

Digital Signal Processing
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Lecture 35:
Properties of the Z-Transform (4)
- Initial value theorem

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(*) Initial Value Theorem

If $x[n] = 0$ for $n < 0$

$$\lim_{z \rightarrow \infty} X(z) = x[0]$$
$$X(z) = x[0] + x[1]z^{-1} + x[2]z^{-2} + \dots$$
$$\lim_{z \rightarrow \infty} X(z) = x[0]$$

So, if $x[n] = 0$ for $n < 0$, then $\lim_{z \rightarrow \infty} X(z) = x[0]$. Now, our discussion on z taking on the value ∞ should make sense now. So, when we really say z tending to *infy*, it does not make sense to talk about the angle, right. So, this is $X(z) = x[0] + x[1]z^{-1} + x[2]z^{-1} + \dots$. And, now if you take the only term that will survive is $x[0]$ because these terms will all vanish when you put $z = \infty$.

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If $x[n] = 0$ for $n < 1$

$$\lim_{z \rightarrow \infty} z X(z) = x[1]$$
$$X(z) = x[1]z^{-1} + x[2]z^{-2} + \dots$$
$$z X(z) = x[1] + x[2]z^{-1} + \dots$$
$$\lim_{z \rightarrow \infty} z X(z) = x[1]$$

Similarly, if $x[n] = 0$ for $n < 1$, then $\lim_{z \rightarrow \infty} zX(z) = x[1]$. Again, this is very easy to see because $X(z)$ after all is $x[1]z^{-1} + x[2]z^{-2} + \dots$ because $x[n] = 0$ for $n < 1$. Therefore, $zX(z) = x[1] + x[2]z^{-1} + \dots$ and then you use the result that is similar to the previous one. Therefore, it is $x[1]$.

Intuitively also this makes sense because $zX(z)$ is a shifting the sequence by one sample to the left. If you multiply the Z-transform by z , you are replacing n by $n + 1$. Therefore, the sequence which was 0 for $n < 1$ right, $zX(z)$ corresponds to the sequence shifted by one sample to the left and then you are using the previous result. That is all; as simple as that.