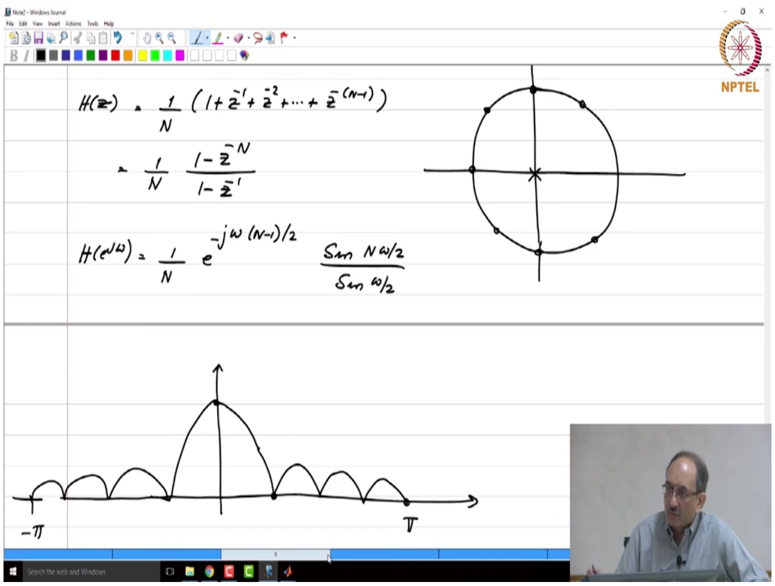
And the frequency response is $\frac{1}{N}(\ )$, you can take $e^{-j\omega N/2}$ outside and if you make the simplification, this will become $(N-1)/2$. The reason why you get $(N-1)/2$ is $e^{j\omega N/2}$ will come from the numerator. Similarly $e^{j\omega/2}$ will come from the denominator; therefore, when you simplify, you will get this as the answer and this will be $\frac{1}{N}e^{-j\omega(N-1)/2}\,\frac{\sin(N\omega/2)}{\sin(\omega/2)}$ and the impulse response is a constant.

So, this is 0, 1, 2 up to $N-1$ and the value of the constant is $\frac{1}{N}$. So, what will happen when you take an input and apply it to this filter, what you are doing is you are taking the input and convolving the input with this impulse response. When you convolve with this impulse response, all you are doing is you take this, flip it and shift it and multiply point by point and summing up all the terms and then dividing by N.

When you take the product and sum, all you are doing is taking N contiguous samples and then averaging them and you will get one number. So, when you apply an input to such a filter, you take N samples and average; that is why it is called as a moving average, because this window slides across the input data. For one fixed position, you will get one number, then you move by one sample, you take the next contiguous N samples, averaged it you will get one number and you keep going.

So, that is why this called as a moving average filter and we have seen this frequency response also before. So, you have for the case of $N = 8$ we have this, and then you have a pole-zero cancellation here; the pole at $z = 1$ cancels zero at $z = 1$; and the pole-zero plot I have plotted is for $N = 8$, therefore, this is how the final pole zero plot looks like.

(Refer Slide Time: 03:53)



And then remember we are looking at magnitude frequency response and we have been introduced to the geometric interpretation of the frequency response and hence if you are on the unit circle here, you will have a certain value.

Therefore, response will have a certain value like this; and then from this point when you move across the unit circle when you approach this zero, the response will fall down and when you hit the first zero as shown here, the response will go to precisely 0 and then similarly when you cross this, you will hit a peak somewhere in between and then hit the next zero when you approach this; and hence the response will look like this. This is how the response will look. So, this is between $-\pi$ and π. In my notes, I will put the precise MATLAB plot corresponding to some value of N, say $N = 8$; I will calculate the exact frequency response and make it as part of my notes.