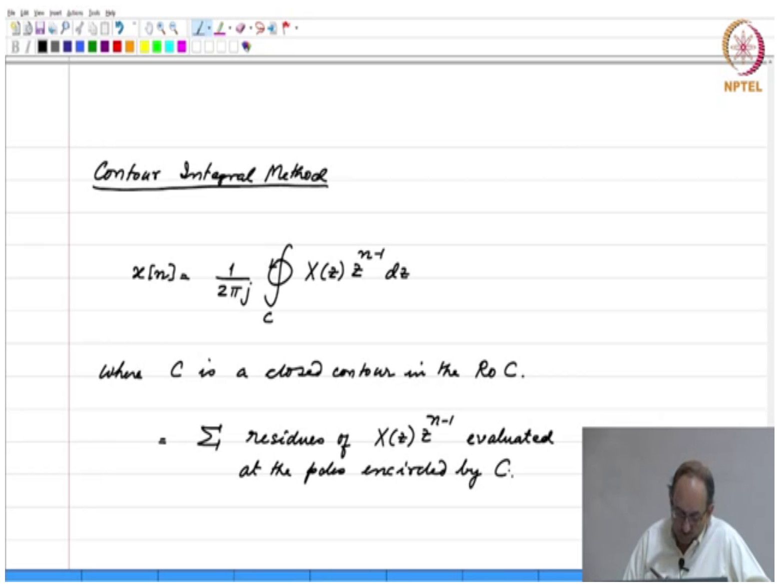
Digital Signal Processing

Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

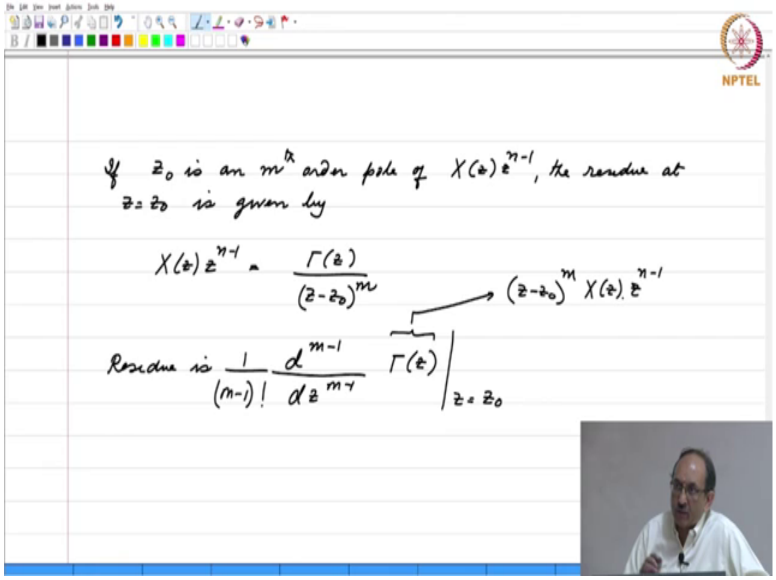
Lecture 41:
Inverse Z-transform (2)
- Contour integral method

(Refer Slide Time: 00:35)



And the final method for finding the Inverse Z-transform is the dreaded contour integral. Actually for rational transfer functions, this will turn out to be one of the easiest methods, all right and the formula is $\frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz$ where C is a closed contour in the region of convergence. And the famous Cauchy residue theorem, you do not have to really evaluate this contour integral; all you need to do is, this is the same as sum of the residues, so, residues of $X(z)z^{n-1}$ at the poles encircled by C.

(Refer Slide Time: 02:29)

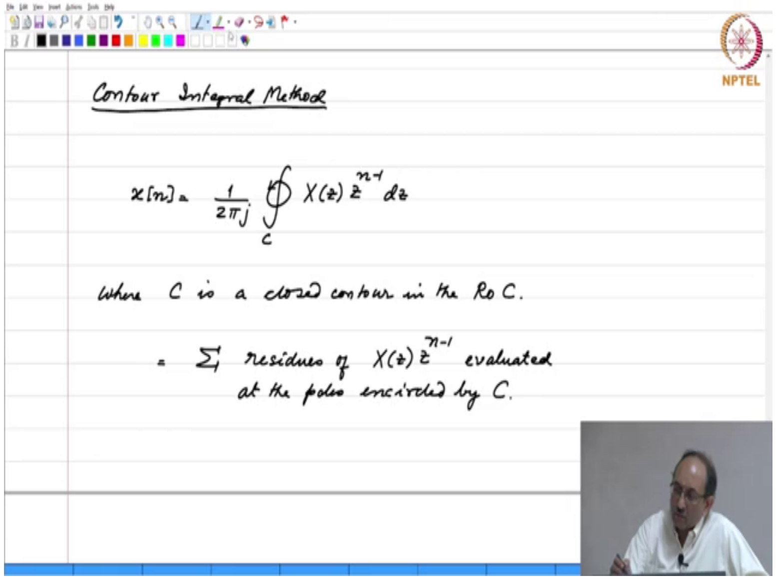


So, this is the this one and as a formula for this, if z_0 is an m^{th} order pole of $X(z)z^{n-1}$, the residue at $z = z_0$ is given by $X(z)z^{n-1} = \dfrac{\Gamma(z)}{(z - z_0)^m}$. So, this is, remember $X(z)z^{n-1}$ 1 can be written as $\dfrac{\Gamma(z)}{(z - z_0)^m}$. All I have done is I have isolated these $(z - z_0)^m$ as separately.

Remember this z_0 is an m^{th} order of pole so, that can be factored out, everything that is remaining is part of $\Gamma(z)$. Therefore, the residue value is given by $\dfrac{d^{m-1}}{dz^{m-1}}\Gamma(z)$ evaluated at $z = z_0$. And this is nothing, but $(z - z_0)^m X(z)z^{n-1}$. Again, this is consistent with what we have seen earlier. For example, if this were a first order pole, then, you also, I also need $\dfrac{1}{(m-1)!}$. Therefore, if this were a first order pole therefore, $(m - 1)!$ is $0! = 1$.

Again, if you put $m - 1$, you get the 0^{th} derivative; 0^{th} derivative means taking no derivative at all, all right. And, this is no different from what we did for partial fraction expansion. All you did was you multiplied by the pole to cancel it out and then evaluated the remaining expression at the location of the pole. Therefore, if you multiply by $(z - z_0)^1$, then you cancel out that simple pole and you evaluate the remaining expression at $z = z_0$. So, this is no different from what we have seen earlier. So, what we will do is we will illustrate this for the $\dfrac{1}{1 - az^{-1}}$ example just to illustrate how this works. And, then you will also show that this indeed gives you back $x[n]$.

(Refer Slide Time: 05:37)



We will start off with this and then show that this indeed gives you $x[n]$ and for that, we will invoke a result from complex variable theory.