Digital Signal Processing

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Lecture 04:
Signal Symmetry, Elementary Signals (1)
Even and Odd Parts of a Signal

Keywords: even signal, odd signal, conjugate symmetric, conjugate anti-symmetric

Now, let us look at some other definition that is similar to what you must have seen in the continuous-time case and here things are very similar, the analogy is very close.

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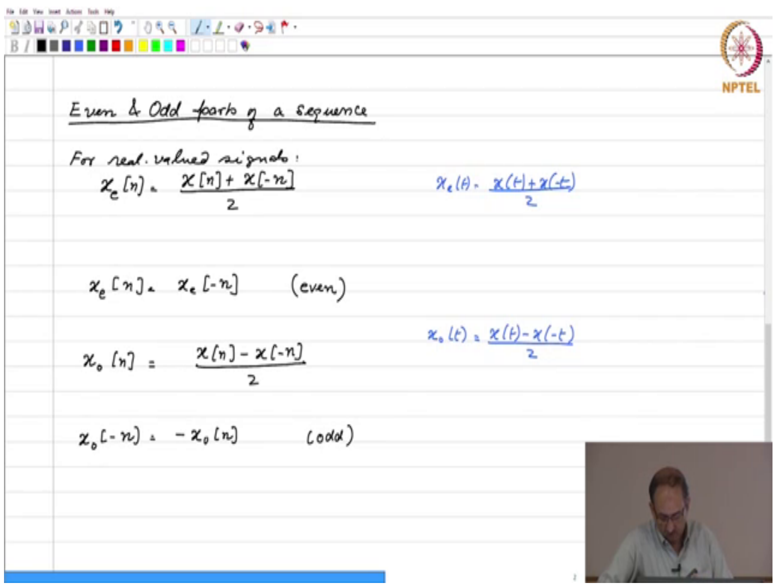

So, we will look at even and odd parts of a sequence

$$x_e[n] = \frac{x[n] + x[-n]}{2}.$$

And this is very similar to

$$x_e(t) = \frac{x(t) + x(-t)}{2},$$

and you can easily verify that $x_e[n] = x_e[-n]$. Any signal that satisfies this relationship namely $x[n]$ being equal to $x[-n]$, it is called an even signal.

And in this particular case, $x_e[n]$ happens to be even part of a given signal $x[n]$ and similarly,

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

and it is easy to see that $x_o[-n] = -x_o[n]$ and this is the odd part of the given sequence $x[n]$. So, any given sequence x can be decomposed into its odd and even parts and the definition of an even signal is $x_e[n] = x_e[-n]$ and the definition for an odd sequences $x_o[n] = -x_o[-n]$. And these are now different from the definitions from their continuous-time counterparts, just to complete this

$$x_o(t) = \frac{x(t) - x(-t)}{2}.$$

So, this we are assuming the signal to be real value. So, this is the case for real-valued signals and there is a corresponding definition for complex-valued signals.

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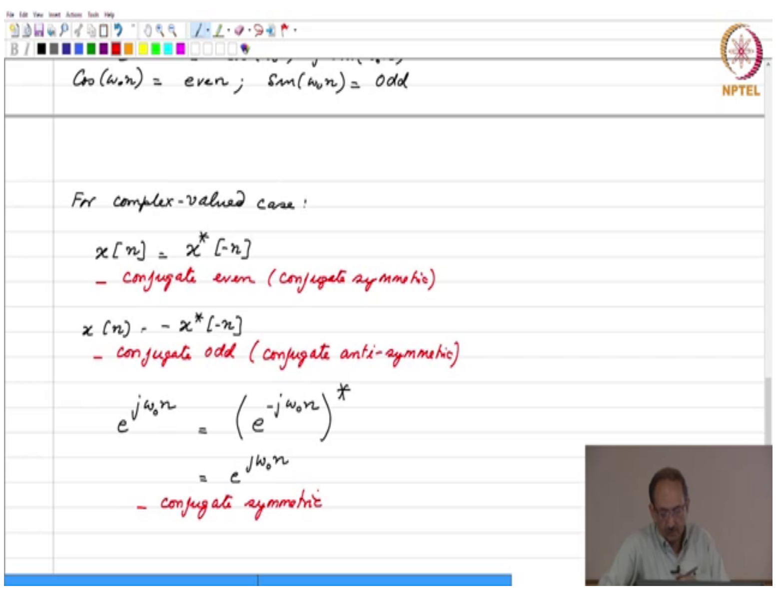

If $x[n] = x^*[-n]$, then this is called conjugate even and if $x[n] = -x^*[-n]$ this is called conjugate odd. The other name that is used for conjugate even is conjugate symmetric and the term that is commonly used for conjugate odd is conjugate anti-symmetric. Exactly the same definitions you must have seen for the continuous-time case.

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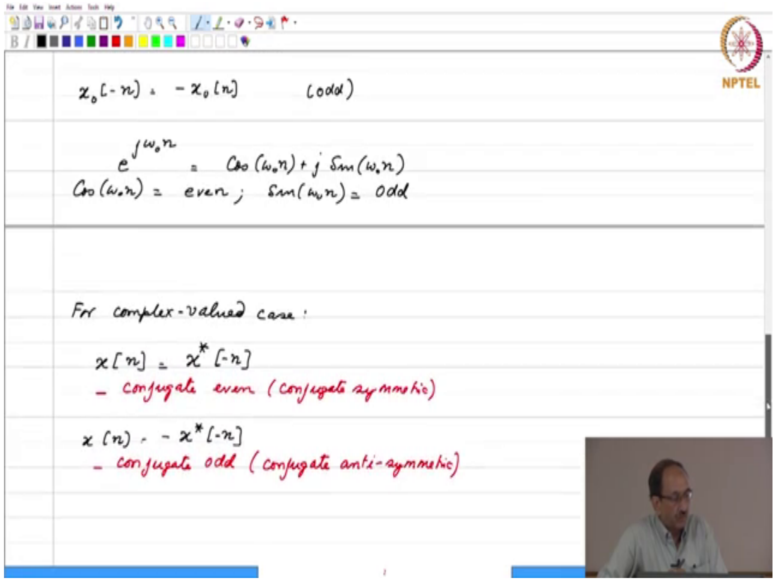

Here, if you note that if you consider $e^{j\omega_o n}$ as a signal and this is $\cos(\omega_0 n) + j\sin(\omega_0 n)$ and it is easy to see that $\cos(\omega_o n)$ is even and $\sin(\omega_0 n)$ is odd symmetric. So, the symmetry associated with these things, the real and imaginary parts are even and odd very simple which you must have seen any number of times before.

On the other hand, if you consider the signal $e^{j\omega_o n}$ so, this is the same as, first of all you need to replace n by $-n$ and then take the complex conjugate. So, if you replace n by $-n$, this becomes $e^{-j\omega_o n}$ and then you need to take the complex conjugate. So, this turns back into the given signal. Therefore, $ee^{j\omega_o n}$ is really conjugate symmetric. So, these are pretty simple and straight forward and follow from the definition. Let me take a simple example that is familiar to you and then point out a difference that you need to pay attention to when it comes to continuous-time versus discrete-time.

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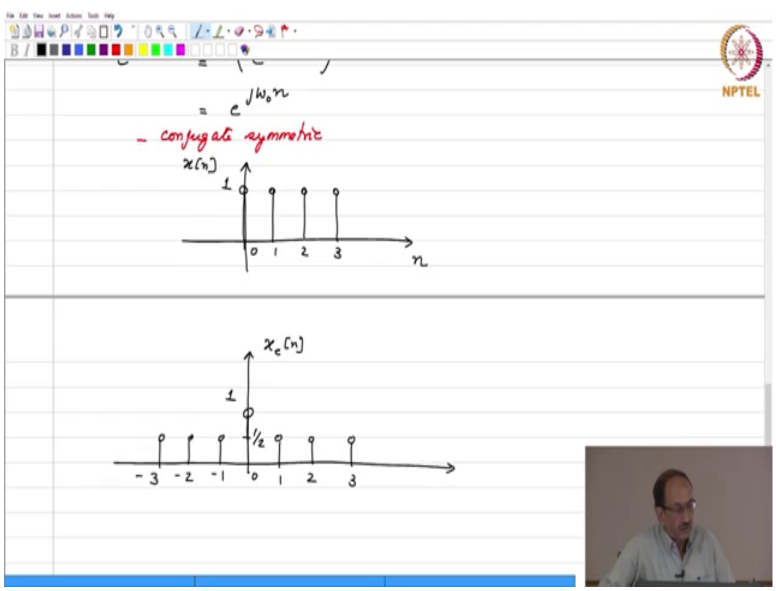

So, suppose let us consider $x[n]$ to be this signal. So, this is my given $x[n]$ and if I want to decompose this into its odd and even parts. So, $x_e[n] = \dfrac{x[n] + x[-n]}{2}$, what happens here is at $n = 0$, $x(0)$ and

$x(-0)$ are the same, if you add them up and then divided by 2, you will get back your sample value at 0. So the remaining three, the even part will look like this and this value is $\frac{1}{2}$.

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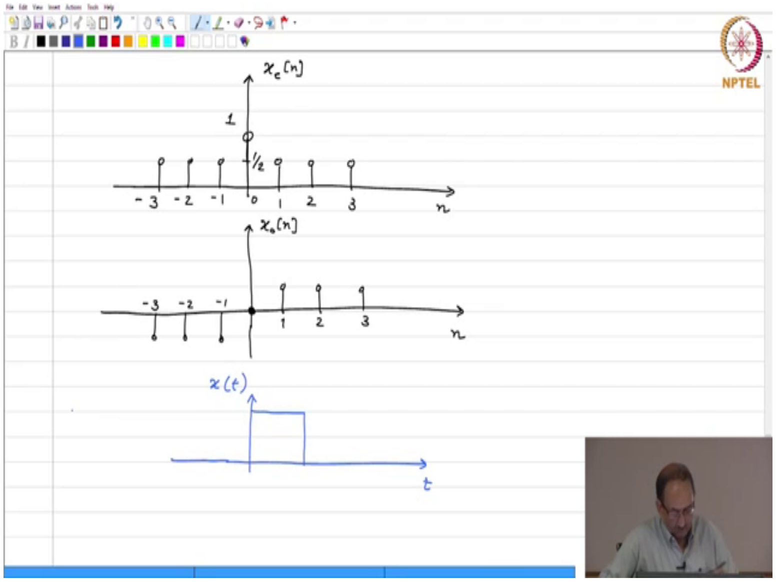

So, this is the even part and the corresponding odd part, this sample value will always be 0 and for $x_o[n]$ if you take $(x[n] - x[-n])/2$, this is the odd part and if you add $x_e[n] + x_o[n]$, you get back your original signal, there are no nothing surprising here. Now, let us look at this for the continuous-time case, suppose you have $x(t)$ like this and if you try to break this up into its odd and even parts.

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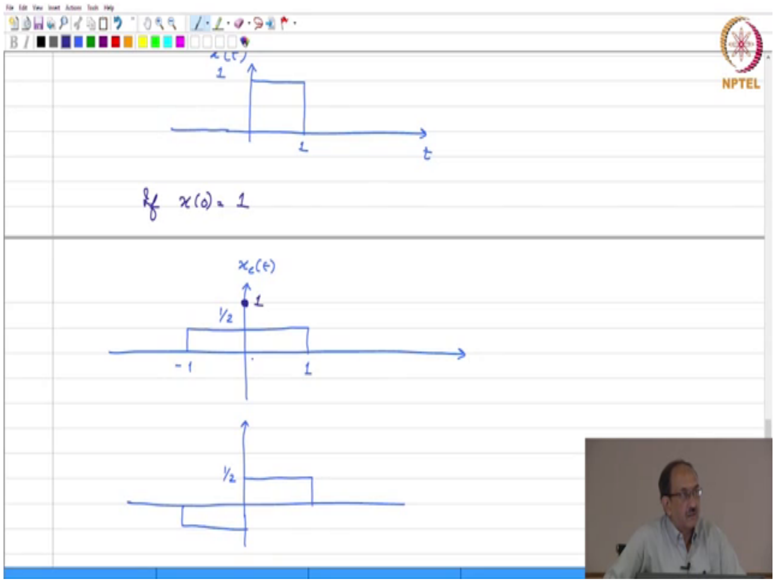

So, this is $\frac{x(t) + x(-t)}{2}$ i.e., the even part. The only thing that you need to pay attention here is; before that let me assume that this is 1 and this is $\frac{1}{2}$. So, this is the even part of the signal and this is the odd part of the signal so that when you add these two, you get back your original $x(t)$. Is there any wrinkle to this, is this fine or should we pay attention to something?

Student: (Refer Time: 10:12) t equal to.

At $t = 0$, it depends on how the signal is defined. If $x(0) = 1$, then for that particular; the even part is really at 0, the value will be 1 because $(x(0) + x(-0))/2$ will give you a value of 1. If $x(0) = 0$, then the original picture stays fine. $x(0)$ can also be undefined, then also this picture is fine except that at 0 the even and odd parts are not defined.

Student: $x(0)$ is (Refer Time: 11:14).

Say that again.

Student: $x(0) = 0$, then even part should be 0 at x equal to (Refer Time: 11:20).

You are right. If $x(0) = 0$, the even part will be 0 at $t = 0$, yes very good. So, the point to be noted is none of these ambiguities come about in the discrete-time case because at the origin this signal is clearly defined in the way that we expect it to be. So, there are similarities between continuous-time and discrete-time, but there are some points at which there will be differences and you need to pay attention to these cases where the differences can manifest themselves.