Digital Signal Processing
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Lecture 37:
Properties of the Z-Transform (5)

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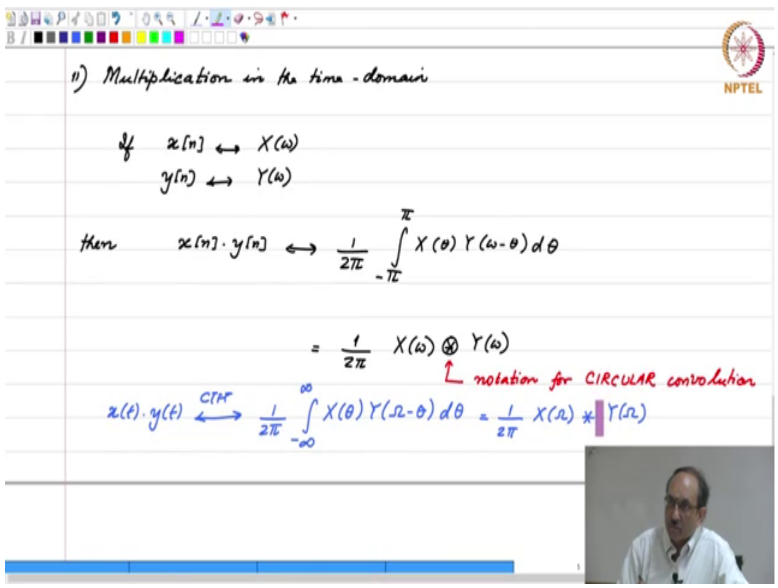

So, if we will first state this for the DTFT case and then we will give the formula for the Z-transform case. And, the Z-transform case, the formula that is given will be understood only when you have the inversion integral. So, if you have $y[n]$ to be a having DTFT $Y(\omega)$ and now, I am using the $X(\omega)$ and $Y(\omega)$ notation rather than $X(e^{j\omega})$ and $Y(e^{j\omega})$ because in this case this is more convenient.

Then, $x[n].y[n]$, this has DTFT $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)Y(\omega-\theta)d\theta$ and this is also notationally written like this. If you look at the expression for the what is there in the integral, this reminds you of convolution. The only difference is the limits are not from $-\infty$ to $+\infty$, but they are between $-\pi$ and π and this is nothing, but $X(\omega)$ convolved with $Y(\omega)$ except that this is not your usual convolution.

The usual convolution is called as linear convolution and this is what is called circular or periodic convolution and that is denoted by an asterisks within a circle. So, $X(\omega) \circledast Y(\omega)$ denotes circular convolution, was the notion of circular convolution introduced in signals and systems? Ok. So, this is an important counter part. We will look at more about this later. And, we will point out the fact that when you have two signals that are periodic, linear convolution does not make sense, but convolution is still an important attribute in those cases. And, the convolution that makes sense when two functions are periodic is circular convolution.

And, notice that here $X(\omega)$ is the DTFT therefore, this is 2π periodic, $Y(\omega)$ is also DTFT it is also 2π periodic. And, when you have two functions that are periodic with the same period, this is the kind of convolution that you can realize using periodic signals. And, we will also see the counterpart for this. Here the independent variable is continuous, ω is a continuous variable. Therefore, you have the convolution integral defined like this, only that the limits are between $-\pi$ to π. This can also be from 0 to 2π.

So, this is circular convolution for functions whose independent variable is continuous, later you will also define a circular convolution for discrete times sequences that are periodic. They are also you will introduce the notion of circular convolution, there it will involve summation. The summation will go from 0 to $N-1$, assuming the sequences are periodic of a period N.

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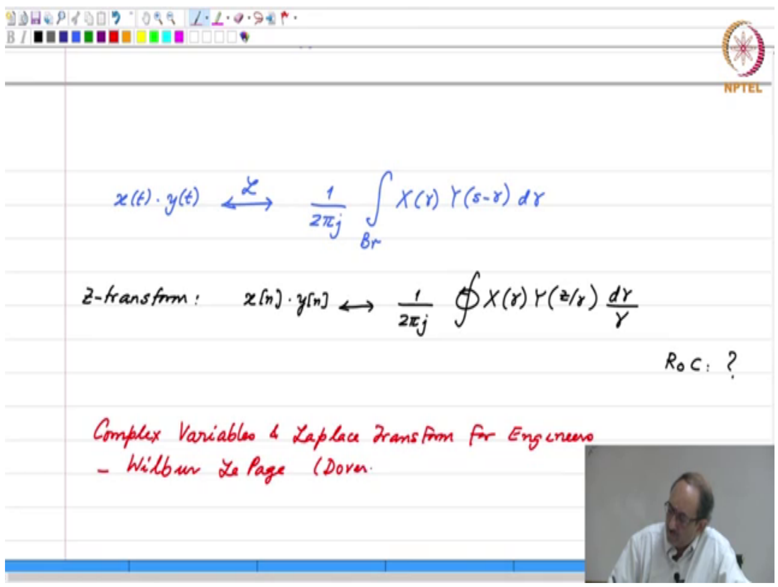

If you now multiply two sequences in the time domain, now we look at the corresponding property for the Z-transform, and as I had mentioned earlier you will understand this better once we introduce the notion of inversion integral.

So, this is $\frac{1}{2\pi j}(\ )$. This is the contour integral over an appropriate contour, $\frac{1}{2\pi j}\oint X(\gamma)Y(z/\gamma)\frac{d\gamma}{\gamma}$ and what is missing here is you also need to specify the corresponding RoC. So, this is very similar to what was happening in the continuous-time case. There if you had $x(t).y(t)$ should multiplied in the time domain, you convolve in the frequency domain, there you convolve the corresponding continuous-time Fourier transforms; there in general, the transform of each of these functions is aperiodic and then when you convolve in the frequency domain, the convolution is your regular convolution; linear convolution.

Therefore, if you recall the continuous-time case so, this was $x(t).y(t)$, your CTFT was $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\theta)Y(\Omega-\theta)d\theta$ and this was nothing, but $\frac{1}{2\pi}X(\Omega) * Y(\Omega)$. And, this is linear convolution. What about the corresponding property for the Laplace? $x(t).y(t) \longleftrightarrow \frac{1}{2\pi j}\int_{B_r} X(\gamma)Y(s-\gamma)d\gamma$. This is complex convolution. Br stands for, you guys have not had a course in a complex, correct. So, Br stands for Bromwich integral, all right.

So, if those of few you who want to know more about this, there is an extremely good book. Yes, this is by Wilbur Le Page. So, this is one of the most accessible books on complex variables and Laplace transform, ok. So, Le Page, Electrical Engineering Professor so, the treatment is fairly rigorous, but not too mathematical, it is very much understandable. So, there he talks about things like complex convolution and so on.

There is also chapter on Z-transform in that book. So, this is an extremely accessible readable book. For those of you are interested in knowing more about complex variables and associated topics. He also talks about convergence of the Laplace besides absolute convergence. So, these are kinds of things if you are interested it is well worth looking this up.