Digital Signal Processing
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Lecture 05:
Signal Symmetry, Elementary Signals (1) -The Unit Step Sequence

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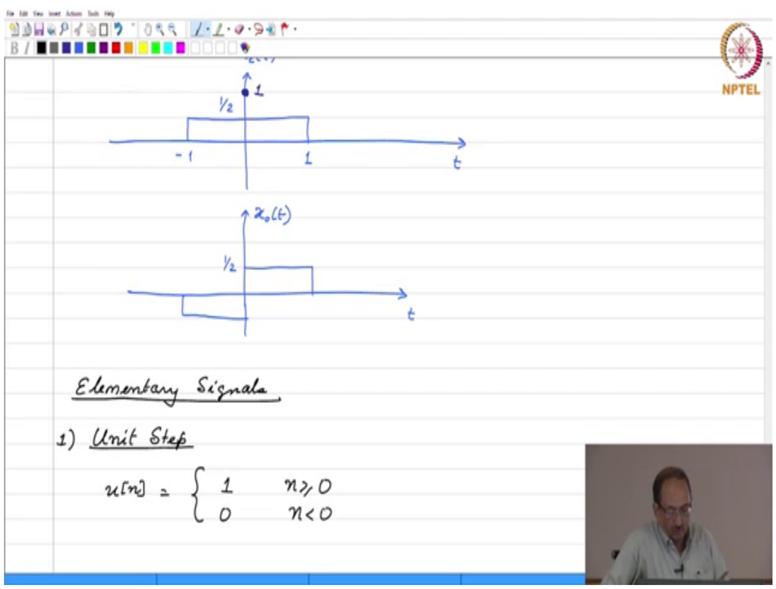

Now, let us move on to Elementary Signals. So, what we will do here is, we will look at again some of these basic signals that you have already seen in signals and systems. So, this will be review if you have already seen them and then we will again point out similarities and differences, just to reinforce these points just in case you have not paid attention to them. So, we will start off with the unit step sequence and then the unit impulse and then the sinusoids. The discrete-time impulse is the most elementary of all signals. It is the simplest discrete-time signal you can think of.

But since what we are going to do now is we are going to compare each elementary signal with its continuous-time counterpart. The discrete-time sequence is the simplest one, but its continuous same counterpart is really complicated. We will come to that after we introduce the unit step sequence. So, the first thing that we want to look at is the unit step and this analogous to the continuous-time case. This is defined as

$$u[n] = \begin{cases} 1 & n \geq 0, \\ 0 & n < 0. \end{cases}$$

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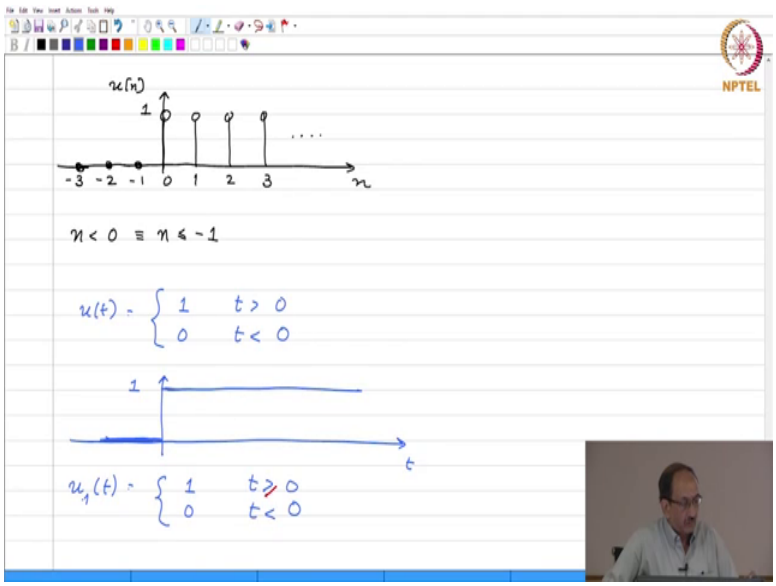

And the picture that is associated with this simple function is this. It is all 1s for $n \geq 0$ and 0 for $n < 0$. And again just to point, make a note of this in case you are not already noticed it; $n < 0$ is the same as $n \leq 1$. So, this is the discrete-time unit step sequence.

The corresponding continuous-time unit step function is

$$u(t) = \begin{cases} 1 & t > 0, \\ 0 & t < 0. \end{cases}$$

and the picture associated with this is; it is 0 for $t < 0$ and 1 for $t > 0$. But you must have perhaps encountered some slightly different definitions. For example,

$$u_1(t) = \begin{cases} 1 & t \geq 0, \\ 0 & t < 0. \end{cases}$$

So, let me call this as $u_1(t)$, all right? So, this is a different definition and once you have seen this variation, you can also guess some of the other variations.

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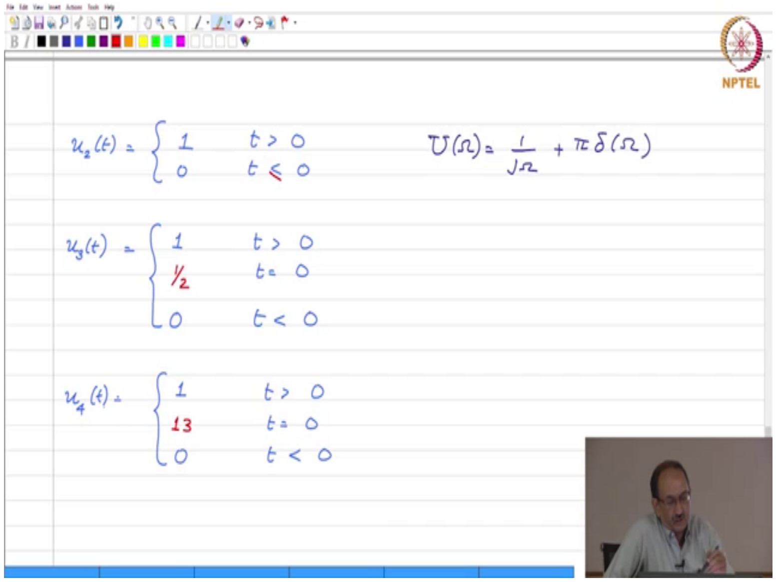

$$u_2(t) = \begin{cases} 1 & t > 0, \\ 0 & t \leq 0. \end{cases}$$

So, one more variant can you suggest?

Student: (Refer Time: 04:45).

Very good;

$$u_3(t) = \begin{cases} 1 & t > 0, \\ 1/2 & t = 0, \\ 0 & t < 0. \end{cases}$$

Then as I always like to say here is my definition of $u(t)$ which you will not find in any book. So, this is

$$u_4(t) = \begin{cases} 1 & t > 0, \\ 13 & t = 0, \\ 0 & t < 0. \end{cases}$$

We will come to this in a minute.

So, let us focus on the CTFT of $u(t)$. So, let me call the CTFT as $U(\Omega)$ and then if you recall your continuous time Fourier transform,

$$U(\Omega) = \frac{1}{j\Omega} + \pi\delta(\Omega).$$

So, now, we have various definitions of $u(t)$ and this is the transform of $u(t)$ which begs the question of which $u(t)$. You say it, this is the transform of $u_1(t)$, why is it that? So, let us defer on whether that is the right answer or not. If you say this is the transform of $u_1(t)$, then what is the transform of $u_2(t)$? Somebody said same, it is very good.

Student: (Refer Time: 07:02) one point would not matter.

One point would not matter, very good; all right. So, all these things have exactly the same Fourier transform, very good. Suppose, I take $U(\Omega)$ which is this and then I take the inverse Fourier transform, right; I should get back to $u(t)$. Which $u(t)$ will I get back?

Student: (Refer Time: 08:12).

$1_1(t)$. Your favorite unit step function, all right. So, let us now;

Student: $u(t)$.

Go ahead.

Student: In which $u(t)$; like (Refer Time: 08:32).

All right, so, 1 for $t > 0$ and 0 for $t < 0$.

Student: (Refer Time: 08:40).

If you look at the inversion integral, it is defined for all t, correct? So, why do you want to leave out $t = 0$ and leave it undefined? All right, so, what is happening here is; in terms of the Fourier transform, the answer that was said saying all these various definitions of $u(t)$ will still give rise to exactly this Fourier transform that is right, because one point is not going to contribute to the integral. If you now take this Fourier transform and then if you do the inverse transform, remember if you recall what is happening in Fourier series, at a point of discontinuity in the signal to what value will the Fourier series converge to.

Student: Average value.

To the average value, right. So, this is no different from what is going to happen with the Fourier transform, all right? So, when you take the inverse Fourier transform of this, because $t = 0$ is a point of discontinuity, what you will get as the inverse Fourier transform of this is $u_3(t)$.

So, this kind of adds some more completion to your understanding of the unit step function in the continuous-time domain even though you have seen this in your earlier course; these are some questions I do not know if they occurred to you to ask. These are the kinds of questions that you should be asking when you look at these things. See these are not complicated questions, they are very simple minded and yet when some of these questions were raised, even though you had seen them before, they were not full or complete understanding.

So, whenever you look at something try to ask these very simple minded questions and make sure you understand things as fully as you can. So, that is the reason why I brought up $u(t)$ even though you had studied unit step function in your earlier course and also its transform.

So, these are some of the similarities and differences that can occur between continuous-time and discrete-time case. As far as $u(t)$ is concerned, at $t = 0$, there are various definitions possible whereas, in the discrete-time case there is no ambiguity in the definition at $n = 0$.