Digital Signal Processing
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Lecture 38:
Properties of the Z-transform (5)
-Parseval's theorem

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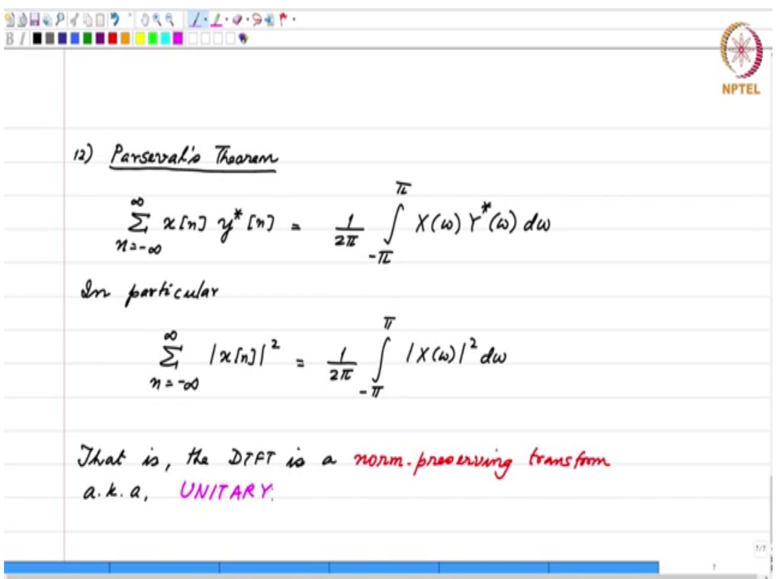

The last properties, Parseval's: so, we had two sequences $x[n]$, $y[n]$, then they are related like $\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)Y^*(\omega)d\omega$. And there is a special case of this general result; if $y[n]$ happens to be the same as $x[n]$, then this becomes $x[n]x^*[n]$ which is nothing, but this magnitude square. On this side also, you can make the same change and you get this result, $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$. And, if you know something about norms of signal, this is the norm of the given signal in the time domain and this is the l_2 norm and is defined like this, $|x[n]|^2$. This is indeed the norm in the transform domain and the norm in the transform domain is really defined like this, $|X(\omega)|^2$. So, what this theorem states is that the norm in the time domain equals the norm in the frequency domain.

Again here, I was stated this; for the DTFT case, and I am again using the $X(\omega)$ notation rather than $X(e^{j\omega})$. And since the norms are equal, this is a statement of the fact that the DTFT is a norm preserving transform. If a transform is norm preserving, this is also called unitary. Also known as unitary.

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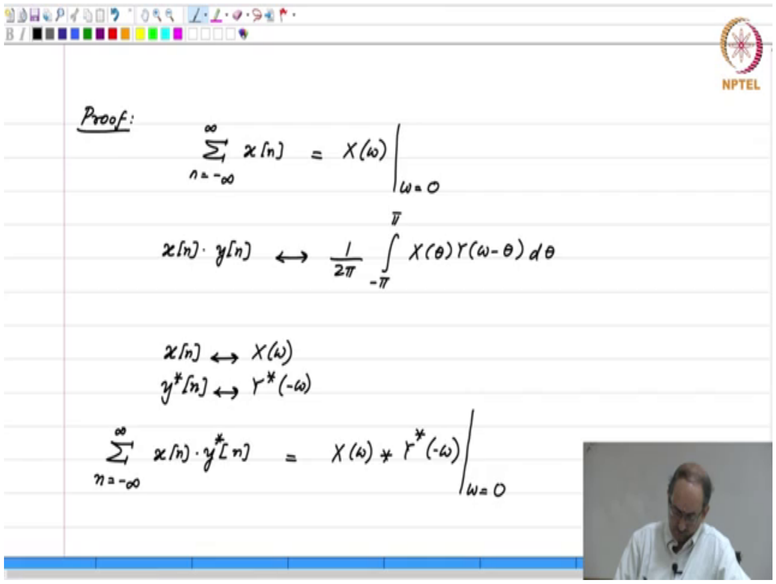

And the proof is really simple. We will use two properties that we have already seen before, under simple application of these two properties will give rise to this result. You also; we already seen that. So, this is this and the other property we have seen is $x[n].y[n] \longleftrightarrow \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(\theta)Y(\omega-\theta)d\theta$. So, these two properties will be used.

So, the first sequence is $x[n]$. So, this has transform $X(\omega)$. The second sequence is $y^*[n]$ and the transform of $y^*[n]$ is $Y^*(e^{(\ )})$. So, this is $Y^*(-\omega)$. You are right, it is $Y*(e^{j\omega})$, but here in this context we are using the omega notation. Therefore, $\displaystyle\sum_{n=-\infty}^{\infty} x[n]y^*[n]$, this is nothing, but $X(\omega) \circledast Y^*(-\omega)\Big|_{\omega=0}$, right.

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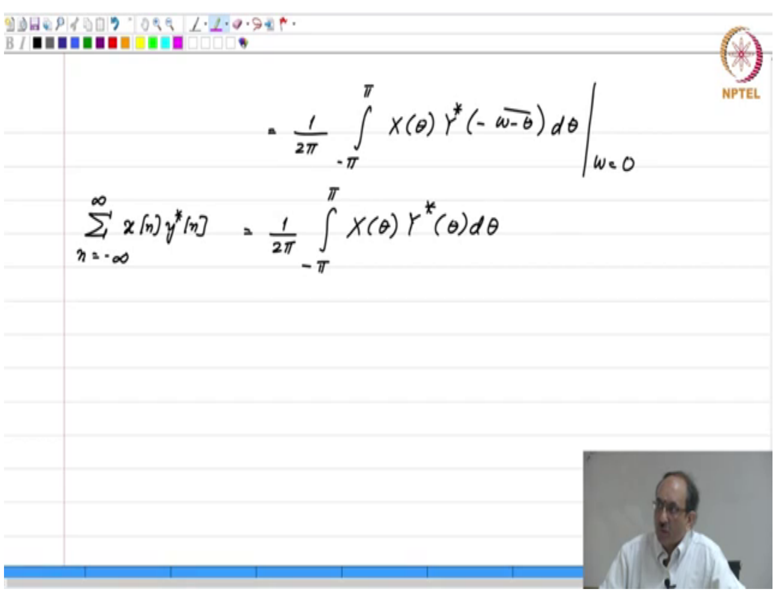

So, this is nothing, but $\frac{1}{2\pi}\int_{-\pi}^{\pi}\Big(\Big)$ and remember this convolution is really circular convolution. Therefore, it is $\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\theta)Y^*(-\overline{\omega-\theta})d\theta$ and all of this has to be evaluated at $\omega = 0$. Therefore, this is $\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\theta)Y^*(\theta)d\theta$. Therefore, $\sum_{n=-\infty}^{\infty} x[n]y^*[n]$ is exactly this.

And what is the corresponding statement for the Z-transform case? That can be actually easily seen from this result, $\frac{1}{2\pi j}\oint X(\gamma)Y(z/\gamma)\frac{d\gamma}{\gamma}$. So, if you now evaluate this at what point in z would this correspond to? Sum up over all n of what is on the left hand side, very good. It is said $z = 1$. If you evaluate the transform at $z = 1$, you are summing up the sequence over all n. Therefore, here if you put $z = 1$, then that will be sum over all n, $x[n].y[n]$.

It will be equal to this convolution evaluated at $z = 1$, that is all. Again, we have to make sure that is part of the region of convergence, only then can evaluate this at $z = 1$. So, we are actually done with all the properties of the Z-transform. So, the next thing to do would be to evaluate the inverse Z-transform.