Digital Signal Processing

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Lecture 18:
LTI Systems (3)- FIR and IIR systems

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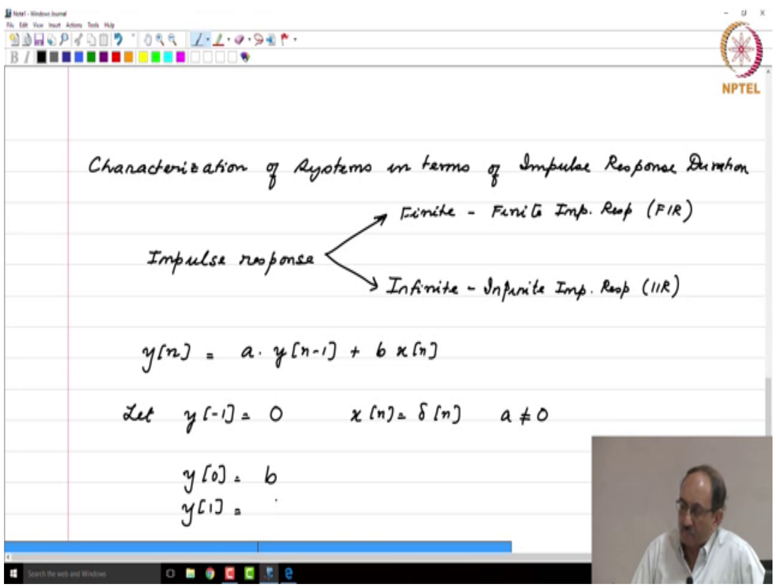

Now, let us further look at characterization of systems along these two lines. So, that is, we want to look at characterization of systems in terms of impulse response duration. So, the impulse response duration can either be finite or infinite. And if the impulse response duration is finite, this is called finite impulse response and this is denoted as FIR. If the impulse response is infinite, just paralleling what was given earlier this is denoted as Infinite Impulse Response and abbreviated as IIR. So, as far as discrete time systems are concerned, among the class that we are interested in, FIR and IIR systems are an important sub classification.

Now, let us look at this simple difference equation, $y[n] = a.y[n-1] + b.x[n]$ and then we will assume $y[-1] = 0$ and let $x[n] = \delta[n]$ and then to satisfy the mathematician we will assume $a \neq 0$. Unlike differential equations which are more difficult to solve, difference equations are easy to solve. All you need to do is start evaluating y for various indices.

For example: if you look at $y[0]$, this is $a.y[-1] + b.x[0]$ and $y[-1] = 0$ as per this assumption and $x[0]$ is for this given input is 1. Therefore, $y[0]$ is very simply b. Similarly you can find out what $y[1]$ is. So, you can recursively evaluate this difference equation and quickly see what the pattern is.

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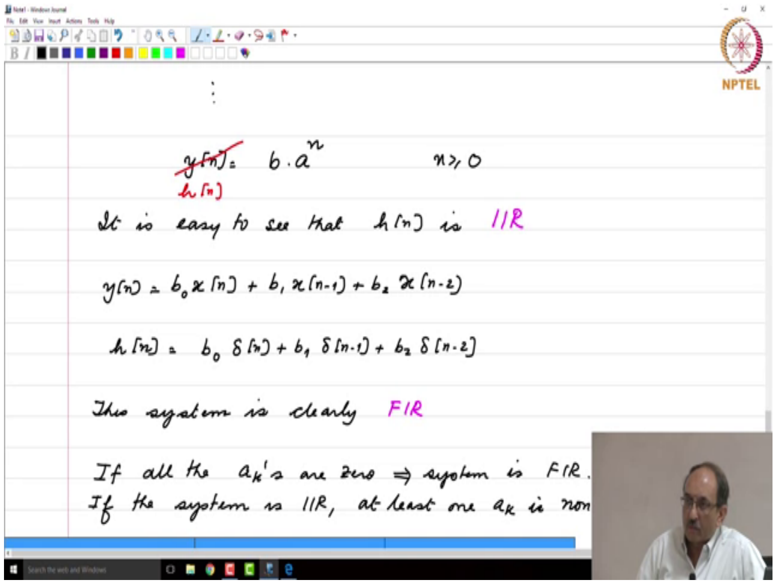

And, if you did this, you will be able to see that $y[n] = b.a^n,\ n \geq 0$. And, it is easy to see, this particular system, that is $y[n] = a.y[n-1] + b.x[n]$, so, clearly the input $x[n]$ is $\delta[n]$ which is the impulse therefore, $y[n]$ that we have obtained here is really not $y[n]$, but $h[n]$. Because, we always use the special notation h to denote impulse response and this is what it is. It is easy to see that, $h[n]$ is just, lets go back and look at the classification we have given in terms of impulse response duration is this IIR or FIR.

So, now whatever it is, is this IIR or FIR? That is the question. This is indeed IIR, because $b.a^n$ is of infinite duration. At no point does this become strictly 0. You should not look at it from the point of it decaying. For example, if $|a| < 1$, that it will decay quickly and beyond a certain point reach a level that is practically 0, that is not the kind of classification we are after. We are looking at it from a theoretical point of view. So, this is clearly an exponential that last forever, for $n \geq 0$ and hence this is IIR.

Now, the question to ask is by looking at the difference equation, can we make some general statements regarding the system being IIR or FIR? So, clearly the system is IIR in this particular case. Now let us look at this particular example, $y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2]$.

So, this is the input output relationship. And what is $h[n]$? All you need to do to get $h[n]$ is replace the input $x[n]$ by $\delta[n]$, right. $h[n]$ is the impulse response, $x[n]$ is the input, input has to be the impulse if you want $y[n]$ to be $h[n]$. So, $h[n]$ in this case happens to be $b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2]$, ok. And this system is clearly IIR or FIR? FIR. Because, the impulse response consists of three samples starting at $n = 0$, it is b_0; at $n = 1$, it is b_1; at $n = 2$, it is b_2. So, this is a system whose impulse response is of finite duration. So, this is an FIR system.

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Now, if you go back and look at this general equation, you can see that if all the a_k's are 0, then the system is FIR, very easily seen from this. That is, if all the a_k's are 0, this implies, system is FIR. Now, let us go back and look at this IIR example that we saw. We know that the system is IIR, based on the example that we just worked out. And we see that, if the system is IIR, at least one a_k is nonzero, alright. So, if the system is IIR, at least one a_k is nonzero.

So, now the question arises, if one a_k is non-zero, does it mean the system is IIR? Notice carefully, what statement we are making here. If the system is IIR, at least one a_k is nonzero. And the previous statement is just the contrapositive of the bottom statement; 'a implies b', the contrapositive is 'not b implies not a'. Therefore, at least one a_k is nonzero, the converse of that is all a_ks are 0. So, not b implies not a, not a is IIR becomes FIR. So, these two statements are, one is the contrapositive of the other. But what we want to ask is, if at least one a_k is nonzero, does it mean the system is IIR?