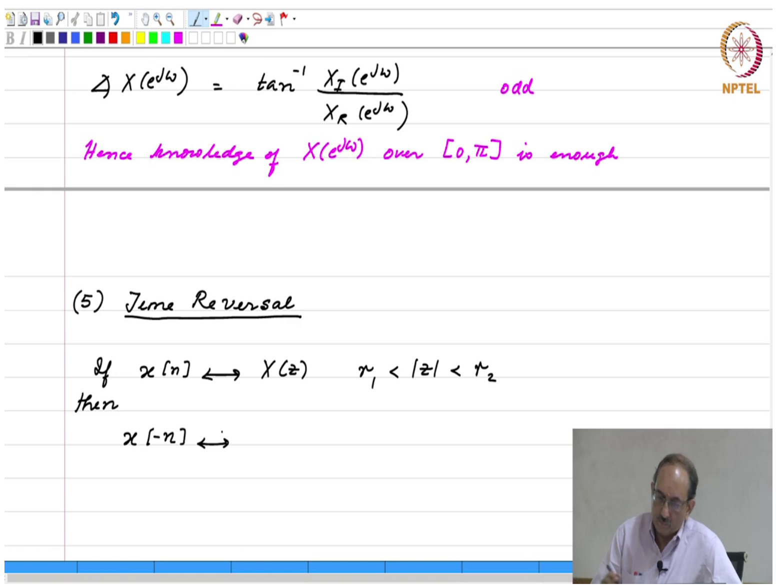
Digital Signal Processing

Prof. C.S. Ramalingam
Department Electrical Engineering
Indian Institute of Technology, Madras

Lecture 31:
Properties of Z-transform (3)
-Time reversal

(Refer Slide Time: 00:17)

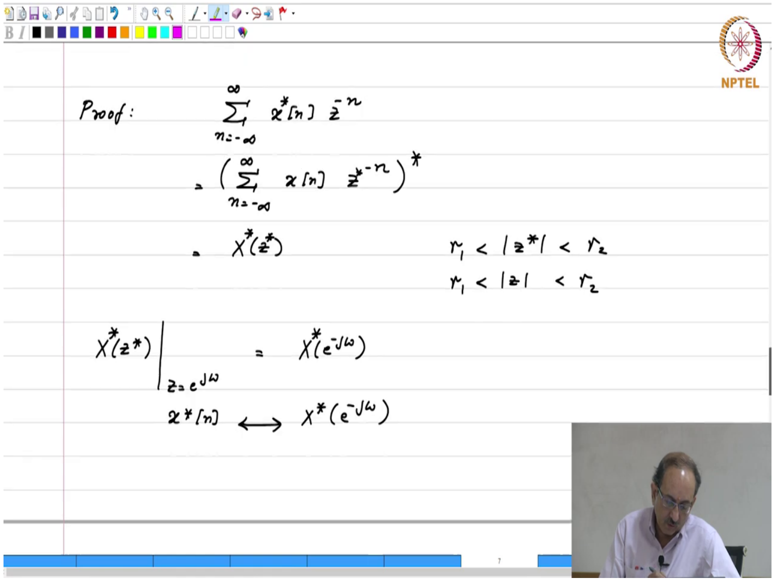


So, the next property we can look at is Time reversal. So, if $x[n]$ has Z-transform, $X(z)$. Yes, question.

Student: (Refer Time: 00:52) Z-transform (Refer Time: 00:54) categorized that for a real function (Refer Time: 01:00).

So, what you can do, that is a good question. So, you need to exploit the fact that $X(z) = X^*(z^*)$, right.

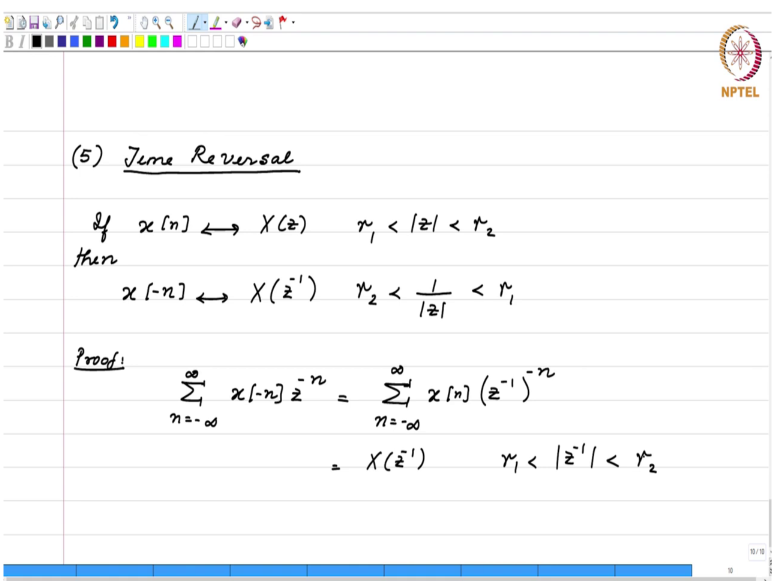
(Refer Slide Time: 01:14)



So, one fall out of this, $X(z)$ being equal to $X^*(z^*)$. So, if z were real, z^* will be the same as z. So, for real z, $X(z)$ will be the same as $X^*(z)$ for a real sequence $x[n]$ because $X(z) = X^*(z^*)$. When the variable z is real, you can conclude that the transform also is real.

So, that is not surprising because if you look at this, just from the plain definition, this is $\sum_{n=-\infty}^{\infty} x^*[n]z^{-n}$; if $x[n]$ were real and if z also were real, the quantity has to be real. So, that is another way of seeing this. So, these are some of the inferences can that can be drawn from the Z-transform. But, typically this property is exploited in the case of DTFT for real valued sequences, where half the information is enough.

(Refer Slide Time: 02:37)



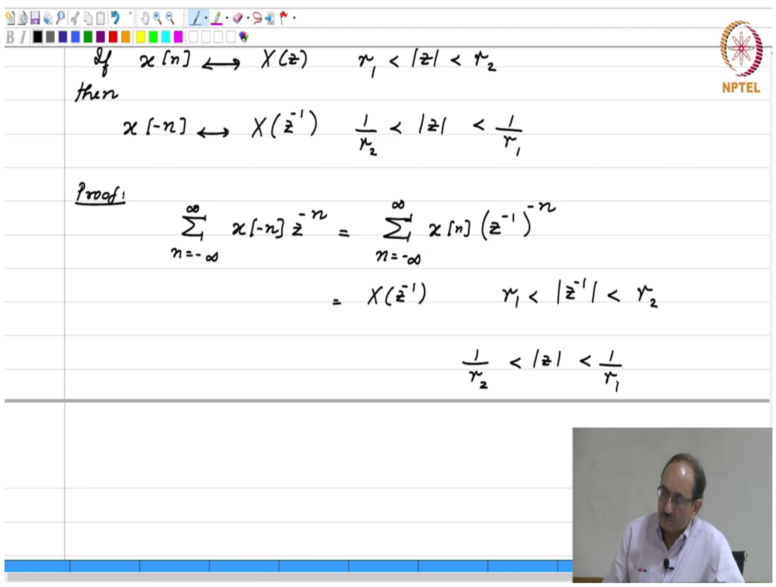
So, $x[-n] \longleftrightarrow X(z^{-1})$ and proof again this pretty straightforward here. All you need to do is

$\sum_{\infty}^{\infty} x[-n]z^{-n}$ and then replace n by $-n$ and re-label this one. So, this becomes $\sum_{\infty}^{\infty} x[n]z^{n}$, but really this is $\sum_{\infty}^{\infty} x[n]\left(z^{-1}\right)^{-n}$, z^n can be written as $\left(z^{-1}\right)^{-n}$ because you want it of the form the complex variable to the $-n$. So, this is really $X(z^{-1})$ and you want $r_1 < |z^{-1}| < r_2$.

Student: (Refer Time: 03:52).

Let me see if I got that right. So, we can see whether I made an error there. So, this is what?

(Refer Slide Time: 04:08)



So, if you look at this. So, this is $r_1 < \frac{1}{|z|}$, correct? Therefore, this implies you have.

Student: (Refer Time: 04:20).

So, this is $|z| < \frac{1}{r_1}$. So, I should have this as $\frac{1}{r_2} < |z| < \frac{1}{r_1}$. So, this is what you meant?

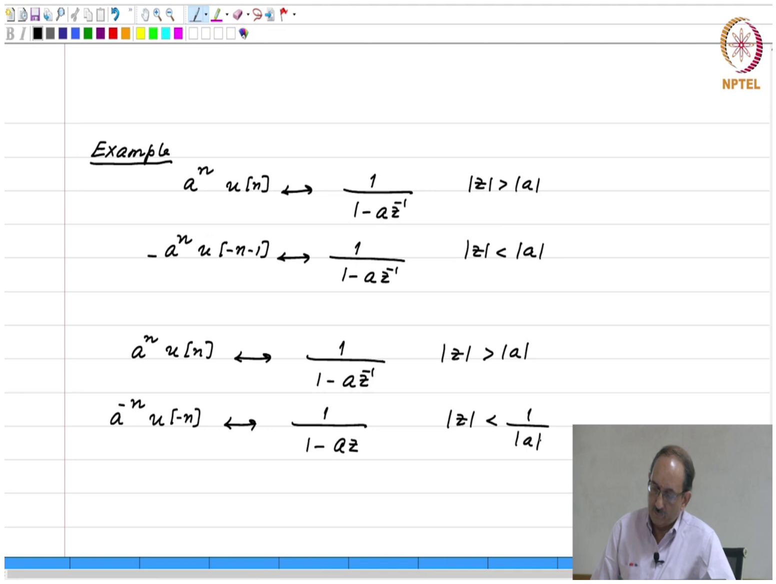
Student: (Refer Time: 04:40).

So, this should be $\frac{1}{r_2}$, correct?

Student: Yes.

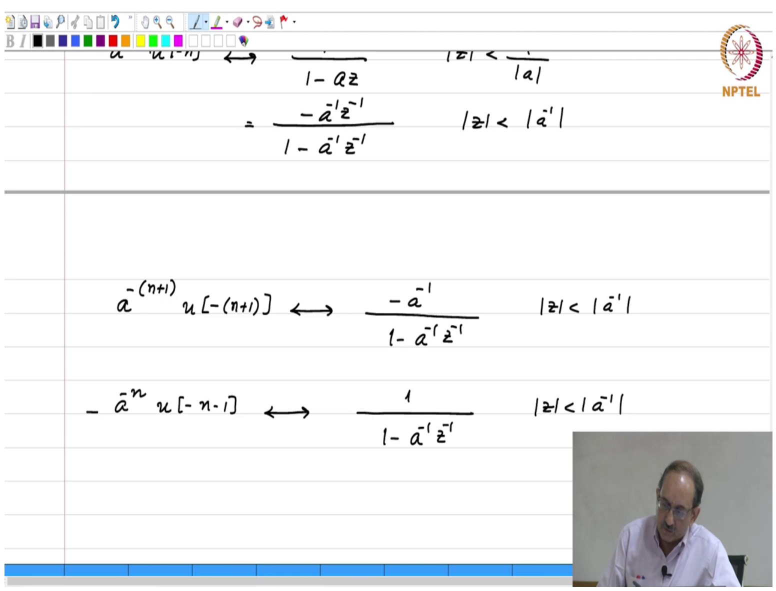
$\frac{1}{r_1}$, did I get it right now? Thank you. So, the RoC should always be of the form $|z|$ between two quantities, you are right. Therefore, this is now $\frac{1}{r_2} < |z| < \frac{1}{r_1}$, ok. Now, it is correct, right?

(Refer Slide Time: 05:27)



And, again we can use this property to derive this transform. We know that $a^n u[n] \longleftrightarrow \dfrac{1}{1-az^{-1}}$, $|z| > |a|$. And, we also know that $-a^n u[-n-1] \longleftrightarrow \dfrac{1}{1-az^{-1}}$, $|z| < |a|$. So, we will derive the second transform starting from the first using the time reversal property. So, $a^n u[n]$ has $\dfrac{1}{1-az^{-1}}$, $|z| > |a|$. Therefore, if you replace n by $-n$, you have $a^{-n}u[-n]$. So, this is $\dfrac{1}{1-az}$, $|z| < \dfrac{1}{|a|}$.

(Refer Slide Time: 06:57)

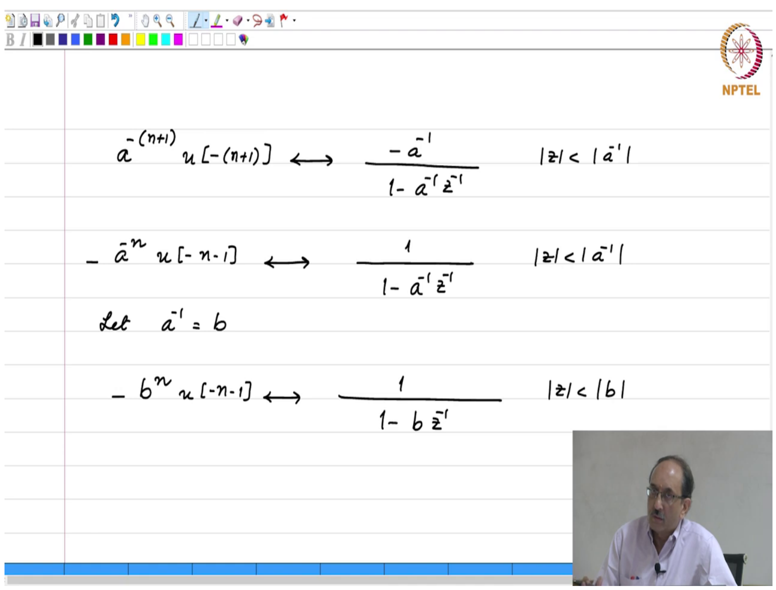


Now, let us multiply numerator and denominator by $-a^{-1}z^{-1}$. Therefore, this becomes $-a^{-1}z^{-1}$ and this is $1 - a^{-1}z^{-1}$. All I have done is multiplied numerator and denominator by $-a^{-1}z^{-1}$. So, this is $|z| < \dfrac{1}{|a|}$ which I have written as $|z| < |a^{-1}|$. Now, I have a factor z^{-1} in the transform, I want to get rid of that. To get rid of that, I need to multiply by z. If I multiply in the transform domain by z, I can get rid of this z^{-1} factor. But, if I multiply the transform domain by z, in the time domain I have to replace wherever n is there, I have to replace n by?

Student: $n - 1$.

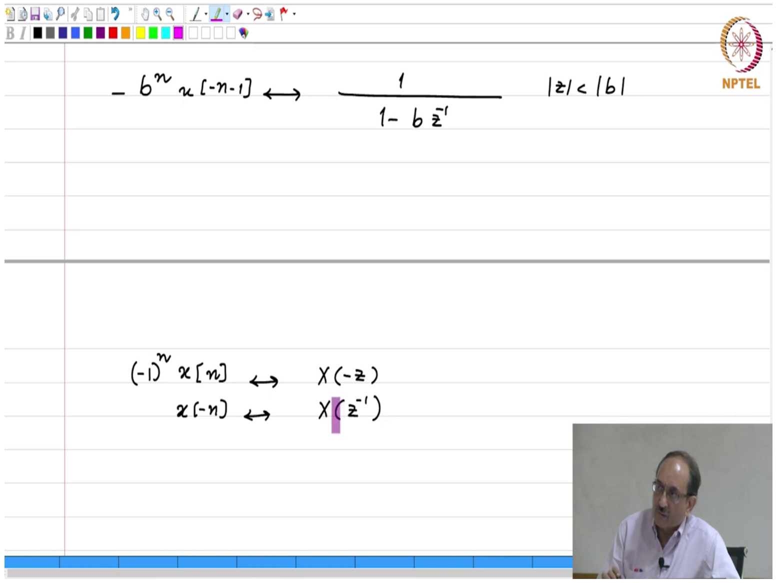
Not $n - 1$. You have to multiplying by z; you have to replace n by $n + 1$, ok. Therefore, I have $-a^{(\ )}$, wherever n is there I am going to replace n by $n + 1$ and this is $a^{-(n+1)}u[-(n + 1)]$. Now, I can get rid of this. And, now I can cancel sorry, I can cancel one a^{-1} factor and take this minus sign to the other side. Therefore, this becomes $-a^{-n}u[-n-1]$. So, I have taken this minus sign to the other side. I have cancelled one a^{-1}. So, therefore, this is $\dfrac{1}{1 - a^{-1}z^{-1}}$, $|z| < |a^{-1}|$.

(Refer Slide Time: 09:17)



So, now let $a^{-1} = b$; therefore, this becomes $-b^n u[-n - 1]$ and this is nothing, but $\dfrac{1}{1 - bz^{-1}}$ with $|z| < |b|$. So, again we have used properties to derive the transform of one starting from the transform of the other. Note that time reversal causes a causal sequence to become anti-causal and vice versa; if $x[n]$ were causal, $x[-n]$ is anti-causal.

(Refer Slide Time: 10:22)



And, also notice this difference between these two properties. You had $(-1)^n x[n]$, this is $X(-z)$ whereas, $x[-n]$ is $X(z^{-1})$. And, time reversal causes a causal sequence to become anti-causal and vice versa.