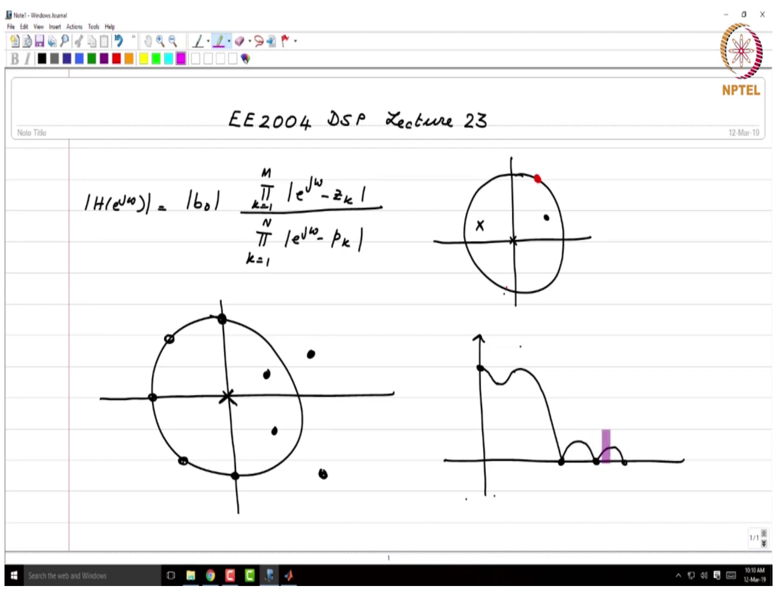
Digital Signal Processing
Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 50:
Magnitude Response (2)
- Magnitude frequency response (cont'd)

Alright, let us a get started. We are looking at Frequency Response.

(Refer Slide Time: 00:51)



And, then we saw the geometric interpretation of the frequency response. So, we saw that the magnitude can be written like this, $|b_0|\frac{\prod_{k=1}^{M}|e^{j\omega} - z_k|}{\prod_{k=1}^{N}|e^{j\omega} - p_k|}$ and immediately the geometric interpretation associated with the difference of two complex numbers gives rise to the picture that we had describe yesterday, that is, you are going along the unit circle.

So, you are, say here on the unit circle and then you have to look at distances to poles and zeros. they may be a trivial pole or a trivial zero here and as an example I have marked this as a trivial pole and we also saw that why this is called as a trivial pole because trivial poles or trivial zeros do not contribute to the magnitude response. And then you find the distances to all the zeros and then the distances to all the poles and the product of these distances to all the zeros to all the poles that gives you the frequency response at that particular point or frequency.

And, then as you go along on the units circle depending upon the position of the poles and zeros, the frequency response will vary and we saw simple example yesterday. And if you go near a pole, the frequency response will be large in magnitude. If you go near a zero, the frequency response will depth. So, in general if there are both poles and zeros; poles tend to provide gain, zeros tend to provide attenuation. And we have a filter typically pass band behavior is given by poles, stop band behavior is given by zeros, because pass band you need gain, stop band you need attenuation.

And, depending upon the order of the filter that is depending up on the number of poles and zeros you have to work with, you can get better and better approximations to the ideal filter. In the case of FIR filters, you only have zeros to work with; there are no non trivial poles for an FIR filter. Therefore, zeros have to provide both pass band gain as well as stop band attenuation. As a simple example so, we will later talk about linear phase FIR filters. In that contexts, you will see plots similar to what I have drawn just now and these are the zeros of an FIR filter and any pole that is there has to necessarily be trivial. So, this is the pole zero plot.

And, now to get a feel for the frequency response, if you are here on the unit circle at omega equal to 0, then you are farthest away from all the zeros. And you go along the unit circle roughly at this location, your closest to these two zeros and hence if you are here, if the magnitude response has this particular value at $\omega = 0$, when you go along the unit circle what will happen is since you are approaching those two these two zeros, what will happen is the magnitude response will dip.

And, then when you go past this pair when you go just past, the magnitude response will again begin to rise and hence the response will be something like this. And again beyond this point, as you keep going further, you are going to approach this zero which means the response will get pulled down and hence you can think of the picture being along these lines. So, the response goes down and when you hit this zero, you will reach this point and hence the response goes to precisely 0. And then when you go past this zero, the response will begin to increase and again when you approach this zero, it will has to it will have to come down and hence we can think of the response having this particular behavior, similar thing happens in this region.

So, the response will look like this. So, here is an FIR filter which is low pass; both pass band and stop band behavior are governed by zeros because there are no poles other than trivial poles to play with. And here if you look at this, zeros of the unit circle govern pass band behavior and as before in the IIR filter case or pole-zero filter case, zeros on the unit circle govern stop band behavior.

Again, in my notes, I will put out a plot that is exact, I will take a FIR filter and then show the exact plot and that will have a response that is similar to this. The way I have drawn this is I have again drawn an equi-ripple response, unless you are told this fact if you did not have numbers to work with, you may not be able to guess the response is equi-ripple.

So, in the case of pole-zero filters, poles contribute to pass band behavior, zeros contribute to stop band behavior and zeros contributing to stop band behavior are zeros that are typically on the unit circle. For the FIR filter case where you do not have poles to play with, pass band and stop band behavior are governed by zeros; pass band behavior is governed by zeros of the unit circle and as before stop band behavior is given by zeros on the unit circle.