Digital Signal Processing

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Lecture 02:
Introduction to Signals
Operations on the independent variable: affine transform

Keywords: independent variable, affine transform, time shifting, time scaling

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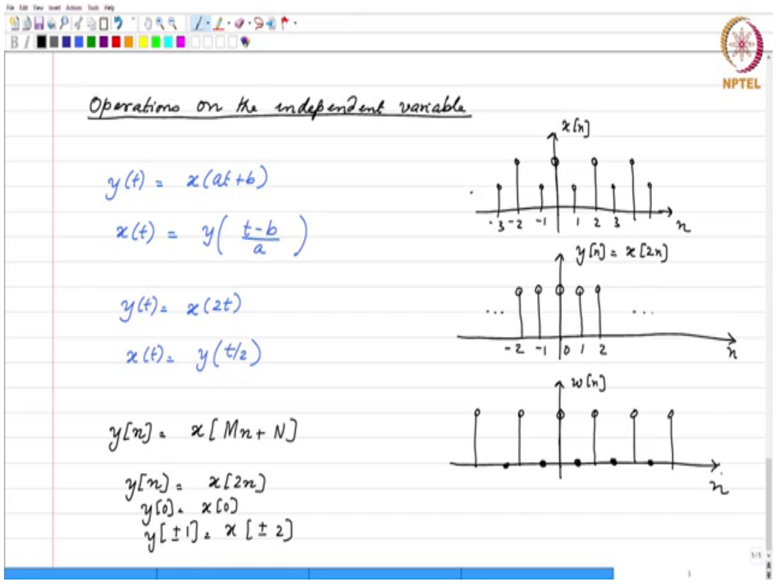
Operations on the independent variable

$y(t) = x(at+b)$

$x(t) = y\left(\frac{t-b}{a}\right)$

$y(t) = x(2t)$

$x(t) = y(t/2)$

$y[n] = x[Mn+N]$

$y[n] = x[2n]$

$y[0] = x[0]$

$y[\pm 1] = x[\pm 2]$

Let us now focus on the operations on the independent variable. The most common operation that you encounter when it comes to operations on the independent variable is what is called the affine transform. It consists of both time shifting as well as time scaling. So,

$$y(t) = x(at+b),$$

is called the affine transform of the independent variable where, you are not only scale but you also shift.

And there are other possibilities of transforming the independent variable other than the affine. There is what is called the Mellin transform and you can compute this by replacing t by e^t. And if you did this, you are non-linearly compressing the independent axis and then if you take the Laplace transform of the non-linearly modified signal, that will be the Mellin transform.

So, this is not the only thing kind of transform that exists, but as far as this course is concerned these are the typical kind of operations that we look at. And if $a > 1$, then this corresponds to time compression. And if $a < 1$, this would correspond to time expansion and if a were negative in addition to it either be in compression or expansion you have reflection.

Now, given $y(t)$ which is the compressed or expanded maybe in reflection is thrown in depending on the value of a. Suppose you want to get $x(t)$ from $y(t)$; $y(t)$ is the affine transformed signal and you want to get back your original signals, so this would correspond to what?

$$x(t) = y\left(\frac{t-b}{a}\right).$$

Because, wherever t is there if you replace t by $\frac{t-b}{a}$, you will get back $x(t)$.

So, if $y(t)$ is $x(2t)$, then $x(t) = y\left(\frac{t}{2}\right)$. So, given the compressed or expanded with a possible reflection and also shift, you can undo the operation of the affine transform and get back your original signal. The corresponding counterpart for this in the discrete-time case is

$$y[n] = x[Mn + N].$$

So, this is the discrete-time counterpart of the affine transform and working on a similar analogy if $y[n] = x[2n]$, that seems like the counterpart of $y(t) = x(2t)$.

So, in this context suppose I have something like this, so this is n and this is $x[n]$ and now I have $y[n]$ which is $x[2n]$. Therefore, $y(0) = x(0)$, $y[1] = x[2]$, $y[2] = x[4]$, $y(-1) = x(-2)$, and so on. So, you see the picture that emerges. This is what the time compressed signal is compressed by a factor of 2. Now, suppose I have $w[n]$ which has these values.

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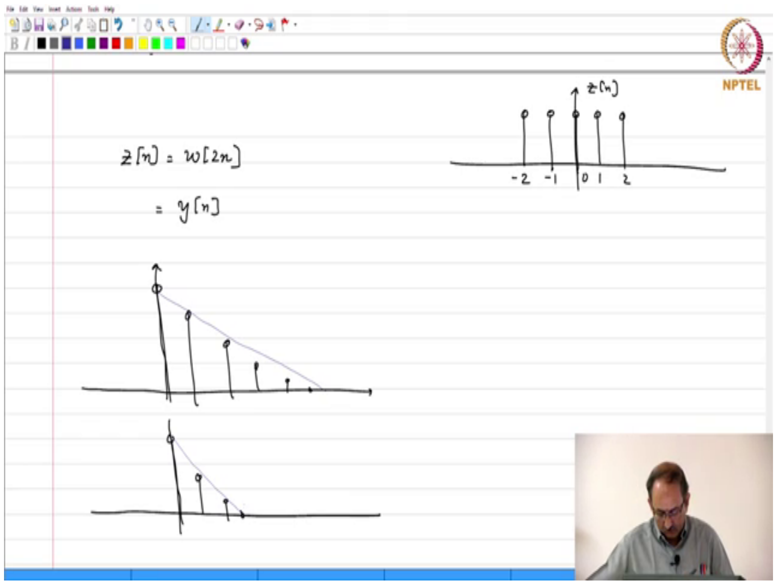

Now, let us look at $z[n] = w[2n]$. Again, what is happening here is all the even indices are picked up. And it is very easy to see that the picture that you get for set of n is this. And the immediate thing that strikes you is that $z[n] = y[n]$. So, the consequence of this is that suppose a given $z[n]$ which is the same as $y[n]$, you cannot tell whether the sequence that it came from was $x[n]$ or $w[n]$.

So, in sharp contrast with respect to the continuous-time case where given $y(t)$ as $x(2t)$, you can go back to x of t by expansion by a factor of 2 whereas here you cannot. The reason why this is happening is when you pick up all the even indices, you are simply discarding the odd indices. Once you have discarded the odd indices, there is no going back.

Therefore, although this appears similar to compression in the continuous-time case, there are important differences. And the reason why this appears similar to the continuous-time compression is suppose you had a waveform like this and then if you picked up all the even indices, so this has an envelope something like this. So, this seems to remind you that this is compression by a factor of 2. So, in that sense this seems similar to that.

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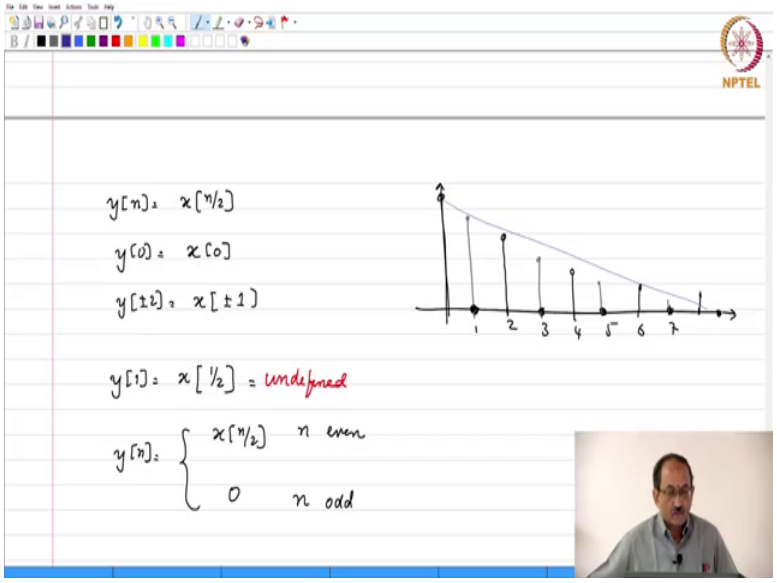

$y[n] = x[n/2]$

$y[0] = x[0]$

$y[\pm 2] = x[\pm 1]$

$y[1] = x[1/2]$ = undefined

$$y[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Now, let us carry this comparison further and then look at the other counterpart that is $y[n] = x[n/2]$. Immediately, you see that $y[0] = x[0]$, $y[\pm 2] = x[\pm 1]$ i.e., $y[2] = x[1]$, $y[-2] = x[-1]$ and so on. So, there are no issues here, but then the moment you talk about $y[1]$, this is $x[1/2]$ and this value is? Student: Undefined.

Undefined. Similarly all the odd indices are undefined, they are not 0. Given this definition, purely if we go by this, all the odd indices are undefined. Therefore, if you look at this particular example that we saw and for this sequence given here,

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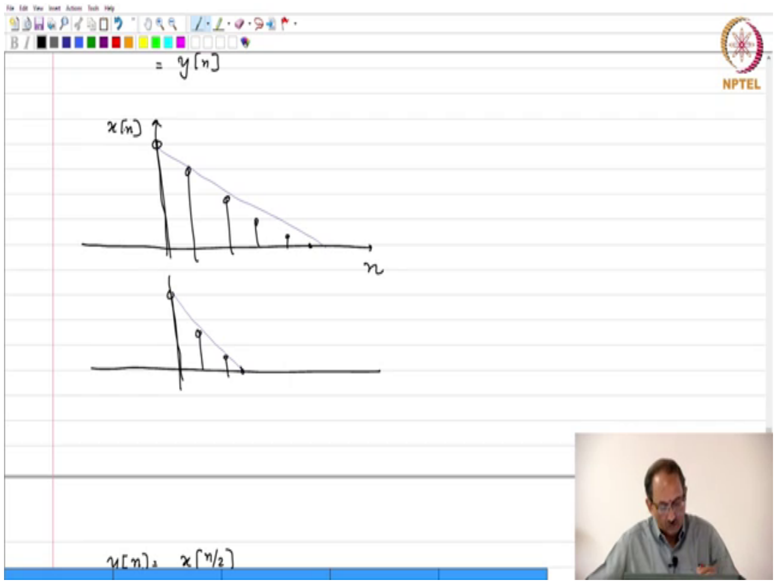

If you look at the $n/2$ sequence i.e., $y[n] = x[n/2]$, if you just focus on this, it does indeed look like an expanded version. But as it is, all the odd indices are undefined but you can go ahead and then define $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd}, \end{cases}$ in which case the picture now becomes like this. But the point is it is not quite the expansion by a factor of 2 that you thought has to be there. And later, we will see that more processing is needed to go from this picture to the picture where all these samples are filled in.

To have a picture that this is expanded by a factor of 2, similar to what was happening in the continuous-time case, more processing is needed. Right now, all you can do is you can only fill in 0s provided you define it like this. And then to go from the 0 filled sequence to a sequence that looks similar to an expanded version, more processing is needed. Therefore, right here you see when it comes to the affine transformation, there are important differences between continuous-time and discrete-time case.