Digital Signal Processing

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Lecture 45:
DTFT of Sequences not in l_1, Response to $\cos(\omega_0 n + \phi)$

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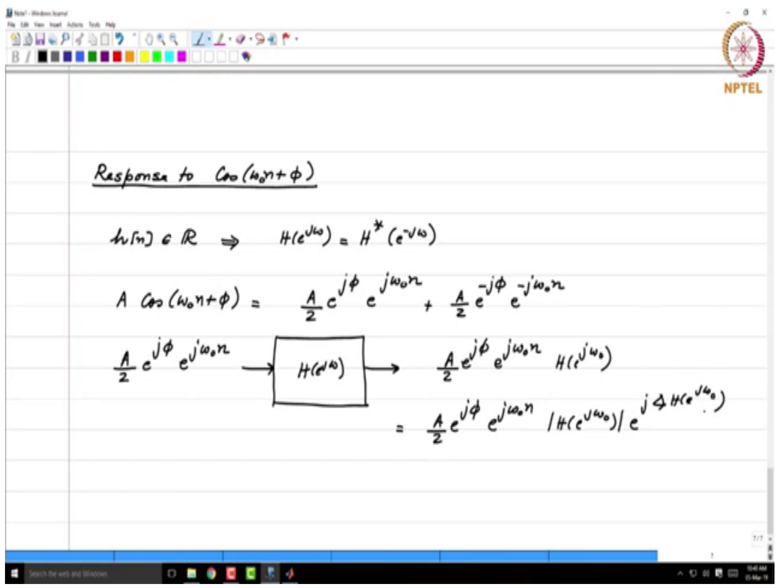

We look at response to $\cos(\omega_0 n + \phi)$. So, what we are going to assume here is we are going to assume $h[n]$ is real value, that is the system that we are interested in has real valued impulse response. So, this implies $H(e^{j\omega})$ is the same as $H^*(e^{-j\omega})$. Therefore, the magnitude response is even and the phase response is odd. Now, let us consider $A\cos(\omega_0 n + \phi)$, so this is nothing but $\dfrac{A}{2}e^{j\phi}e^{j\omega_0 n} + \dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}$.

And the response to $\dfrac{A}{2}e^{j\phi}e^{j\omega_0 n}$, if this were applied to the system, you had this frequency response to be this, output would be $\dfrac{A}{2}e^{j\phi}e^{j\omega_0 n}H(e^{j\omega_0})$. And then you have to evaluate $H(e^{j\omega})$ at $\omega = \omega_0$, this after all is an eigen signal. If you apply an eigen signal to any LTI system, output will be exactly the same thing except will be scale by the frequency transfer function evaluated at that particular frequency.

Therefore, this will be $H(e^{j\omega_0})$. And, this in turn can be written as $\dfrac{A}{2}e^{j\phi}e^{j\omega_0 n}H(e^{j\omega_0})$ and $H(e^{j\omega_0})$ we will write it as $|H(e^{j\omega_0})|e^{j\angle H(e^{j\omega_0})}$. So, this will be $\dfrac{A}{2}e^{j\phi}e^{j\omega_0 n}|H(e^{j\omega_0})|e^{j\angle H(e^{j\omega_0})}$. So, in this context, we had raised the question is z_0^n, the only eigen signal for an LTI system, right.

And z_0 being $e^{j\omega_0}$ is a special case, but for z_0^n is an eigen signal, $e^{j\omega_0 n}$ also an eigen signal. Remember, I had asked whether this is the only eigen signal for an LTI system. The corresponding counterpart question was, Is $e^{s_0 t}$ the only eigen signal for an LTI system? That question is still open, just wanted to remind you of that that is all.

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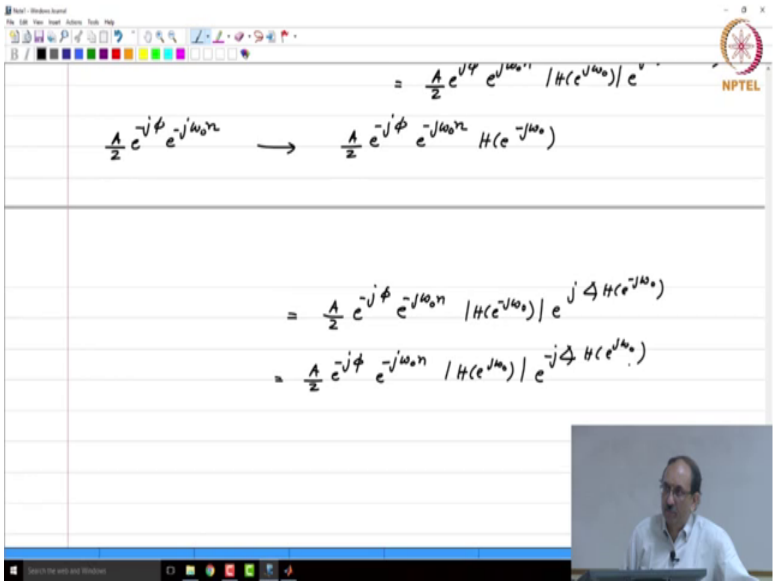

Therefore, the other signal is $\dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}$. Now this when applied to this system will give you, $\dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}H(e^{-j\omega_0})$. Because now you have to evaluate this transfer function at the frequency of the input, the input frequency is $-\omega_0$. And this of course is $\dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}(\ )$.

Now, I will write this as for exactly like what I did for the previous term, magnitude times $e^{j\angle(\ )}$. So, it will be this. And, now we are going to use the fact the impulse response is real and the transfer function process symmetry. So, this in turn becomes $\dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}|H(e^{-j\omega_0})|e^{j\angle H(e^{-j\omega_0})}$ and magnitude is even. Therefore, this becomes $|H(e^{j\omega_0})|$. And the phase angle is odd, therefore, this is $e^{-j\angle H(e^{j\omega_0})}$. And, now I can combine these two terms.

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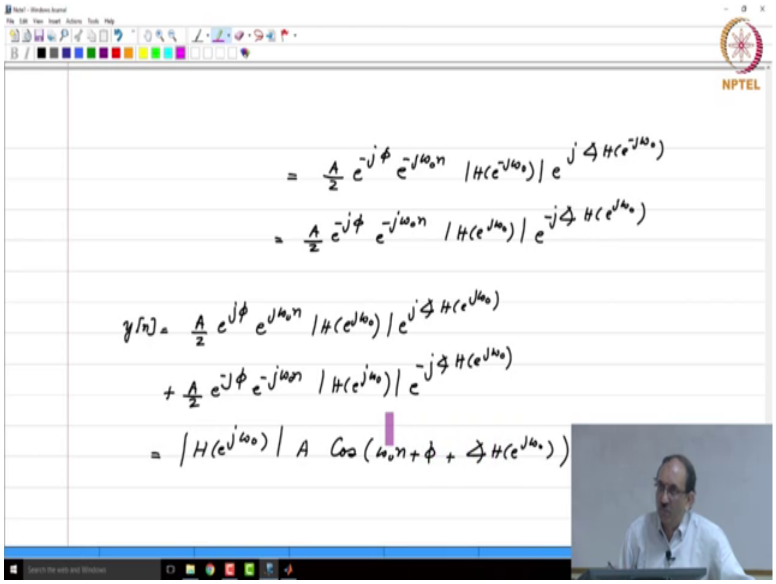

Therefore, the output is $y[n] = \dfrac{A}{2}e^{j\phi}e^{j\omega_0 n}|H(e^{j\omega_0})|e^{j\angle H(e^{j\omega_0})} + \dfrac{A}{2}e^{-j\phi}e^{-j\omega_0 n}|H(e^{j\omega_0})|e^{-j\angle H(e^{j\omega_0})}$, this term stays as it is because of the even symmetry. And, then this is this and now when you add this is the same as. This of course comes out and the remaining term is A as it is, and then this will be nothing but $|H(e^{j\omega_0})|A\cos(\omega_0 n + \phi + \angle H(e^{j\omega_0}))$. And, this is exactly what was happening in the continuous-time case as well.

We had $A\cos(\Omega_0 t + \theta)$ applied to a continuous-time system whose impulse response was real value, you would get A times magnitude of the transfer function evaluated at that frequency times $\cos(\Omega_0 t + \theta +$ a term that is the counterpart of this). So, this is no different from what was happening in the continuous time case, in fact, the derivation that we did just now mimicked exactly those same steps, nothing this derivation is specific to the discrete-time case.

So, the implication of this is important to realize, if you give cosine to the, a steady state sinusoid to the system, the output also will be a sinusoid, but two things will happen, one it will get amplitude scale, and the amplitude scaling is governed by this factor; the other thing is phase shift. And, one important consequence of this is that suppose the frequency transfer function had a value of 0 at that particular frequency, then that sinusoid will get filtered out.

Therefore, if the frequency response is 0 at $\omega = \omega_0$, then this system will completely eliminate this particular input, that is all. And, this is exactly what is happening in the continuous-time case as well. In the continuous-time case, if the frequency response had a 0 at that particular frequency, output will be 0. Notice carefully that input is $A\cos(\omega_0 n + \phi)$, and this is a sinusoid that is of infinite duration, it is an everlasting sinusoid.

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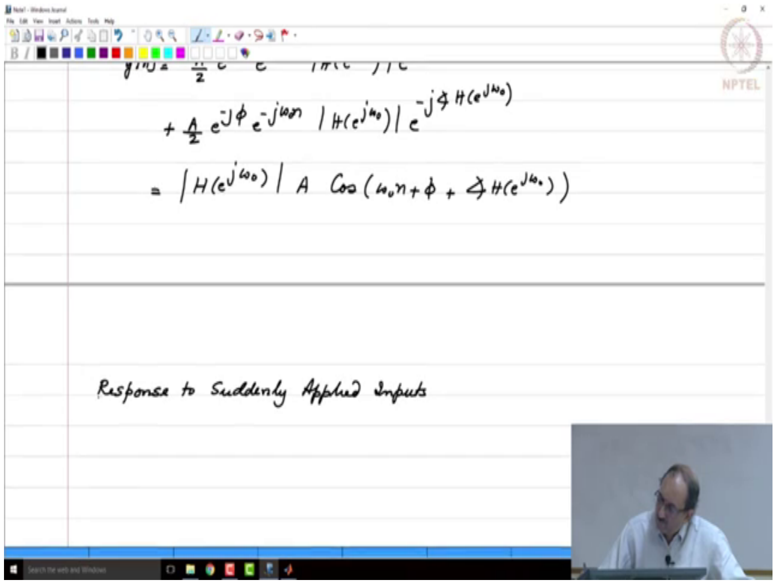

And, the reason I am mentioning that is, we are now going to consider response to suddenly applied inputs.