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## Lecture 48: Causality & Stability, Response to Suddenly Applied Inputs, Frequency Response (1)

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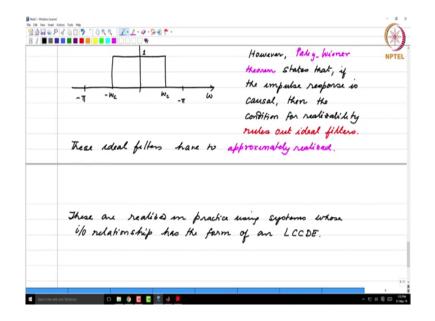
Now, let us move on to the next equally important topic of frequency domain response of LTI systems. So moment you talk about frequency response, remember frequency response is a transform evaluated on the unit circle therefore, we assume that e to the j omega belongs to the Region of Convergence and in practice; we will almost always want causality. So, we will also assume causality and moment you talk about frequency response, you are reminded of frequency domain filtering. And you always start off with ideal filters and you must have encountered ideal high pass, low pass, band pass and band stop filters in signals and systems.

And we have seen this before; so this is an ideal low pass filter; so this is the low pass filters response in the discrete time case. So, this is between  $-\omega_c$  and  $+\omega_c$  and this pass band gain is 1; this is called the pass band and this is the stop band. And for ideal filters, the transition from pass band to stop band happens abruptly; this is the ideal low pass filters frequency response. And similarly, you can have pictures of low pass, band pass and band stop.

Remember, we also want causality in practice because if the independent variable is time for realizability; you need causality; non causal systems are not realizable. You cannot realize non causal systems in

practice if you allow for a certain time delay, but in general non causal systems are non realizable therefore, we want causality. But, so causality is desirable, but there is bad news here; almost expected, ideal things do not exist in practice all right; except there is one exception right, ideal students do exist all right.

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So, ideal teacher never does. Paley Wiener theorem states that; so if the impulse response is causal, then the condition for realizability rules out ideal filters. So, we are not going to get into what the statement of the Paley Wiener theorem is and proof and so on. All we need to know for our purposes is that; if you want the system to have a causal impulse response, there is a certain condition for realizability due to Paley Wiener and ideal filters violate them.

So, ideal filters cannot be realized in practice; therefore, these ideal filters, they are also called as brick wall filters because the shape remains you brick walls. So, these ideal filters have to be approximately realized. That is, we can have filters that are practical, that are reasonably close to the ideal response, that is the passband will not exactly be 1; will roughly be 1.

The stopband will not have 0 gain, but will have a very small gain and the transition from passband to stop band will not happen abruptly but will happen over a certain frequency interval. And smaller the transition bandwidth is, the better the approximation; closer the pass band, the better is the filter closer to ideal approximation and you want stop band rejection to be as much as possible.

And these are realized in practice using systems whose input output relationship has the form of; if you want to guess what would the I/O relationship be in what form; no, what is the general class that this has to belong to? Rational or in the time domain, we are talking about input-output relationship therefore, it has to be of the form of an LCCDE. So, we will look at what this form takes in frequency domain and then we will build up ideas of magnitude response, phase response and so on.