Digital Signal Processing
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Lecture 25:
Z-transform (3)
-Properties of the RoC

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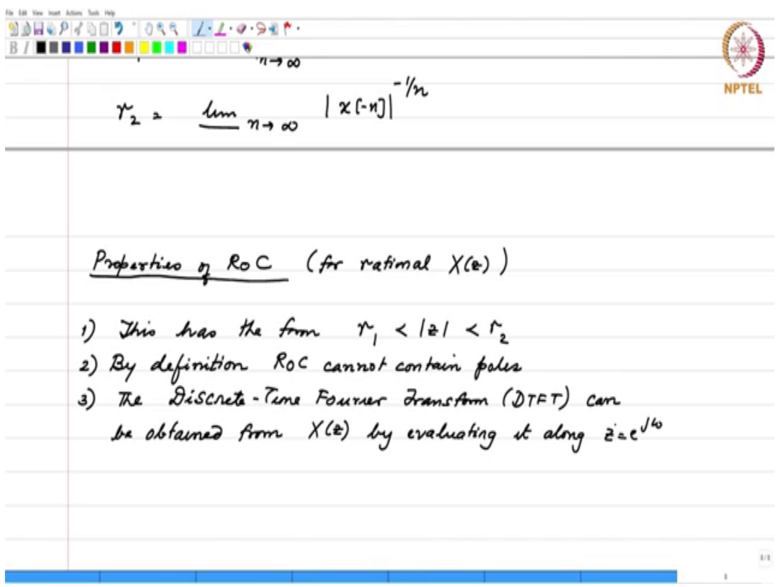

We will now look at the properties of RoC. This will basically summarize what we may have seen before, all right. So, we limit ourselves to systems having or signals having a rational Z-transform. So, this has the form $r_1 < |z| < r_2$. By definition, RoC cannot contain poles.

We will later define what is called the Discrete Time Fourier transform. And the Discrete Time Fourier transform, this is abbreviated as the DTFT which is the counterpart to the CTFT, which is the Continuous Time Fourier transform. So, the discrete time Fourier transform can be obtained from $X(z)$ by evaluating it along the unit circle. So, this should remind you of evaluating the continuous time Fourier transform by taking the bilateral Laplace and replacing s by $j\omega$.

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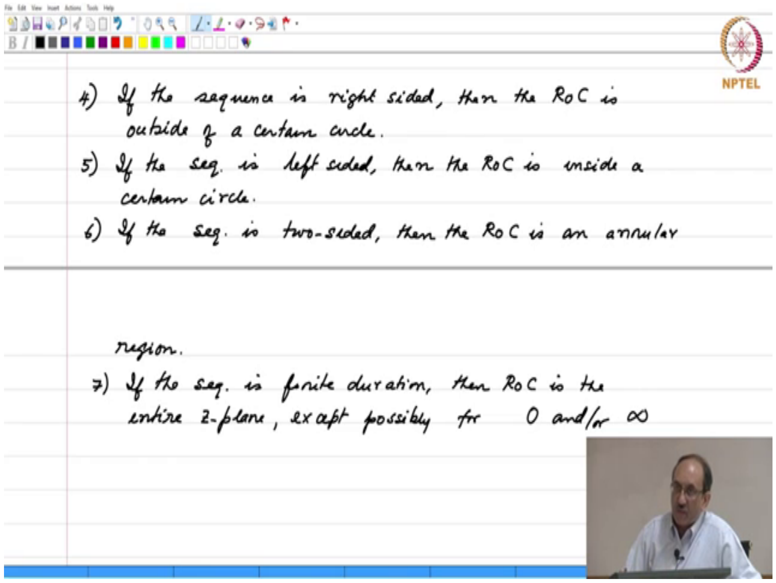

And if the sequence is right sided, then the RoC is outside of a certain circle. So, this is the counterpart of what was happening in the Laplace case. If the function is right sided, then the bilateral Laplace transform the region of convergence was to the right of a certain vertical line. If the sequence is left sided, then the RoC is inside a certain circle.

So, if the function is left sided the bilateral Laplace, the RoC was to the left of a certain vertical line. What that line is depends upon the function. And, if the sequence is two-sided, then the RoC is an annular region. By two-sided, here we mean infinite duration on both sides. And if the sequence is finite duration, then the RoC is the entire z-plane except possibly for 0 and or ∞.

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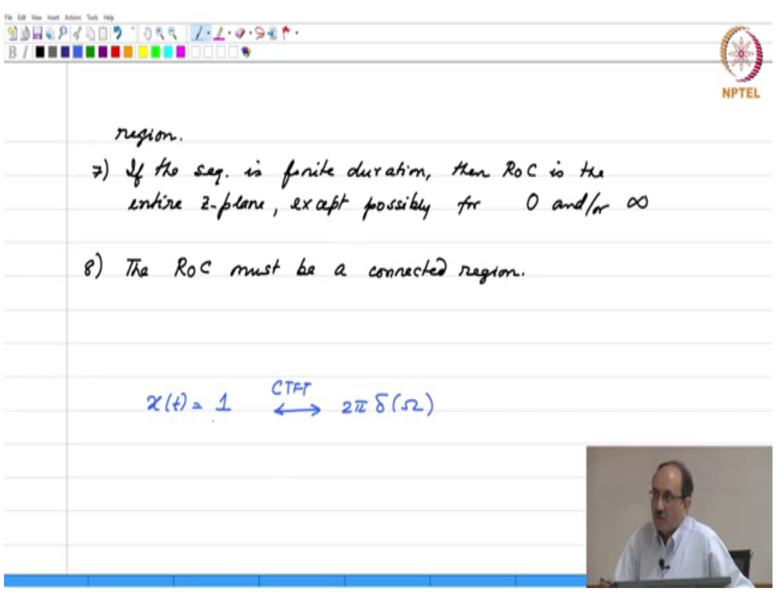

On the other hand, if you had a continuous-time function that is absolutely integrable, that is a finite duration, the RoC is the entire s-plane. And similar to the Laplace transform case, the RoC must be a connected region. That is why if you had $r_1 > r_2$, then for the right sided part, you will have a region that is outside this radius, for the left sided sequence, the region will be inside of a certain circle. And if there is no overlap, these two distinct regions are not connected and RoC cannot be a disconnected region. RoC has to be necessarily a connected region which is why Z-transform in those cases do not exist. And these properties are very similar to the Laplace case. You can see the parallels between whats happening there versus here, so if you understand the your Laplace very well, you can see the close connection between Laplace and Z-transform. Yeah, question.

Student: (Refer Time: 06:27).

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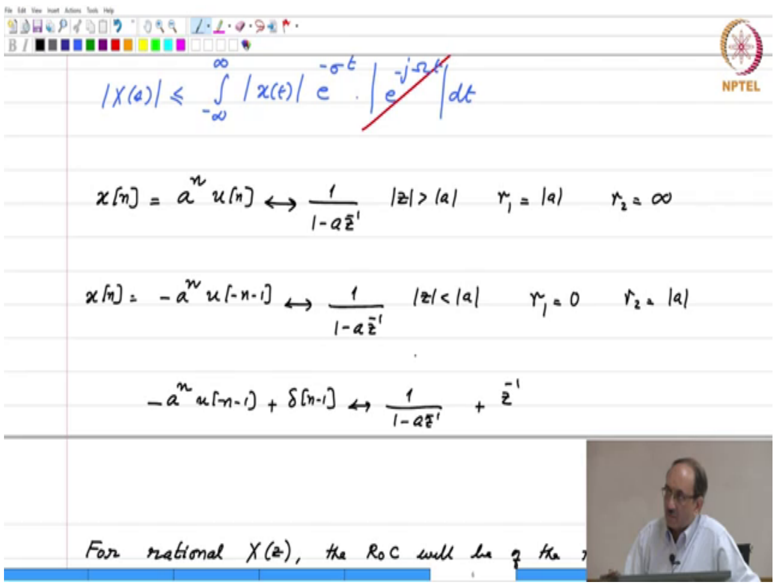

Yes, whether for example, if you look at this particular case, $-a^n u[-n-1]$, so here you had $|z| < |a|$. And in this case, $r_1 = 0$ was part of the region of convergence, that is $z = 0$ is part of the region of convergence. On the other hand, suppose I take this same sequence, $-a^n u[-n-1]$, so this is purely left sided and then I add $\delta[n-1]$. So, now this transform is $\dfrac{1}{1-az^{-1}} + z^{-1}$. Now, you are introducing a pole at $z = 0$. Now, in this case, the region of convergence will be $0 < |z| < |a|$. So, there are examples where 0 may be present, 0 may not be present.

What we will do next is, we look at properties of the Z-transform and we will also consider the DTFT as a special case of the Z-transform. We will see what the implication of the unit circle being part of the region of convergence means. We look at that. So, this is similar to the $j\omega$ axis being part of the region of convergence, and then we will develop properties of the Z-transform and for each property we will also give the property of the DTFT. When we give the property of the DTFT, we are assuming the region of convergence includes the unit circle. We will formally define what the DTFT is, and then when we discuss properties we will develop both these topics side by side. And later we will focus on some sequences that have DTFT, but not Z-transform.

So, this is no different from what is happening the CTFT case. You can always consider the CTFT as a special case of Laplace with s being replaced by $j\omega$. But, there are functions that have Fourier transform, but no Laplace transform. And one immediate example that you can think of is which you have already seen that has CTFT, but no Laplace. Yeah, give me a time domain function that has CTFT, but no Laplace. By Laplace, we mean bilateral.

This should not take long, this one of the simplest functions that you have seen any number of times in that course. What about Laplace, what about the bilateral Laplace transform of $x(t) = 1$? So, this can be written as $u(t) + u(-t)$, $u(t)$ will have Laplace $1/s$ with $Real\{s\} > 0$, $u(-t)$ will have Laplace $1/s$ with $Real\{s\} < 0$. So, together, these two cannot exist because there is no common overlap region. Therefore, here is an example of a function that has continuous time Fourier transform, but no Laplace.